A new approach for the load calculation of the most loaded rolling element for the rolling bearing with internal radial clearance - The case study

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Abstract: In this paper is presented a case study which has the goal to show the benefits of the application of a new approach for the calculation of load of the most loaded rolling element at the rolling bearing with the internal radial clearance. The calculation is based on the so-called load factors. By multiplication load factors with the value of the external radial load, the load which is transferred by the most loaded rolling element of the bearing is obtained. The case study is made for two types of bearings, the ball, and roller bearing. Obtained results were compared with the results obtained based on the calculation using some of the most commonly used methods so far. The analysis showed greater precision of the considered model with the same or much simpler application. For this reason, the proposed model is considered very suitable for practical application.

Keywords: rolling bearing, most loaded rolling element, load factor, internal load distribution, rolling elements, radial clearance, bearing deflection.

1. INTRODUCTION

A load calculation which is transferred by the most loaded rolling element has occupied a special place since the beginning of the use of the rolling bearings in machines. This is one very important characteristics of the roller bearing, especially for the calculation of the static load capacity, the rating life, but also the overall load distribution on the rolling elements inside the bearing. However, due to the complexity of the bearing construction, way of the functioning, and many other influential factors, this problem has not yet been fully resolved. So far, the Stribeck's method has been most commonly used to calculate this load [1].

The calculation based on the Stribeck numbers is the most commonly used method for the calculation of the load which transfers the most loaded rolling element. *Stribeck's* terms are still used today to calculate the static load capacity of a rolling bearing [2, 3]. However, Stribeck's method has one essential drawback, it does not take into account the influence of the size of the internal clearance on the load distribution within the bearing.

In this paper is shown a new mathematical model for the calculation of the load which is transferred by the most loaded rolling element of the rolling bearing with internal radial clearance. A detailed elaboration of the model is presented in [4], and the model is purposed for radially loaded rolling bearing, with balls or rollers. Based on the papers of Mitrovic et al. [5, 6], the model is using the so-called load factors, which show which part of the external load is transferred by the most loaded rolling element of the bearing.

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Nomenclature

K	–	effective coefficient of stiffness due to <i>Hertz's</i> contact effect
z	–	total number of rolling elements
z_s	–	the maximum number of rolling elements that can be found in the loaded zone
t_q	–	coefficient of the boundary bearing deflection
s_q	–	coefficient of the boundary radial load
k_{max}	–	the load factor of the most loaded rolling element
k_{maxq}	–	the load factor of the most loaded rolling element in the boundary position of support
q	–	number of active rolling elements
Q	–	external radial load of bearing
Q_q	–	boundary external radial load of bearing
e	–	internal radial clearance
F_i	–	load of the <i>i-th</i> rolling element
F_{max}	–	load of the most loaded rolling element
S	–	the Stribeck number

Indexes

i	–	index of the rolling element
e	–	boundary case of support on an even number of rolling elements
o	–	boundary case of support on an odd number of rolling elements

By simple multiplication of the load factor with the value of the external radial load, the load which is transferred by the most loaded rolling element of the bearing is obtained:

$$F_{max} = k_{max} \cdot Q \quad (1)$$

where is: F_{max} – the load which is transferred by the most loaded rolling element of the bearing, k_{max} – the load factor, Q – the total external radial load.

The values of the load factor of the most loaded rolling element (k_{maxq}) which were derived in literature [4] correspond to the boundary values of the bearing deflection and the boundary external radial loads which are defined in the papers [7, 8]. Those values are shown in *Tables A1. and A2. in Appendix A.*

If the right side of the equation (1) is multiplied and divided with the total number of the rolling elements of the bearing and the product $k_{max} \cdot z$ is marked with the S , it can be obtained:

$$F_{max} = k_{max} \cdot z \cdot \frac{Q}{z} = S \cdot \frac{Q}{z} \quad (2)$$

The factor S shows how many times the greater load is transferred by the most load rolling element relative to the case when the load would be even, ie. when every ball would transfer the load of the size Q/z . In the literature, this factor is often called the *factor of the non-uniformity load distribution* [9].

Stribeck determined that with the increase of the total number of the ball in the bearing with zero clearance the factor of the non-uniformity load distribution asymptotically strive to constant $S = 4.37$ [1]. This number is called the Stribeck number and in the literature is taken like relevant value for the calculation of the load of the most loaded ball at the bearings with zero clearance, e.i. at the calculation of the static load rating of the bearing. [10]. *Palmgren* later proposed that in the bearings with the rollers and zero-clearance Stribeck number amounts $S = 4.08$ [11]. For the bearings with radial clearance greater than zero, the impact of the clearance on the load distribution is approximated by increasing value of the Stribeck number on the value $S = 5$, independently on the size of the clearance. This applies to every type of ball and roller bearing with the internal radial clearance.

However, this has the justification only for orientational calculations because this only makes a difference whether or not the clearance is present in the bearing, and does not take into account the influence of the clearance size or even the type of bearing on the internal load distribution.

The model which is proposed in this paper take into account the impact of the clearance and the type of the bearings on the internal load distribution. The impact of the clearance is taken through the number of active rolling elements in the bearing. By its simplicity, it can be compared to the Stribeck method. The proposed model, like the Stribeck one, is based on simple mathematical operations, does not require the usage of the computer and it can be very useable for fast calculations and work in the field.

The aim of the study presented in this paper is to verificate the mathematical model presented in [4]. For this purpose, a load calculation of the most loaded rolling element was carried for two different types of a radial rolling bearings: ball bearing 6206 and bearing with the cylindrical rollers NU 2205 EC. For both bearing types, the analysis was made for different values of the internal radial clearance. The obtained results were compared with results obtained based on Stribeck coefficients. Also, both methods were compared with the results obtained by solving the system of static equilibrium equations [4]. The precision of the results obtained by solving the system of equations is solely depended on the precision of the numerical procedure for solving them. In this regard, these results were considered absolutely accurate and compared to them the calculation error according to the proposed method and the Stribeck method was analyzed.

The problem of calculating the internal load distribution is a statically indeterminate problem. Therefore, the system of static equilibrium equations needs to be extended with additional equations, which are based on the relationship between contact stresses and deformations. A detailed description of the calculation is presented in the references [7, 8].

Since this is a calculation of load for the most loaded rolling element, an analysis was done only for the BSO boundary position of supporting. This is justified because, from the aspect of load distribution, this is the most unfavorable position. Then the most loaded rolling element transfers the largest part of the external load [12]. Almost all of the so far published studies have limited their analysis to only this position of support [13-18]. The obtained results are sufficient and applicable for practical use in most cases. Besides, the coefficients of Stribeck refer to only this position of supporting.

