Simplified Beam-Column Joint Model for RC Moment Resisting Frame

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Simplified Beam-Column Joint Model for RC Moment Resisting Frame

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Abstract

During strong seismic events, inelastic shear deformation occurs in beam-column joints. Several researchers have provided different joint models to incorporate its inelastic properties in analytical modelling. However, these models require cumbersome calculations and tedious modelling techniques, which are time-consuming thus limit their practical application. To overcome these limitations, a new analytical model has been developed using a rotational spring and rigid links, which incorporates the inelastic shear deformation of the joint. The stiffness properties of the rotational spring element have been assigned in terms of a moment rotation curve developed from the joint shear stress-strain backbone curve. The inelastic joint rotation behaviour has been categorised in three stages viz. cracking, yielding and ultimate. The joint shear stress has been estimated using an analytical model based on the hybrid approach. The strain values corresponding to these damage stages have been evaluated based on the assessment of experimental data. Further, to account for the effect of adjoining beam and column on the joint rotation behavior, a geometry factor has been proposed to modify the stiffness properties of rotational spring. The response of the proposed analytical model has been validated at the component and structural levels. It has been observed that the proposed simplified beam-column joint model effectively emulates the inelastic behaviour of the joint in comparison to the existing models.

Keywords Beam-column joint, analytical model, inelastic behaviour, rotational spring elements
1. Introduction

Post-earthquake reconnaissance reports (EERI 1994 and EERI 1999 a, b) have shown that failure of beam-column joint resulted in the partial or total collapse of RC structures. Inadvertent damage to the beam-column joint alleviates the seismic performance of an RC structure, which may lead to undesirable brittle failure. To achieve the desired mode of hinge formation in a structure in order to dissipate the maximum amount of earthquake input energy, it is necessary to comprehend joints behavior and its impact on the overall performance of the structure. Many researchers (Filiatrault et al. 1998a-b; Calvi et al, 2002; Sharma et al. 2011; Park and Mosalam, 2013; and Ning et al., 2016) conducted experimental and analytical studies at both component and structural levels. Most of the joint failures have been attributed due to either excessive shear deformation of the joint panel or anchorage failure of longitudinal reinforcement (Paulay and Priestley, 1992; Shin and LaFave, 2004 and Celik and Ellingwood, 2008). Internal forces in structural members are commonly estimated from analytical study performed using computer-based programmes for design and assessment of RC frame structures. However, in the current design practice, generally, the deformation of the joint is neglected and considered as a rigid element (Pampanin et al., 2002; Celik and Ellingwood, 2008; Favvata et al., 2008; and Unal and Burak, 2013). If the beam-column joint modelled as a rigid element, the analytical model of the structure becomes stiff and yields an inappropriate response.

In the design of RC frame structures, strong column-weak beam design philosophy is recommended in the codes to ensure the formation of plastic hinges in beams rather in columns. In addition, beam-column joints are expected to behave elastically. However, experimental studies demonstrated that the joint panel undergoes large inelastic deformation even if the strong-column-weak-beam design philosophy is followed. Therefore, to obtain the realistic behavior of the structure from the analytical study, it is necessary to incorporate the inelastic properties of the joint instead of assuming it as a rigid element or eliminating its behavior.

Various national codes prescribe basic equations for calculating joint shear capacity (Parate and Kumar, 2019); however, few codes such as ASCE-41 and FEMA 356 provide joint rigidity factors to be considered in the modelling. Besides, researchers developed various analytical models based on different modelling approaches, like lumped plasticity approach (i.e. single spring approach), multi-spring approach, and finite element method. In the lumped plasticity approach, the inelastic deformation behavior of beam-column joints has been modelled using the inelastic rotational spring element (Giberson, 1969; Otani, 1974; Anderson and Townsend, 1977 and Raffaelle et al., 1992). Further, several researchers (Alath and Kunnath, 1995; Lowes and Altoontash, 2002; Pampanin et al, 2003; Shin and LaFave, 2004; Favatta et al, 2008; Sharma et al, 2012; Unal and Burak, 2013; Risi et al, 2014; Shayanfar et al, 2016) proposed multi-spring joint models to account the shear and bond-slip behavior separately. Finite element method based joint modelling (Eligehausen et al., 2006; Sagbas et al., 2011 and Omidi and Behnamfer, 2015) provide a comprehensive understanding of beam-column joint behavior and necessitate extensive computational modelling. These approaches show good agreement with the experimental results of joint, though it requires time-consuming computational effort.