2. THE STATIC EQUILIBRIUM EQUATIONS FOR THE BSO POSITION OF SUPPORT

In *Fig.1* the scheme of the internal load distribution for the BSO position of support for the rolling bearing with internal radial clearance is presented. The force equilibrium condition for this position can be set as:

$$Q = F_0 + 2 \cdot \sum_{i=1}^{\frac{q-1}{2}} F_i \cdot \cos(i\gamma) \quad (3)$$

Where: Q -external radial load, F_0 –load of the most loaded rolling element, F_i -load of i -th rolling element, γ -angular spacing between rolling bearing elements, q - number of active rolling elements.

Fig 1. Internal radial load distribution at rolling bearing

Equation (3) is a statically indeterminate equation. Unknown magnitudes in the equation are loads which transfer the rolling elements of the bearing (F_i). By taking into account the symmetry of the problems the number of unknown magnitudes is equal to $(q+1)/2$. So, besides the equilibrium equation (3) it is necessary to set up more $(q-1)/2$ additional equations, based on the relationship between the contact stresses and the deformations of the coupled parts of the bearing.

According to [4] the total deflection of the bearing is equal to the sum of the boundary deflection until the entry of the i -th rolling element in contact with the bearing rings, and the projection of the total contact deformation of the i -th rolling element on the direction of the action of external load, ie.:

$$w_o = a_{2i+1} + \frac{\delta_i}{\cos(i\gamma)} \quad (4)$$

Where is a_{2i+1} – the boundary deflection until the entry of the i -th rolling element in contact with the bearing rings. The boundary deflection is directly dependent on the type of bearing and the size of the internal radial clearance, and it can be easily determined based on the procedure shown in [7].

According to the Hertz theory [8] the size of the contact deformations on the place of the i -th rolling element can be calculated based on the equation:

$$\delta_i = \left(\frac{F_i}{K} \right)^{\frac{1}{n}} \quad (5)$$

Where is K – Effective coefficient of the bearing stiffness, and n - exponent which is dependent on the type of the bearing ($n = 3/2$ for a ball bearing and $n = 10/9$ for a bearing with the rollers).

Based on Fig.1.b it is clear that the total deflection of the bearing is equal to the contact deformation on the place of the most loaded rolling element, ie. $w_o = \delta_0$. Based on that it can be written that is:

$$w_o = \left(\frac{F_0}{K} \right)^{\frac{1}{n}} \quad (6)$$

Taking into account the equations (3) - (6) of the system of equations for the calculation of the internal load distribution for the BSO support position it can be written as:

$$\begin{aligned}
Q &= F_0 + 2 \cdot \sum_{i=1}^{\frac{q-1}{2}} F_i \cdot \cos(i\gamma) \\
\left(\frac{F_0}{K}\right)^{\frac{1}{n}} &= a_3 + \frac{\delta_1}{\cos \gamma} \\
&\text{---} \\
\left(\frac{F_0}{K}\right)^{\frac{1}{n}} &= a_{2i+1} + \frac{\delta_i}{\cos(i\gamma)} \\
&\text{---} \\
\left(\frac{F_0}{K}\right)^{\frac{1}{n}} &= a_q + \frac{\delta_{(q-1)/2}}{\cos(\frac{q-1}{2}\gamma)}
\end{aligned} \tag{7}$$

The system of equations (7) consists of $(q+1)/2$ non-linear equations. By solving this system it can be possible to obtain the loads which transfer the individual rolling elements. Since the system is non-linear it can be only solved numerically. The total number of active rolling elements of the bearing (q) can be easily determined based on the procedure shown in [8].

2. A CASE STUDY FOR THE BALL BEARING

The radial ball bearing 6206 was taken as an example for the calculation. The effective coefficient of the bearing stiffness is $K=3.41 \cdot 10^5 \text{ N/mm}^{3/2}$. The total number of rolling elements is $z = 9$, and the maximum possible number of the active rolling elements is $z_s = 5$ [10]. The values of the coefficient of boundary bearing deflection (t_q) and the coefficient of boundary external radial load (s_q) were determined from the tables which are presented in the references [7] and [8] and shown in *Table 1*. The values of the load factor for the most loaded rolling element k_{maxq} are also shown in *Table 1*. These values were determined based on *Table A1*. from *Appendix A* [4].

Table 1. The coefficient of boundary bearing deflection (t_q) and boundary external radial load (s_q) and the load factor of the most loaded rolling element (k_{maxq}) for ball bearing 6206.

	$q=3$	$q=4$	$q=5$
t_q	0.3054	0.9358	4.7588
s_q	0.1688	1.5498	20.0348
k_{maxq}	1,0000	0,5321	0,5182

The results of load calculation for the most loaded rolling element for the radial single row 6206 ball bearing are shown in *Tables 2-9*. The results are shown for different values of the internal radial clearance, which were varied in the range from 0 to 50 μm , which covers the full range of standard recommended values of the internal radial clearance for this bearing, for all classes (C2 to C5) [19]. The load values for the most loaded rolling element were calculated and shown in relation to three different methods:

- by solving the systems of nonlinear equations (equations of static equilibrium plus additional equations based on the relation of contact stresses and deformations. In *Tables 2-9* - *exact value*)
- by calculation using the Stribeck's number according to equation (2),
- by applying the load factor of the most loaded rolling element according to equation (1).

To get an effective analysis, the calculation by using the load factor was made especially for all three values of this factor shown in *Table 1*. Below the results are shown errors expressed in percentages, which were obtained by comparing with the exact value. The external load values were varied in the range from 10 to 10 000 N, which is very close to the static load rating of the bearing which is 11 200 N. In addition, the calculation was made also with respect to the boundary values of external load (Q_q). These values in the tables are bold. The boundary values of the external load were calculated in relation to the following equation [8]:

$$Q_q = K \cdot s_q \cdot \left(\frac{e}{2} \right)^n \quad (8)$$

The results are shown in *Tables 2-9*. and graphically presented in *Figures 2-9*. The calculation results of the load on the most loaded rolling element are shown on the left side. The errors in a percentage are shown on the right side.

Table 2. Load of the most loaded rolling element for bearing 6206 with radial clearance $e = 0 \mu\text{m}$

Q	5,0	8,0	10,0	20,0	50,0	80,0	100,0	200,0	500,0	800,0	1000,0	2000,0	5000,0	8000,0	10000,0
<i>exact result</i>	2,9	4,4	5,4	10,5	25,4	40,3	50,1	99,2	246,0	392,6	490,3	978,3	2441,4	3904,0	4879,0
q_3	5,0	8,0	10,0	20,0	50,0	80,0	100,0	200,0	500,0	800,0	1000,0	2000,0	5000,0	8000,0	10000,0
%	75,4	82,2	84,5	91,0	96,6	98,8	99,6	101,6	103,2	103,8	104,0	104,4	104,8	104,9	105,0
q_4	2,7	4,3	5,3	10,6	26,6	42,6	53,2	106,4	266,1	425,7	532,1	1064,2	2660,5	4256,8	5321,0
%	-6,6	-3,0	-1,8	1,6	4,6	5,8	6,2	7,3	8,1	8,4	8,5	8,8	9,0	9,0	9,1
q_5	2,6	4,1	5,2	10,4	25,9	41,5	51,8	103,6	259,1	414,6	518,2	1036,4	2591,0	4145,6	5182,0
%	-9,1	-5,6	-4,4	-1,0	1,9	3,0	3,4	4,5	5,3	5,6	5,7	5,9	6,1	6,2	6,2
<i>Stribek</i>	2,4	3,9	4,9	9,7	24,3	38,8	48,6	97,1	242,8	388,4	485,6	971,1	2427,8	3884,4	4855,6
%	-14,8	-11,5	-10,4	-7,2	-4,5	-3,5	-3,1	-2,1	-1,3	-1,1	-1,0	-0,7	-0,6	-0,5	-0,5

Fig 2. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 0 \mu\text{m}$

Table 3. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 5 \mu\text{m}$

Q	5,0	7,2	10,0	50,0	66,1	80,0	100,0	200,0	500,0	800,0	854,8	1000,0	5000,0	8000,0	10000,0
<i>exact result</i>	5,0	7,2	9,4	32,9	41,6	49,1	59,7	111,7	264,4	415,4	442,9	515,5	2487,2	3957,9	4937,2
q_3	5,0	7,2	10,0	50,0	66,1	80,0	100,0	200,0	500,0	800,0	854,8	1000,0	5000,0	8000,0	10000,0
%	0,0	0,0	6,6	52,2	58,8	63,0	67,5	79,0	89,1	92,6	93,0	94,0	101,0	102,1	102,5
q_4	2,7	3,8	5,3	26,6	35,2	42,6	53,2	106,4	266,1	425,7	454,8	532,1	2660,5	4256,8	5321,0
%	-46,8	-46,8	-43,3	-19,0	-15,5	-13,3	-10,9	-4,8	0,6	2,5	2,7	3,2	7,0	7,6	7,8
q_5	2,6	3,7	5,2	25,9	34,3	41,5	51,8	103,6	259,1	414,6	442,9	518,2	2591,0	4145,6	5182,0
%	-48,2	-48,2	-44,8	-21,2	-17,7	-15,5	-13,2	-7,2	-2,0	-0,2	0,0	0,5	4,2	4,7	5,0
<i>Stribek</i>	2,8	4,0	5,6	27,8	36,7	44,4	55,6	111,1	277,8	444,4	474,9	555,6	2777,8	4444,4	5555,6
%	-44,4	-44,4	-40,8	-15,5	-11,8	-9,4	-6,9	-0,6	5,1	7,0	7,2	7,8	11,7	12,3	12,5

Fig 3. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 5 \mu\text{m}$

Table 4. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 10 \mu\text{m}$

Q	10,0	20,0	20,4	50,0	100,0	187,1	200,0	500,0	800,0	1000,0	2000,0	2417,6	5000,0	8000,0	10000,0
<i>exact result</i>	10,0	20,0	20,4	40,6	69,9	117,8	124,8	282,2	436,2	538,1	1043,0	1252,7	2535,7	4016,0	5000,2
q_3	10,0	20,0	20,4	50,0	100,0	187,1	200,0	500,0	800,0	1000,0	2000,0	2417,6	5000,0	8000,0	10000,0
%	0,0	0,0	0,0	23,0	43,1	58,8	60,3	77,2	83,4	85,8	91,8	93,0	97,2	99,2	100,0
q_4	5,3	10,6	10,8	26,6	53,2	99,6	106,4	266,1	425,7	532,1	1064,2	1286,4	2660,5	4256,8	5321,0
%	-46,8	-46,8	-46,8	-34,5	-23,9	-15,5	-14,7	-5,7	-2,4	-1,1	2,0	2,7	4,9	6,0	6,4
q_5	5,2	10,4	10,6	25,9	51,8	97,0	103,6	259,1	414,6	518,2	1036,4	1252,8	2591,0	4145,6	5182,0
%	-48,2	-48,2	-48,2	-36,2	-25,8	-17,7	-16,9	-8,2	-5,0	-3,7	-0,6	0,0	2,2	3,2	3,6
<i>Stribeck</i>	5,6	11,1	11,3	27,8	55,6	103,9	111,1	277,8	444,4	555,6	1111,1	1343,1	2777,8	4444,4	5555,6
%	-44,4	-44,4	-44,4	-31,6	-20,5	-11,8	-10,9	-1,6	1,9	3,2	6,5	7,2	9,5	10,7	11,1

Fig 4. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 10 \mu\text{m}$

Table 5. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 15 \mu\text{m}$

Q	10,0	20,0	37,4	50,0	100,0	200,0	343,6	500,0	800,0	1000,0	2000,0	4441,4	5000,0	8000,0	10000,0
<i>exact result</i>	10,0	20,0	37,4	47,3	79,6	137,6	216,3	299,8	456,9	560,4	1071,2	2301,3	2580,7	4071,6	5061,1
q_3	10,0	20,0	37,4	50,0	100,0	200,0	343,6	500,0	800,0	1000,0	2000,0	4441,4	5000,0	8000,0	10000,0
%	0,0	0,0	0,0	5,7	25,6	45,4	58,8	66,8	75,1	78,4	86,7	93,0	93,7	96,5	97,6
q_4	5,3	10,6	19,9	26,6	53,2	106,4	182,8	266,1	425,7	532,1	1064,2	2363,3	2660,5	4256,8	5321,0
%	-46,8	-46,8	-46,8	-43,8	-33,2	-22,6	-15,5	-11,3	-6,8	-5,1	-0,7	2,7	3,1	4,5	5,1
q_5	5,2	10,4	19,4	25,9	51,8	103,6	178,0	259,1	414,6	518,2	1036,4	2301,5	2591,0	4145,6	5182,0
%	-48,2	-48,2	-48,2	-45,2	-34,9	-24,7	-17,7	-13,6	-9,3	-7,5	-3,2	0,0	0,4	1,8	2,4
<i>Stribeck</i>	5,6	11,1	20,8	27,8	55,6	111,1	190,9	277,8	444,4	555,6	1111,1	2467,5	2777,8	4444,4	5555,6
%	-44,4	-44,4	-44,4	-41,3	-30,2	-19,2	-11,8	-7,4	-2,7	-0,9	3,7	7,2	7,6	9,2	9,8