Past studies (Kim and LaFave, 2007; Lima et al., 2012 and Parate and Kumar, 2016) shows that the joint shear strength is affected by various governing parameters viz., the geometry of beam and column, longitudinal and shear reinforcement, grade of concrete and column axial load. The main limitation of existing joint shear strength
The proposed model is based on the lumped plasticity approach. To simulate the inelastic response of the joint, a rotational spring element (to incorporate the relative rotation of the beam and column) with rigid links (to define size of the joint panel) has been proposed and collectively referred as the joint panel zone. The characteristic envelope curve of the shear stress-strain for joint has been developed from the earlier proposed model based on a hybrid approach (Parate and Kumar, 2019). The proposed analytical joint model has been validated at the component level with the exterior and interior joints for different materials (viz. concrete and rebar grade), geometrical configurations (viz., joint with different beam depth and width), shear reinforcement and axial loads. The efficacy of the proposed model has been assessed in comparison with other simplified beam-column joint models by Sharma et al. (2011), ASCE 41 (2013) and Unal and Burak (2013).

The proposed beam-column joint model has been validated at the structural level using an experimentally tested RC frame by Filiatrault et al. (1998a). The tested RC frames were designed and detailed for varying ductility level according to the prevailing Canadian code of practice. The analytical model of experimentally tested RC frames has been developed using the proposed beam-column joint model. The nonlinear analysis of the frame has been performed for the same ground motion. Further, the results have been compared with the experimental and analytical study performed by Filiatrault et al. (1998a-b). In present study, the analytical modelling has been carried out using SAP 2000 structural analysis programme. Overall the present study provides a simplified and reliable beam-column joint model which attempts to overcome the limitations of existing models.

2. Proposed beam-column joint model

Under gravity loads, the tensile forces in rebar and compressive forces in concrete of adjoining members act in the opposite direction and therefore no shear action develops in the joint. On contrary, during a seismic event, the direction of the forces developed in the rebar and concrete become same in the adjoining members and due to that push-pull effect the joint experiences shear deformation (Ghobarah and Biddah, 1999; Calvi et al. 2002 and Park S, 2010). In the present study, to capture the joint shear deformation and to define the finite size of the joint panel zone, a rotational spring along with rigid links has been proposed respectively to model the beam-column joint as shown in Fig. 2.1. Researchers, Celik and Ellingwood (2008) and Risi et al. (2014 and 2016) identified that due to shear deformation the joint panel zone undergoes different stages of failure namely cracking, yielding, and ultimate stage. The first stage represents the cracking of the core concrete in the direction of the diagonal compressive strut. In second stage, the longitudinal beam bars begin to yield at the joint face.
In the intervening stage, the joint reaches its ultimate strength and beyond that point, the shear strength of joint starts degrading as shown in Fig. 2.2. The proposed model consists of the rotational sping and its stiffness characteristics have been assigned in terms of shear stress-strain backbone curve. In the backbone curve four damage states, viz. initial cracking, flexural yielding, ultimate stress, and residual stress has been demarked based on experimental observations from Celik and Ellingwood (2008) and Risi et al. (2014) as shown in Fig. 2.2. Further, the shear stress at those corresponding stages has been determined based on experimental and analytical studies.
2.1 Joint shear stress

The formulation adopted for estimating the shear capacity of the joint at the aforementioned four damage stages viz., cracking, yielding, cracking and residual stages.

2.1.1 Cracking stage

Under cyclic loading initially, hairline cracks develop in the core concrete of the joint. Various researchers proposed equations to estimate joint shear stress corresponding to this initial cracking. However, Uzumeri et al. (1977) consider the influence of compressive strength of concrete and column axial load on the initial cracking of the joint effectively. Further, Risi et al. (2014) reported that the equation of Uzumeri et al. (1977) shows the lower mean error with the test results. Therefore, in the proposed model to estimate the joint shear stress corresponding to the initial cracking stage, the following equation proposed by Uzumeri et al. (1977) (i.e., Eq. 1) has been adopted.

$$
\tau_{jcr} = 0.29\sqrt{f_c} \sqrt{1 + 0.29 \frac{N_c}{A_j}}
$$

2.1.2 Yielding stage

The yielding stage differs for different failure modes i.e. beam flexural yielding joint shear failure (BFJS) mode and joint shear failure (JS) mode. In case of the BFJS failure mode, the yielding stage represents the stage at which the beam longitudinal bars begin to yield. Whereas in case of JS failure mode, yielding corresponds to the widening of diagonal cracks in core concrete of joint. It is difficult to identify the beginning of beam bar yielding as they are embedded within the joint and cannot be visualized. Risi et al. (2014) reported that, the yield strength of the joint can be considered as 0.85 times of the ultimate shear strength of the joint. Accordingly, in the present study, the joint shear strength at yielding stage is considered to be 0.85 times the ultimate shear strength, as given in Eq. (2).