Fig 5. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 15 \mu\text{m}$

Table 6. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 20 \mu\text{m}$

Q	10,0	20,0	50,0	57,6	100,0	200,0	500,0	529,0	800,0	1000,0	2000,0	5000,0	6838,0	8000,0	10000,0
<i>exact result</i>	10,0	20,0	50,0	57,6	88,6	150,1	317,4	333,1	477,6	582,7	1099,4	2619,6	3543,1	4123,9	5119,3
q_3	10,0	20,0	50,0	57,6	100,0	200,0	500,0	529,0	800,0	1000,0	2000,0	5000,0	6838,0	8000,0	10000,0
%	0,0	0,0	0,0	0,0	12,8	33,3	57,5	58,8	67,5	71,6	81,9	90,9	93,0	94,0	95,3
q_4	5,3	10,6	26,6	30,7	53,2	106,4	266,1	281,5	425,7	532,1	1064,2	2660,5	3638,5	4256,8	5321,0
%	-46,8	-46,8	-46,8	-46,8	-40,0	-29,1	-16,2	-15,5	-10,9	-8,7	-3,2	1,6	2,7	3,2	3,9
q_5	5,2	10,4	25,9	29,9	51,8	103,6	259,1	274,1	414,6	518,2	1036,4	2591,0	3543,5	4145,6	5182,0
%	-48,2	-48,2	-48,2	-48,2	-41,5	-30,9	-18,4	-17,7	-13,2	-11,1	-5,7	-1,1	0,0	0,5	1,2
<i>Stribek</i>	5,6	11,1	27,8	32,0	55,6	111,1	277,8	293,9	444,4	555,6	1111,1	2777,8	3798,9	4444,4	5555,6
%	-44,4	-44,4	-44,4	-44,4	-37,3	-26,0	-12,5	-11,8	-6,9	-4,7	1,1	6,0	7,2	7,8	8,5

Fig 6. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 20 \mu\text{m}$

Table 7. Load of the most loaded rolling element for ball bearing 6206 with internal clearance $e = 30 \mu\text{m}$

Q	10,0	20,0	50,0	80,0	100,0	105,8	200,0	500,0	800,0	971,8	1000,0	2000,0	5000,0	8000,0	10000,0
<i>exact result</i>	10,0	20,0	50,0	80,0	100,0	105,8	173,5	352,0	518,6	611,9	627,1	1155,6	2696,1	4215,0	5222,3
q_3	10,0	20,0	50,0	80,0	100,0	105,8	200,0	500,0	800,0	971,8	1000,0	2000,0	5000,0	8000,0	10000,0
%	0,0	0,0	0,0	0,0	0,0	0,0	15,3	42,0	54,3	58,8	59,5	73,1	85,5	89,8	91,5
q_4	5,3	10,6	26,6	42,6	53,2	56,3	106,4	266,1	425,7	517,1	532,1	1064,2	2660,5	4256,8	5321,0
%	-46,8	-46,8	-46,8	-46,8	-46,8	-46,8	-38,7	-24,4	-17,9	-15,5	-15,1	-7,9	-1,3	1,0	1,9
q_5	5,2	10,4	25,9	41,5	51,8	54,8	103,6	259,1	414,6	503,6	518,2	1036,4	2591,0	4145,6	5182,0
%	-48,2	-48,2	-48,2	-48,2	-48,2	-48,2	-40,3	-26,4	-20,1	-17,7	-17,4	-10,3	-3,9	-1,6	-0,8
<i>Stribek</i>	5,6	11,1	27,8	44,4	55,6	58,8	111,1	277,8	444,4	539,9	555,6	1111,1	2777,8	4444,4	5555,6
%	-44,4	-44,4	-44,5	-44,4	-44,4	-44,4	-36,0	-21,1	-14,3	-11,8	-11,4	-3,9	3,0	5,4	6,4

Fig 7. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 30 \mu\text{m}$

Table 8. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 40 \mu\text{m}$

Q	10,0	20,0	50,0	80,0	100,0	163,0	200,0	500,0	800,0	1000,0	1496,1	2000,0	5000,0	8000,0	10000,0
exact result	10,0	20,0	50,0	80,0	100,0	163,0	193,1	385,6	559,1	671,0	942,0	1211,7	2772,6	4304,5	5318,8
q_3	10,0	20,0	50,0	80,0	100,0	163,0	200,0	500,0	800,0	1000,0	1496,1	2000,0	5000,0	8000,0	10000,0
%	0,0	0,0	0,0	0,0	0,0	0,0	3,6	29,7	43,1	49,0	58,8	65,1	80,3	85,9	88,0
q_4	5,3	10,6	26,6	42,6	53,2	86,7	106,4	266,1	425,7	532,1	796,1	1064,2	2660,5	4256,8	5321,0
%	-46,8	-46,8	-46,8	-46,8	-46,8	-46,8	-44,9	-31,0	-23,9	-20,7	-15,5	-12,2	-4,0	-1,1	0,0
q_5	5,2	10,4	25,9	41,5	51,8	84,4	103,6	259,1	414,6	518,2	775,3	1036,4	2591,0	4145,6	5182,0
%	-48,2	-48,2	-48,2	-48,2	-48,2	-48,2	-46,3	-32,8	-25,8	-22,8	-17,7	-14,5	-6,5	-3,7	-2,6
Stribek	5,6	11,1	27,8	44,4	55,6	90,5	111,1	277,8	444,4	555,6	831,2	1111,1	2777,8	4444,4	5555,6
%	-44,4	-44,4	-44,5	-44,4	-44,4	-44,4	-42,5	-28,0	-20,5	-17,2	-11,8	-8,3	0,2	3,3	4,5