$$
v_{yield} = 0.85v_{ultimate}
$$

2.1.3 Ultimate stage

As previously stated, the calculation of yield strength is dependent on the ultimate shear strength of the joint, and hence the accuracy of the joint model is governed by the ultimate shear strength prediction. Parate and Kumar (2019) highlighted the limitations of the various joint shear strength prediction models proposed by various researchers and national codes. Further, they proposed ultimate shear strength prediction equation based on hybrid approach (i.e. combination of strut-and-tie and empirical approaches) for RC beam-column joints. This equation takes into account the effects of many governing parameters and has been found to be effective in accurately calculating the joint shear strength using a wide database of experimental results. Hence, Eq. (3) has been used to estimate the value of ultimate shear stress on the backbone curve.
\[ v_{\text{ultimate}} = v_{\text{jh}} = (\chi \alpha \beta \lambda \kappa c^0.6 A_{\text{strut}} \cos \theta) + \phi A_{sj} f_{sj} \]  

where, \( \chi \) represents a factor for type of joint taken as 1.00 and 1.20 for exterior (├) and interior (┼) joint respectively; factors ‘\( \alpha \)’ and ‘\( \beta \)’ indicates the influence of amount of beam and column longitudinal reinforcement respectively; the factor ‘\( \lambda \)’ indicate the influence of presence of transverse beam and slab taken equal to 1.20 for one transverse beam and 1.00 for no transverse beam; the factor ‘\( \kappa \)’ is to consider the effect of wide beam/column, taken equal to the ratio of width of beam to column (\( b_b/b_c \)).

2.1.4 Residual stage

It is considered that thirty percent of the ultimate strength of the joint as residual strength and determined using Eq. (4),

\[ v_{\text{residual}} = 0.30 v_{\text{ultimate}} \]  

2.2 Joint shear strain

Various researchers (Meinheit and Jirsa, 1981; Watanabe et al., 1988; Fuji and Morita, 1991; Kaku and Asakusa, 1991; Noguchi and Kashiwazaki, 1992; Raffaelle and Wight, 1995; Chen and Chen, 1999; Clude et al. 2000; Pantelides et al. 2002; Teng and Zhou, 2003; Hwang et al, 2005 and Burak and Wight, 2005) had evaluated the shear strain corresponding to the various damage stages of the joint from the experimental results. These strain values vary due to changes in various geometrical and material properties. Therefore, a comprehensive study has been performed for a set of strain values provided by the aforementioned researcher. The experiments were performed with or without considering the column axial load. In actual structures, joints are always subjected to column axial loads. Hence, the strain values of those specimens subjected to column axial load only have been considered in the present study. The strain data has been segregated for three damage states (i.e., cracking, yielding and ultimate) for interior and exterior joint as shown in Fig. 2.3.
Fig. 2.3: Joint shear strain at three different stages; (a) cracking, (b) yielding, (c) ultimate for interior joint; and (d) cracking, (e) yielding, (f) ultimate for exterior joints

The limiting strain value corresponding to the damage state has been evaluated by calculating the mean strain value of the segregated strain values. The optimum number of strain values have been obtained by neglecting outliers to get the minimum coefficient of variation. The proposed limiting shear strain ($\gamma$) values for interior and exterior joints correspond to four damage states have been tabulated in Table 2.1.

Table 2.1 Limiting ranges of joint shear deformations for different stages of backbone curve

<table>
<thead>
<tr>
<th>Limiting stages</th>
<th>Joint shear deformation (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Interior Joint</td>
</tr>
<tr>
<td>(1) Cracking</td>
<td>0.001</td>
</tr>
<tr>
<td>(2) Yielding</td>
<td>0.0051</td>
</tr>
<tr>
<td>(3) Ultimate</td>
<td>0.0131</td>
</tr>
<tr>
<td>(4) Residual</td>
<td>0.300</td>
</tr>
</tbody>
</table>
2.3 Conversion of joint shear stress-strain into joint moment-rotation

To convert the shear stress-strain curve of joint into moment-rotation of RC beam-column joint, several formulations are available in the literature. Various researchers (viz. Celik and Ellingwood 2008; Risi et al. 2014; Omidi and Behnamfar, 2015; Unal and Burak 2013, and Shin and Lafave, 2004) have used different conversion factors. The conversion factors utilized by the aforementioned researchers are presented in Table 2.2. The researchers proposed these conversion models based on few experimental results. While performing the validation study it has been found that the formulation proposed by Celik and Ellingwood (2008) is more appropriate amongst all. Celik and Ellingwood (2008) account for the geometry of the beam-column joint however it neglects the effect of the geometry of adjoining beams and columns on the rotation behavior of joint. Therefore, a new factor ‘’ψ’’ has been introduced in the framework of Celik and Ellingwood (2008) based on the ratio of width and depth of adjoining beams and columns.