Fig 8. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 40 \mu\text{m}$

Table 9. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 50 \mu\text{m}$

Q	10,0	20,0	50,0	80,0	100,0	200,0	227,7	500,0	800,0	1000,0	2000,0	2090,9	5000,0	8000,0	10000,0
exact result	10,0	20,0	50,0	80,0	100,0	200,0	227,7	417,7	598,7	714,2	1267,4	1316,5	2849,0	4394,0	5415,2
q_3	10,0	20,0	50,0	80,0	100,0	200,0	227,7	500,0	800,0	1000,0	2000,0	2090,9	5000,0	8000,0	10000,0
%	0,1	0,0	0,0	0,0	0,0	0,0	0,0	19,7	33,6	40,0	57,8	58,8	75,5	82,1	84,7
q_4	5,3	10,6	26,6	42,6	53,2	106,4	121,2	266,1	425,7	532,1	1064,2	1112,6	2660,5	4256,8	5321,0
%	-46,7	-46,8	-46,8	-46,8	-46,8	-46,8	-46,8	-36,3	-28,9	-25,5	-16,0	-15,5	-6,6	-3,1	-1,7
q_5	5,2	10,4	25,9	41,5	51,8	103,6	118,0	259,1	414,6	518,2	1036,4	1083,5	2591,0	4145,6	5182,0
%	-48,1	-48,2	-48,2	-48,2	-48,2	-48,2	-48,2	-38,0	-30,8	-27,4	-18,2	-17,7	-9,1	-5,7	-4,3
Stribek	5,6	11,1	27,8	44,4	55,6	111,1	126,5	277,8	444,4	555,6	1111,1	1161,6	2777,8	4444,4	5555,6
%	-44,4	-44,4	-44,4	-44,5	-44,4	-44,4	-44,4	-33,5	-25,8	-22,2	-12,3	-11,8	-2,5	1,1	2,6

Fig 9. Load of the most loaded rolling element for ball bearing 6206 with radial clearance $e = 50 \mu\text{m}$

3. A CASE STUDY FOR BEARING WITH ROLLERS

As an example for the calculation, a radial roller bearing with cylindrical rollers NU 2205 EC was taken. The effective coefficient of the stiffness of this bearing is $K=3.14 \cdot 10^5 \text{ N/mm}^{10/9}$. The total number of rolling elements is $z = 13$, and a maximal possible number of active rolling elements which can occur in the loaded zone is $z_s = 7$ [7]. Based on tables shown at references [7] and [8], the values of the coefficient of boundary bearing deflection (t_q) and the coefficient of boundary external radial load (s_q), which are shown in Table 10. The values of the load factor of the most loaded rolling element k_{maxq} are also shown in Table 10. These values are determined according to Table A2. from Appendix A [4].

Table 10. The coefficient of boundary bearing deflection (t_q) and boundary external radial load (s_q) and the load factor of the most loaded rolling element (k_{maxq}) for roller bearing NU 2205 EC

	$q=3$	$q=4$	$q=5$	$q=6$	$q=7$
t_q	0,1294	0,3061	0,7604	1,7901	7,2962
s_q	0,1031	0,5043	1,6650	5,2715	27,7786
k_{maxq}	1,0000	0,5150	0,4430	0,3506	0,3276

The results of the calculation of load for the most loaded roller are shown in Tables 11-18. The analysis is made for different values of the internal radial clearance which were varied between 0 to $50 \mu\text{m}$, which covers the whole range of standard recommended values for an internal radial clearance, for all classes (from C2 to C5), for this bearing type [19]. For this example too, the values of the load were calculated and shown regarding the three above mentioned methods. The values of the external load were varied in the range from 10 do 30 000 N, which is close to the static load rating of this bearing, which is 34 100 N. Also, the results acquired in relation to the values of the boundary external load (Q_q) are bolded in these tables. The value of the boundary external load (Q_q) was obtained based on equation (8). Results from Tables 11-18. are graphically shown in Figs. 10-17.

Table 11. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 0 \mu\text{m}$

Q	10,0	20,0	50,0	80,0	100,0	200,0	500,0	800,0	1000,0	2000,0	5000,0	8000,0	10000,0	15000,0	20000,0	30000,0
<i>exact result</i>	6,2	10,2	22,2	32,4	38,9	71,2	167,1	262,7	326,1	641,9	1587,7	2532,9	3162,9	4737,8	6312,5	9461,7
q_3	10,0	20,0	50,0	80,0	100,0	200,0	500,0	800,0	1000,0	2000,0	5000,0	8000,0	10000,0	15000,0	20000,0	30000,0
%	62,3	95,3	124,8	147,1	156,9	180,9	199,2	204,5	206,6	211,6	214,9	215,8	216,2	216,6	216,8	217,1
q_4	5,2	10,3	25,8	41,2	51,5	103,0	257,5	412,0	515,0	1030,0	2575,0	4120,0	5150,0	7725,0	10300,0	15450,0
%	-16,4	0,6	15,8	27,2	32,3	44,6	54,1	56,8	57,9	60,5	62,2	62,7	62,8	63,1	63,2	63,3
q_5	4,4	8,9	22,2	35,4	44,3	88,6	221,5	354,4	443,0	886,0	2215,0	3544,0	4430,0	6645,0	8860,0	13290,0
%	-28,1	-13,5	-0,4	9,5	13,8	24,4	32,5	34,9	35,8	38,0	39,5	39,9	40,1	40,3	40,4	40,5
q_6	3,5	7,0	17,5	28,0	35,1	70,1	175,3	280,5	350,6	701,2	1753,0	2804,8	3506,0	5259,0	7012,0	10518,0
%	-43,1	-31,5	-21,2	-13,4	-9,9	-1,5	4,9	6,8	7,5	9,2	10,4	10,7	10,8	11,0	11,1	11,2
q_7	3,3	6,6	16,4	26,2	32,8	65,5	163,8	262,1	327,6	655,2	1638,0	2620,8	3276,0	4914,0	6552,0	9828,0
%	-46,8	-36,0	-26,3	-19,1	-15,8	-8,0	-2,0	-0,2	0,5	2,1	3,2	3,5	3,6	3,7	3,8	3,9
<i>Stribek</i>	3,1	6,3	15,7	25,1	31,4	62,8	156,9	251,1	313,8	627,7	1569,2	2510,8	3138,5	4707,7	6276,9	9415,4
%	-49,1	-38,7	-29,4	-22,5	-19,4	-11,9	-6,1	-4,4	-3,8	-2,2	-1,2	-0,9	-0,8	-0,6	-0,6	-0,5

Fig 10. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 0 \mu\text{m}$

Table 12. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 5 \mu\text{m}$

Q	10,0	41,7	100,0	200,0	203,7	500,0	672,6	1000,0	2000,0	2129,6	5000,0	8000,0	10000,0	11222,1	20000,0	30000,0
<i>exact result</i>	10,0	41,7	67,4	108,6	110,1	229,2	298,0	407,3	732,0	773,8	1693,3	2650,1	3287,0	3675,9	6449,9	9605,3
q_3	10,0	41,7	100,0	200,0	203,7	500,0	672,6	1000,0	2000,0	2129,6	5000,0	8000,0	10000,0	11222,1	20000,0	30000,0
%	0,0	0,0	48,4	84,1	85,0	118,1	125,7	145,5	173,2	175,2	195,3	201,9	204,2	205,3	210,1	212,3
q_4	5,2	21,4	51,5	103,0	104,9	257,5	346,4	515,0	1030,0	1096,7	2575,0	4120,0	5150,0	5779,4	10300,0	15450,0
%	-48,5	-48,5	-23,6	-5,2	-4,7	12,3	16,3	26,4	40,7	41,7	52,1	55,5	56,7	57,2	59,7	60,8
q_5	4,4	18,5	44,3	88,6	90,3	221,5	298,0	443,0	886,0	943,4	2215,0	3544,0	4430,0	4971,4	8860,0	13290,0
%	-55,7	-55,7	-34,3	-18,4	-18,0	-3,4	0,0	8,8	21,0	21,9	30,8	33,7	34,8	35,2	37,4	38,4
q_6	3,5	14,6	35,1	70,1	71,4	175,3	235,8	350,6	701,2	746,6	1753,0	2804,8	3506,0	3934,5	7012,0	10518,0
%	-64,9	-64,9	-48,0	-35,4	-35,1	-23,5	-20,9	-13,9	-4,2	-3,5	3,5	5,8	6,7	7,0	8,7	9,5
q_7	3,3	13,6	32,8	65,5	66,7	163,8	220,4	327,6	655,2	697,7	1638,0	2620,8	3276,0	3676,4	6552,0	9828,0
%	-67,2	-67,2	-51,4	-39,7	-39,4	-28,5	-26,0	-19,6	-10,5	-9,8	-3,3	-1,1	-0,3	0,0	1,6	2,3
<i>Stribek</i>	3,8	16,0	38,5	76,9	78,4	192,3	258,7	384,6	769,2	819,1	1923,1	3076,9	3846,2	4316,2	7692,3	11538,5
%	-61,5	-61,5	-42,9	-29,2	-28,8	-16,1	-13,2	-5,6	5,1	5,9	13,6	16,1	17,0	17,4	19,3	20,1

Fig 11. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 5 \mu\text{m}$

Table 13. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 10 \mu\text{m}$

Q	50,0	89,97	100,0	200,0	440,1	500,0	800,0	1000,0	1453,0	2000,0	4600,2	5000,0	8000,0	20000,0	24241,1	30000,0
exact result	50,0	89,96	95,0	138,8	237,9	262,2	383,1	463,2	643,6	827,1	1671,5	1800,2	2762,5	6590,4	7940,4	9762,4
q_3	50,0	89,97	100,0	200,0	440,1	500,0	800,0	1000,0	1453,0	2000,0	4600,2	5000,0	8000,0	20000,0	24241,1	30000,0
%	0,0	0,01	5,3	44,1	85,0	90,7	108,8	115,9	125,7	141,8	175,2	177,8	189,6	203,5	205,3	207,3
q_4	25,8	46,33	51,5	103,0	226,6	257,5	412,0	515,0	748,3	1030,0	2369,1	2575,0	4120,0	10300,0	12484,2	15450,0
%	-48,5	-48,49	-45,8	-25,8	-4,7	-1,8	7,5	11,2	16,3	24,5	41,7	43,0	49,1	56,3	57,2	58,3
q_5	22,2	39,86	44,3	88,6	195,0	221,5	354,4	443,0	643,7	886,0	2037,9	2215,0	3544,0	8860,0	10738,8	13290,0
%	-55,7	-55,70	-53,4	-36,2	-18,0	-15,5	-7,5	-4,4	0,0	7,1	21,9	23,0	28,3	34,4	35,2	36,1
q_6	17,5	31,54	35,1	70,1	154,3	175,3	280,5	350,6	509,4	701,2	1612,8	1753,0	2804,8	7012,0	8498,9	10518,0
%	-64,9	-64,94	-63,1	-49,5	-35,1	-33,2	-26,8	-24,3	-20,9	-15,2	-3,5	-2,6	1,5	6,4	7,0	7,7
q_7	16,4	29,47	32,8	65,5	144,2	163,8	262,1	327,6	476,0	655,2	1507,0	1638,0	2620,8	6552,0	7941,4	9828,0
%	-67,2	-67,24	-65,5	-52,8	-39,4	-37,5	-31,6	-29,3	-26,0	-20,8	-9,8	-9,0	-5,1	-0,6	0,0	0,7
Stribek	19,2	34,60	38,5	76,9	169,3	192,3	307,7	384,6	558,8	769,2	1769,3	1923,1	3076,9	7692,3	9323,5	11538,5
%	-61,5	-61,53	-59,5	-44,6	-28,8	-26,7	-19,7	-17,0	-13,2	-7,0	5,9	6,8	11,4	16,7	17,4	18,2