Table 2.2 Models for conversion from joint shear stress-strain into joint moment-rotation

<table>
<thead>
<tr>
<th>Researchers</th>
<th>Proposed model for Moment ($M_j$)</th>
<th>Rotation ($\theta_j$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Celik and Ellingwood (2008)</td>
<td>$M_j = \frac{\tau_j A_j}{1 - \frac{b_j}{L_b} \frac{1}{L_c}}$</td>
<td>$\theta_j = \gamma_j$</td>
</tr>
<tr>
<td>Risi et al. (2014)</td>
<td>$M_j = \frac{\tau_j A_j}{1 - \frac{h_c}{2L_b} \frac{1}{2L_c}}$</td>
<td>$\theta_j = \gamma_j$</td>
</tr>
<tr>
<td>Omidi and Behnamfar (2015)</td>
<td>$M_j = X_v A_j$</td>
<td>$\theta_j = X_y A_j$</td>
</tr>
<tr>
<td>Unal and Burak (2013)</td>
<td>$M_j = \tau_j A_j j_d$</td>
<td>$\theta_j = \gamma_j$</td>
</tr>
<tr>
<td>Shin and Lafave (2004)</td>
<td>$M_j = \tau_j A_j j_d$</td>
<td>$\theta_j = \gamma_j$</td>
</tr>
</tbody>
</table>

The following revised formulation (i.e. Eq. 5 and Eq. 6) for converting the stress-strain behavior of joint into moment rotation behavior has been used.

$$M_j = \psi \cdot \tau_j A_j \frac{\eta}{\lambda}$$  \hspace{1cm} (5)

where, $\psi = \left(\frac{b_b}{b_c}\right) \left(\frac{h_b}{h_c}\right)$, $\eta = 1 - \frac{h_j}{L_c} \frac{b_j}{L_b} - \frac{1}{1 - \frac{b_j}{L_b} \frac{1}{L_c}}$, and $\lambda = \frac{1 - b_j}{L_b} \frac{1}{j_d} - \frac{1}{L_c}$.
The rotation \((\theta_j)\) of two rigid links representing the deformation of the finite size of joint panel zone, has been considered equal to the joint shear strain \((\gamma_j)\) multiplied by adjoining beam-column geometry factor ‘\(\psi\)’ as shown in Eq. (6).

\[
\theta_j = \psi \cdot \gamma_j
\]

3. Hysteresis modelling

A link element from the library of SAP computer program has been utilized to examine the inelastic behavior of beam-column joints (CSI Analysis Reference Manual for SAP2000). The nonlinear cyclic behavior of joints has been modelled using pivot and Takeda hysteresis models. The pivot model offers additional parameters to regulate the hysteretic loop among the existing models for RC structural sections (such as kinematic, Takeda, and pivot). Amongst them, the pivot model which is based on the parameters that determine the unloading and reverse loading trends is well suited to describe the response of RC beam-column joints under cyclic loading. In the pivot model, the unloading and reverse loading tend to be directed toward specific points, called pivot points. These pivot points control the effect of strength degradation through the parameter ‘\(\alpha\)’ and pinching of hysteresis loops through the parameter ‘\(\beta\)’. The parameter ‘\(\alpha\)’ is a primary pivot point that controls the unloading stiffness of the member and the parameter ‘\(\beta\)’ is a pinching pivot point that controls the pinching behavior of the element.

The pivot model is first introduced by Dowell et al. in 1998; however, its applicability was limited only to the circular columns of bridges. The controlling points of the hysteresis loop had been derived by considering only two parameters i.e. column axial load (ALR) and longitudinal reinforcement \((p_t)\). Sharma et al. (2013) extended the applicability of Dowell’s model for RC columns of rectangular shape and beam-column joints. Sharma et al. (2013) have proposed some formulations empirically to calculate the two pivot parameters ‘\(\alpha\)’ and ‘\(\beta\)’ by considering three parameters i.e. column axial load \((ALR)\), longitudinal reinforcement \((p_t)\), and shear reinforcement \((p_{sh})\), as given in following Eq. (7) and Eq. (8).