Fig 12. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 10 \mu\text{m}$

Table 14. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 15 \mu\text{m}$

Q	100,0	141,2	200,0	500,0	690,5	800,0	1000,0	2000,0	2279,9	5000,0	7218,3	8000,0	10000,0	15000,0	20000,0	30000,0
exact result	100,0	141,2	168,7	295,3	373,3	417,7	498,5	898,6	1009,9	1905,9	2622,7	2874,3	3516,7	5117,3	6713,9	9901,0
q_3	100,0	141,2	200,0	500,0	690,5	800,0	1000,0	2000,0	2279,9	5000,0	7218,3	8000,0	10000,0	15000,0	20000,0	30000,0
%	0,0	0,0	18,6	69,3	85,0	91,5	100,6	122,6	125,7	162,3	175,2	178,3	184,4	193,1	197,9	203,0
q_4	51,5	72,7	103,0	257,5	355,6	412,0	515,0	1030,0	1174,1	2575,0	3717,4	4120,0	5150,0	7725,0	10300,0	15450,0
%	-48,5	-48,5	-38,9	-12,8	-4,7	-1,4	3,3	14,6	16,3	35,1	41,7	43,3	46,4	51,0	53,4	56,0
q_5	44,3	62,5	88,6	221,5	305,9	354,4	443,0	886,0	1010,0	2215,0	3197,7	3544,0	4430,0	6645,0	8860,0	13290,0
%	-55,7	-55,7	-47,5	-25,0	-18,0	-15,2	-11,1	-1,4	0,0	16,2	21,9	23,3	26,0	29,9	32,0	34,2
q_6	35,1	49,5	70,1	175,3	242,1	280,5	350,6	701,2	799,3	1753,0	2530,7	2804,8	3506,0	5259,0	7012,0	10518,0
%	-64,9	-64,9	-58,4	-40,6	-35,1	-32,9	-29,7	-22,0	-20,9	-8,0	-3,5	-2,4	-0,3	2,8	4,4	6,2
q_7	32,8	46,3	65,5	163,8	226,2	262,1	327,6	655,2	746,9	1638,0	2364,7	2620,8	3276,0	4914,0	6552,0	9828,0
%	-67,2	-67,2	-61,2	-44,5	-39,4	-37,3	-34,3	-27,1	-26,0	-14,1	-9,8	-8,8	-6,8	-4,0	-2,4	-0,7
Stribek	38,5	54,3	76,9	192,3	265,6	307,7	384,6	769,2	876,9	1923,1	2776,3	3076,9	3846,2	5769,2	7692,3	11538,5
%	-61,5	-61,5	-54,4	-34,9	-28,8	-26,3	-22,9	-14,4	-13,2	0,9	5,9	7,0	9,4	12,7	14,6	16,5

Fig 13. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 15 \mu\text{m}$

Table 15. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 20 \mu\text{m}$

Q	100,0	194,4	200,0	500,0	800,0	950,4	1000,0	2000,0	3138,6	5000,0	8000,0	9937,0	10000,0	15000,0	20000,0	30000,0
exact result	100,0	194,3	197,4	328,4	452,4	513,8	534,0	936,4	1390,3	2010,2	2985,5	3610,5	3630,8	5236,8	6837,1	10029,6
q_3	100,0	194,4	200,0	500,0	800,0	950,4	1000,0	2000,0	3138,6	5000,0	8000,0	9937,0	10000,0	15000,0	20000,0	30000,0
%	0,0	0,0	1,3	52,3	76,8	85,0	87,3	113,6	125,7	148,7	168,0	175,2	175,4	186,4	192,5	199,1
q_4	51,5	100,1	103,0	257,5	412,0	489,4	515,0	1030,0	1616,4	2575,0	4120,0	5117,5	5150,0	7725,0	10300,0	15450,0
%	-48,5	-48,5	-47,8	-21,6	-8,9	-4,7	-3,5	10,0	16,3	28,1	38,0	41,7	41,8	47,5	50,6	54,0
q_5	44,3	86,1	88,6	221,5	354,4	421,0	443,0	886,0	1390,4	2215,0	3544,0	4402,1	4430,0	6645,0	8860,0	13290,0
%	-55,7	-55,7	-55,1	-32,5	-21,7	-18,1	-17,0	-5,4	0,0	10,2	18,7	21,9	22,0	26,9	29,6	32,5
q_6	35,1	68,1	70,1	175,3	280,5	333,2	350,6	701,2	1100,4	1753,0	2804,8	3483,9	3506,0	5259,0	7012,0	10518,0
%	-64,9	-64,9	-64,5	-46,6	-38,0	-35,1	-34,3	-25,1	-20,9	-12,8	-6,1	-3,5	-3,4	0,4	2,6	4,9
q_7	32,8	63,7	65,5	163,8	262,1	311,3	327,6	655,2	1028,2	1638,0	2620,8	3255,4	3276,0	4914,0	6552,0	9828,0
%	-67,2	-67,2	-66,8	-50,1	-42,1	-39,4	-38,6	-30,0	-26,0	-18,5	-12,2	-9,8	-9,8	-6,2	-4,2	-2,0
Stribek	38,5	74,8	76,9	192,3	307,7	365,5	384,6	769,2	1207,2	1923,1	3076,9	3821,9	3846,2	5769,2	7692,3	11538,5
%	-61,5	-61,5	-61,0	-41,4	-32,0	-28,9	-28,0	-17,9	-13,2	-4,3	3,1	5,9	5,9	10,2	12,5	15,0

Fig 14. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 20 \mu\text{m}$

Table 16. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 30 \mu\text{m}$

Q	100,0	200,0	305,0	500,0	800,0	1000,0	1491,7	2000,0	4924,8	5000,0	8000,0	10000,0	15000,0	15592,4	20000,0	30000,0
<i>exact result</i>	100,0	200,0	304,9	394,2	521,8	604,9	806,3	1012,3	2181,6	2208,0	3204,8	3857,1	5474,5	5665,4	7082,7	10286,2
q_3	100,0	200,0	305,0	500,0	800,0	1000,0	1491,7	2000,0	4924,8	5000,0	8000,0	10000,0	15000,0	15592,4	20000,0	30000,0
%	0,0	0,0	0,0	26,8	53,3	65,3	85,0	97,6	125,7	126,5	149,6	159,3	174,0	175,2	182,4	191,7
q_4	51,5	103,0	157,1	257,5	412,0	515,0	768,2	1030,0	2536,3	2575,0	4120,0	5150,0	7725,0	8030,1	10300,0	15450,0
%	-48,5	-48,5	-48,5	-34,7	-21,0	-14,9	-4,7	1,7	16,3	16,6	28,6	33,5	41,1	41,7	45,4	50,2
q_5	44,3	88,6	135,1	221,5	354,4	443,0	660,8	886,0	2181,7	2215,0	3544,0	4430,0	6645,0	6907,4	8860,0	13290,0
%	-55,7	-55,7	-55,7	-43,8	-32,1	-26,8	-18,0	-12,5	0,0	0,3	10,6	14,9	21,4	21,9	25,1	29,2
q_6	35,1	70,1	106,9	175,3	280,5	350,6	523,0	701,2	1726,6	1753,0	2804,8	3506,0	5259,0	5466,7	7012,0	10518,0
%	-64,9	-64,9	-64,9	-55,5	-46,2	-42,0	-35,1	-30,7	-20,9	-20,6	-12,5	-9,1	-3,9	-3,5	-1,0	2,3
q_7	32,8	65,5	99,9	163,8	262,1	327,6	488,7	655,2	1613,4	1638,0	2620,8	3276,0	4914,0	5108,1	6552,0	9828,0
%	-67,2	-67,2	-67,2	-58,5	-49,8	-45,8	-39,4	-35,3	-26,0	-25,8	-18,2	-15,1	-10,2	-9,8	-7,5	-4,5
<i>Stribek</i>	38,5	76,9	117,3	192,3	307,7	384,6	573,7	769,2	1894,2	1923,1	3076,9	3846,2	5769,2	5997,1	7692,3	11538,5
%	-61,5	-61,5	-61,5	-51,2	-41,0	-36,4	-28,8	-24,0	-13,2	-12,9	-4,0	-0,3	5,4	5,9	8,6	12,2