\[
\alpha = 0.170k_\alpha + 0.415
\]

\[
\beta = 0.485k_\beta + 0.115
\]

where, \(k_\alpha = \frac{\rho_t}{ALR}\) and \(k_\beta = (ALR)^{0.25} \times (\rho_{sh})^{0.20}\)

The present study aims to reduce joint modelling computation effort while maintaining an acceptable level of accuracy in the results. However, implementation of the pivot hysteresis model will not fulfil this objective as it demands to compute two pivot parameters ‘\(\alpha\)’ and ‘\(\beta\)’ for each joint. On the other hand, the Takeda hysteresis model does not require additional parameters and is appropriate for reinforced concrete (Takeda et al. 1970). Therefore, Takeda hysteresis model has been used for simulating the degrading hysteretic behavior of beam-column joint in nonlinear analysis. In the validation part of the proposed joint model at the component level, results have been reported using both pivot and Takeda hysteresis models. This comparative assessment shows that the pivot model gives better insight into the cyclic behavior of the joint compared to Takeda model. However, overall the Takeda
model also gives sufficient accuracy and can be considered for the joint modelling. Therefore in the case of the structural level validation part Takeda hysteresis model has been adopted.

4. Validation of proposed joint model at the component level

The joint inelastic behavior prediction of the proposed model has been compared initially with experimental results and then with other analytical joint models.

4.1 Validation with experimental results

The proposed analytical model has been validated with six joint specimens that were designed with different material and geometric configurations. Three exterior joint specimens of Wong and Kuang, (2008) (viz. 'BS-L-300', 'BS-L-450', and 'BS-L-600'), two exterior joint specimens of Behnam et al., (2017) (viz. 'S1-BC1' and 'S3-BC2') and one interior joint specimen of Fan et al. (2014) (viz. 'JM2-14') have been used for the validation. The details of specimen is presented in Table 4.1.

Table 4.1 Details of six joint specimens considered for analytical verification

<table>
<thead>
<tr>
<th>Sr. No.</th>
<th>Researchers</th>
<th>Specimen</th>
<th>Joint Type</th>
<th>Beam width ($b_b$)</th>
<th>Beam depth ($h_b$)</th>
<th>Column width ($b_c$)</th>
<th>Column depth ($h_c$)</th>
<th>Concrete strength ($f_c$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Wong and Kuang (2008)</td>
<td>BS-L-300</td>
<td>Exterior</td>
<td>260</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>38.80</td>
</tr>
<tr>
<td>2.</td>
<td>Wong and Kuang (2008)</td>
<td>BS-L-450</td>
<td>Exterior</td>
<td>260</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>38.80</td>
</tr>
<tr>
<td>4.</td>
<td>Behnam et al. (2017)</td>
<td>S1-BC1</td>
<td>Exterior</td>
<td>300</td>
<td>300</td>
<td>300</td>
<td>360</td>
<td>36.10</td>
</tr>
<tr>
<td>5.</td>
<td>Behnam et al. (2017)</td>
<td>S3-BC2</td>
<td>Exterior</td>
<td>600</td>
<td>300</td>
<td>300</td>
<td>360</td>
<td>36.10</td>
</tr>
</tbody>
</table>

4.2 Comparison with other joint models

Researchers Sharma et al. (2011) and Unal and Burak (2013) have proposed joint shear strength prediction formulation along with joint rotation behavior models similar to the proposed rotational spring-based model. Therefore in the present study, models proposed by Sharma et al. (2011) and Unal and Burak (2013) have been considered for comparison. In addition, the joint shear strength prediction model proposed by ASCE 41 (2013)
The ultimate shear strength prediction models proposed by these researchers and ASCE 41 (2013) guideline have been discussed along with formulations.

### 4.2.1 Model of Sharma et al. (2011)

Sharma et al. (2011) used the principal tensile stress as failure criteria for the development of analytical model for the exterior type of beam-column joints as shown in Eq. (9). In this model only concrete compressive strength \( f_c \) and concrete tensile strength \( p_t \), width of beam \( b_b \) and column \( b_c \), and column axial stress \( N_c/A_c \) have been considered. The longitudinal reinforcement from the beam and column, as well as the shear reinforcement inside the joint, are not taken into account in the model.

\[
V_{jh} = \left( \frac{\sigma - \sigma_a}{\alpha} \right) b_c h_c
\]  
(9)

The coefficient \( \alpha \) is the aspect ratio of joint \( h_b/h_c \) and \( \sigma_a \) is vertical joint shear stress \( (N_c/A_c) \) due to column axial load \( (N_c) \). The axial stress \( \sigma \) is given by the following equation,

\[
\sigma = 2\sigma_a + \alpha^2 p_t + \alpha \sqrt{\alpha^2 p_t^2 + 4 p_t (\sigma_a + p_t)}
\]

and the principal tensile stress of concrete \( (p_t) \) is,

\[
p_t = \frac{\sigma}{2} \frac{\sigma}{2} \left[ 1 + \frac{4(\sigma - \sigma_a)^2}{\alpha^2 \sigma^2} \right]
\]

### 4.2.2 Model of Burak and Unal (2013)

Unal and Burak (2013) developed a parametric Eq. (10) based on the correlation of various parameters with the shear stress of the joint \( (V_j) \), with an emphasis on joint geometry. The model considers the effect of concrete strength \( (f_c) \), beam width \( (b_b) \), column width \( (b_c) \), beam \( (h_b) \), column depth \( (h_c) \), column axial stress \( (N_c/A_c) \), joint eccentricity \( (e) \) and joint volumetric ratio for one layer of transverse reinforcement \( (\rho_j) \). Longitudinal reinforcement of beams and columns is not taken into account.

\[
V_j (MPa) = JT \left( f_c, f_y \right) b_j \rho_j \left[ 1 + \frac{1}{1 + e/b_c} \left( b_b \left( 1 + \frac{N_c}{A_c f_c} \right) \right) \right] WBCISI
\]  
(10)

Where, JT is the joint types taken equal to 1.00 for exterior joint and CI is column index depending on the column aspect ratio taken equal to \( \sqrt{b_c/h_c} \) when \( b_c/h_c < 1.00 \) and 1.00 for other cases. SI is slab index taken equal to 1.00 when slab is not present. When wide beam is present in the loading direction the effect is considered equal to the expression as \( WB = \left( 1 - \frac{h_b}{b_b} \frac{b_j}{b_b} \right) \) otherwise taken equal to 1.00.
4.2.3 Model of ASCE 41 (2013)

ASCE-41 (2013) recommends an Eq. (11) for the prediction of joint shear strength that depends on the type of joint and transverse reinforcement ratio. The joint shear strength predicting equation is as,

\[ V_j = 0.083 \lambda \gamma \sqrt{f_c'} A_j \quad \text{(MPa units)} \]  

(11)

where, \( \lambda = 0.75 \) for lightweight aggregates concrete and 1.0 for normal-weight aggregate concrete; \( A_j \) is the effective horizontal joint area and the constant \( \gamma \) depends on the joint transverse reinforcement and type of joint.

5. Analysis results at component level

The test specimens have been modelled using SAP 2000 software along with the proposed and considered joint models, and analysed for the identical loading and boundary conditions. Further, the experimentally obtained response of joint specimens under cyclic loading has been compared with the analytical results of proposed and considered models. For each specimen, the predictions from various models have been compared in terms of the load-deformation hysteresis plot, as well as the ultimate shear strength (Table 5.1).

<table>
<thead>
<tr>
<th>Sr. No</th>
<th>Name of researcher</th>
<th>Joint details</th>
<th>Joint shear strength (Expt.) (kN)</th>
<th>Proposed model</th>
<th>% Error</th>
<th>Sharma et al. (2011) % Error</th>
<th>ASCE 41-13 % Error</th>
<th>Unal &amp; Burak (2013) % Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Wong and Kuang</td>
<td>BS-L-300</td>
<td>505.00</td>
<td>1.19</td>
<td>18.54</td>
<td>1.44</td>
<td>0.96</td>
<td>1.01</td>
</tr>
<tr>
<td>2.</td>
<td>Wong and Kuang</td>
<td>BS-L-450</td>
<td>341.42</td>
<td>1.16</td>
<td>15.74</td>
<td>0.90</td>
<td>-10.02</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>(2008)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Wong and Kuang</td>
<td>BS-L-600</td>
<td>283.90</td>
<td>1.26</td>
<td>26.18</td>
<td>0.60</td>
<td>-40.30</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>(2008)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>Behnam et al.</td>
<td>S1-BC1</td>
<td>483.00</td>
<td>1.13</td>
<td>12.90</td>
<td>1.21</td>
<td>20.75</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>(2017)</td>
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<td>Behnam et al.</td>
<td>S3-BC2</td>
<td>1034.00</td>
<td>1.12</td>
<td>12.34</td>
<td>2.54</td>
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<td>6.</td>
<td>Fan et al.</td>
<td>JM2-14</td>
<td>434.67</td>
<td>1.22</td>
<td>21.56</td>
<td>1.06</td>
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5.1 Test specimen of Wong and Kuang (2008)

Three full-scale specimens, designated as BS-L-300, BS-L-450, and BS-L-600, were used in this investigation to validate the efficacy of the proposed joint model in case of the different beam to column depth ratios (i.e. 1.0, 1.5 and 2.0) shown in Fig. 5.1. In considered three joint specimens the material properties and reinforcement are identical (Table 5.1). The experimental results of three specimens have been compared with the corresponding applied force versus deformation response curves of the analytical models developed using the proposed joint model as shown in Fig. 5.2. Further, Table 5.1 compares the joint shear strength of the proposed model to the other three considered models and the experimental results.

![Fig. 5.1 Details of joint specimen tested by Wong and Kuang (2008) for the specimens (a) BS-L-300, (b) BS-L-450, and (c) BS-L-600](image)

Based on the experimental results authors reported that with the increase in beam to column depth ratio, the joint shear strength decreases. However, the models proposed by Sharma et al. (2011), ASCE 41 (2013) and Unal and Burak (2013) do not consider the effect of beam for estimation of joint shear strength. ASCE 41 (2013) and Unal and Burak (2013) models estimate joint shear strength close to the experimental results only for joint specimen BS-L-300 with an equal depth ratio ($h_b/h_c=1.00$). In the case of BS-L-450 ($h_b/h_c=1.50$) and BS-L-600 ($h_b/h_c=2.00$) joint specimens, considered three models overestimate joint shear strength compared to experimental results. On the other hand, the prediction of the proposed model has been found to be more rational than the other three models for all three joint specimens. As shown in Fig. 5.2, joint specimens modelled with full rigidity have higher stiffness than experimental results. Hysteresis response of the proposed analytical model shows good agreement with the experimental results of all three joint specimens.
Fig. 5.2 Validation of proposed model with the test specimens of Wong and Kuang (2008) for (a) BS-L-300, (b) BS-L-450, and (c) BS-L-600.
5.2 Test specimen of Behnam et al. (2017)

A validation study for four specimens with beam to column width ratios 1.0, 1.5, 2.0, and 2.5 has been performed to assess the efficacy of the proposed model in predicting the joint rotation behavior of the beam-column joint for the different width ratios. As illustrated in Fig. 5.3, two joint specimens, 'S1-BC1' and 'S3-BC2,' with width ratios of 1.0 and 2.0, have been considered in this study.

![Fig. 5.3 Details of joint specimen tested by Behnam et al. (2017)](image)

(a) joint specimen with different beam widths; (b) typical column section; (c) beam section for S1-BC1 specimen; and (d) beam section for S3-BC2 specimen.

The joint shear strength prediction of the proposed model found to be more close to experimental results with fewer percentage errors than existing models for the specimen ‘S1-BC1’ with equal width of beam and column. The models of ASCE 41 (2013) and Unal and Burak (2013) predict higher joint shear strength than experimental results, but the model of Sharma et al. (2011) predicts moderately higher joint shear strength than the proposed model. All three considered joint models anticipate much higher joint shear strength for the joint specimen ‘S3-BC2’, which has a beam width twice the column width. The proposed model prediction, on the other hand, show good agreement with the experimental results and is relatively more accurate than the other three models as shown in Fig. 5.4.
5.3 Test specimen of Fan et al. (2014)

Fan et al. (2014) investigated fifteen full-scale internal beam-column junctions that were built according to the ACI code. One typical interior joint specimen, 'JM2-14' has been selected from the tested specimens and details of the specimen has been shown in Fig. 5.5.
Fig. 5.5 Details of joint specimen tested by Fan et al. (2014) for the specimen JM2-14 (a) joint specimen; (b) beam section; and (c) column section

Fig. 5.6 Validation of proposed model with the test specimens JM2-14 of Fan et al. (2014)

Experimental results of the test specimen have been compared with the analytical results of the proposed model along with three selected models. In this case (i.e. interior joint) also the proposed joint model wisely predicts the joint behavior than that of the other three models as shown in Fig 5.6. Among three models, Sharama et al. (2011) show the least percentage difference in joint shear strength, whereas, ASCE 41 (2013) and Unal and Burak (2013) models show a relatively higher difference as reported in Table 5.1.
6. Validation of proposed joint model at the structural level

The experimental investigation by Filiatrault et al. (1998a) has been considered to determine the effectiveness of the proposed joint model in achieving the realistic behavior of the structure in the software model. The experiment was conducted on two half-scale RC moment resistant frames with varied ductility factors, viz (a) nominal ductility (R = 2) and (b) ductile structure (R = 4) and were designed according to Canadian code. Both are two-storey two bays frames of 5 m in width and 3 m in height. In addition, Filiatrault et al. (1998b) conducted an analytical study to evaluate the influence of inelastic deformations of beam-column joints on the overall performance of RC frames previously tested by Filiatrault et al (1998a). Filiatrault et al. (1998b) used the computer program RUAUMOKO to perform the nonlinear time history analysis in the analytical investigation. The experimental results were compared to the analytical responses of two frames obtained from nonlinear time history analysis.

The same two RC frames have been analysed in this study using nonlinear time history analysis using SAP 2000 software. The software-based models of the two frames have been created using actual material properties, member properties, and applied loads identical to Filiatrault et al (1998a). The fibre hinge model, as shown in Fig. 6.1, has been used to consider the nonlinear behavior of beam and column members.

![Fig. 6.1 Typical modeling details for RC frame](image)

The inelastic properties of the beam-column joint have been modelled using the proposed analytical model, which is similar to the analytical study presented by Filiatrault et al. (1998b). The response of frame with the proposed analytical joint model has been compared with the response obtained from the experimental and analytical study of Filiatrault et al. (1998a-b). Moreover, commonly adopted two different modelling approaches viz. centerline model and rigid joint model have been considered for comparative assessment of the behavior of the frames under monotonic loading. For the two RC frames with different ductility (i.e. R=2 and R=4), a nonlinear static pushover
analysis has been performed. The capacity curve prediction with the proposed analytical joint model differs from the prediction of Filiatrault et al. (1998b) in the post-yield region for both the frames (Fig. 6.2 a-b).

**Fig. 6.2** Comparison of capacity curves for both RC frames with, a) nominal ductility and b) ductile frame
Fig. 6.3 Top storey acceleration and relative displacement time histories for RC frames with (a) nominal ductility \((R = 2)\) and (b) ductile frame \((R = 4)\)
The rigid joint assumption predicts a stiffer structure with increased strength and ductility (Fig. 6.2 a-b). The centre line model (i.e. the analytical model without joint consideration) predicts similar behavior with lower stiffness and strength. Filiatrault et al. (1998b) modelled the behavior of the joint with a spring element at the end of the structural element. The stiffness parameters of the corresponding springs have been iterated in such a way that the experimental and analytical results match. Furthermore, in the case of a nominal ductility frame, the joint behavior is assumed to be brittle with very low post-failure stiffness, and the yield/failure moment is calculated using trial analysis. In the case of a ductile frame, the analytical results of the elastic spring model for joint proposed by Filiatrault et al. (1998b) show good agreement with the experimental results but have no rational basis. The proposed joint model, on the other hand, takes into account the rotation behavior of the joint at cracking, yielding, and ultimate joint shear strength level thus, resulting in a different capacity curve prediction than Filiatrault et al. (1998b) analytical model. Other results, such as nonlinear top storey acceleration and relative displacement for intensity level 2, show good agreement for both nominal ductility and ductile frame (refer Fig. 6.3).

7. Conclusions

The present study deals with the development of an analytical model that can capture the inelastic behavior of RC beam-column joints at the component and structural level. The analytical model has been defined by assigning a rotational spring at the connection of beam and column element along with the rigid links that represent the finite size of the joint panel zone. The proposed analytical model for joint has been described by a quadrilinear moment–rotation spring characterized separately in three stages i.e. cracking, yielding, and ultimate. The joint shear strength at the ultimate stage has been evaluated based on the hybrid approach which overcomes the limitations of existing models and also validated for the large experimental database. Further, the strain values at these three stages have been decided based on the assessment of experimental observations. To convert the estimated joint stress-strain behavior of the joint into moment rotation behavior the existing framework has been modified with the proposed beam to column geometry factor. The efficacy of the proposed joint model has been verified by comparing the analytical response with experimental results for various joint specimens with different material and geometric configurations. Furthermore, the nonlinear response of RC frame structure modelled with the proposed analytical joint model has been investigated. For all considered cases proposed analytical model shows good agreement with experimental results. Overall the proposed simplified beam-column joint model seems to be rational in comparison to other similar joint models.
References


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**Competing Interests**

The authors have no relevant financial or non-financial interests to disclose.

**Author Contributions**

All authors contributed to the study conception and design. Material preparation, data collection and analysis were performed by Dr. Kanak Parate, Dr. Onkar Kumbhar and Dr. Ratnesh Kumar. The first draft of the manuscript was written by Dr. Kanak Parate and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.