Fig 15. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 30 \mu\text{m}$

Table 17. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 40 \mu\text{m}$

Q	100,0	200,0	419,8	500,0	800,0	1000,0	2000,0	2053,5	5000,0	6779,7	8000,0	10000,0	15000,0	20000,0	21465,1	30000,0
exact result	100,0	200,0	419,8	458,8	590,9	675,8	1088,3	1110,0	2294,4	3003,2	3417,4	4079,4	5710,4	7327,0	7799,3	10542,0
q_3	100,0	200,0	419,8	500,0	800,0	1000,0	2000,0	2053,5	5000,0	6779,7	8000,0	10000,0	15000,0	20000,0	21465,1	30000,0
%	0,0	0,0	0,0	9,0	35,4	48,0	83,8	85,0	117,9	125,7	134,1	145,1	162,7	173,0	175,2	184,6
q_4	51,5	103,0	216,2	257,5	412,0	515,0	1030,0	1057,5	2575,0	3491,6	4120,0	5150,0	7725,0	10300,0	11054,5	15450,0
%	-48,5	-48,5	-48,5	-43,9	-30,3	-23,8	-5,4	-4,7	12,2	16,3	20,6	26,2	35,3	40,6	41,7	46,6
q_5	44,3	88,6	186,0	221,5	354,4	443,0	886,0	909,7	2215,0	3003,4	3544,0	4430,0	6645,0	8860,0	9509,0	13290,0
%	-55,7	-55,7	-55,7	-51,7	-40,0	-34,4	-18,6	-18,0	-3,5	0,0	3,7	8,6	16,4	20,9	21,9	26,1
q_6	35,1	70,1	147,2	175,3	280,5	350,6	701,2	719,9	1753,0	2377,0	2804,8	3506,0	5259,0	7012,0	7525,7	10518,0
%	-64,9	-64,9	-64,9	-61,8	-52,5	-48,1	-35,6	-35,1	-23,6	-20,9	-17,9	-14,1	-7,9	-4,3	-3,5	-0,2
q_7	32,8	65,5	137,5	163,8	262,1	327,6	655,2	672,7	1638,0	2221,0	2620,8	3276,0	4914,0	6552,0	7032,0	9828,0
%	-67,2	-67,2	-67,2	-64,3	-55,6	-51,5	-39,8	-39,4	-28,6	-26,0	-23,3	-19,7	-13,9	-10,6	-9,8	-6,8
Stribek	38,5	76,9	161,5	192,3	307,7	384,6	769,2	789,8	1923,1	2607,6	3076,9	3846,2	5769,2	7692,3	8255,8	11538,5
%	-61,5	-61,5	-61,5	-58,1	-47,9	-43,1	-29,3	-28,8	-16,2	-13,2	-10,0	-5,7	1,0	5,0	5,9	9,5

Fig 16. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 40 \mu\text{m}$

Table 18. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 50 \mu\text{m}$

Q	100,0	200,0	500,0	537,9	800,0	1000,0	2000,0	2631,3	5000,0	8000,0	8687,4	10000,0	15000,0	20000,0	27504,9	30000,0
exact result	100,0	200,1	500,0	537,9	659,6	746,5	1164,3	1422,4	2377,5	3574,9	3848,3	4294,9	5943,7	7569,8	9993,8	10797,0
q_3	100,0	200,0	500,0	537,9	800,0	1000,0	2000,0	2631,3	5000,0	8000,0	8687,4	10000,0	15000,0	20000,0	27504,9	30000,0
%	0,0	0,0	0,0	0,0	21,3	34,0	71,8	85,0	110,3	123,8	125,7	132,8	152,4	164,2	175,2	177,9
q_4	51,5	103,0	257,5	277,0	412,0	515,0	1030,0	1355,1	2575,0	4120,0	4474,0	5150,0	7725,0	10300,0	14165,0	15450,0
%	-48,5	-48,5	-48,5	-48,5	-37,5	-31,0	-11,5	-4,7	8,3	15,2	16,3	19,9	30,0	36,1	41,7	43,1
q_5	44,3	88,6	221,5	238,3	354,4	443,0	886,0	1165,7	2215,0	3544,0	3848,5	4430,0	6645,0	8860,0	12184,7	13290,0
%	-55,7	-55,7	-55,7	-55,7	-46,3	-40,7	-23,9	-18,0	-6,8	-0,9	0,0	3,1	11,8	17,0	21,9	23,1
q_6	35,1	70,1	175,3	188,6	280,5	350,6	701,2	922,5	1753,0	2804,8	3045,8	3506,0	5259,0	7012,0	9643,2	10518,0
%	-64,9	-65,0	-64,9	-64,9	-57,5	-53,0	-39,8	-35,1	-26,3	-21,5	-20,9	-18,4	-11,5	-7,4	-3,5	-2,6
q_7	32,8	65,5	163,8	176,2	262,1	327,6	655,2	862,0	1638,0	2620,8	2846,0	3276,0	4914,0	6552,0	9010,6	9828,0
%	-67,2	-67,3	-67,2	-67,2	-60,3	-56,1	-43,7	-39,4	-31,1	-26,7	-26,0	-23,7	-17,3	-13,4	-9,8	-9,0
Stribek	38,5	76,9	192,3	206,9	307,7	384,6	769,2	1012,0	1923,1	3076,9	3341,3	3846,2	5769,2	7692,3	10578,8	11538,5
%	-61,5	-61,6	-61,5	-61,5	-53,3	-48,5	-33,9	-28,8	-19,1	-13,9	-13,2	-10,4	-2,9	1,6	5,9	6,9

Fig 17. Load of the most loaded rolling element for roller bearing NU 2205 EC with clearance $e = 50 \mu\text{m}$

4. DISCUSSION OF ACQUIRED RESULTS

The part of the load which transfer the most loaded rolling element of the rolling bearing is possible to get simply, by multiplication of load factor k_{maxq} with the value of the external radial load. On the level of load of the most loaded rolling element, the largest influence has the number of active rolling elements, which is again primarily dependant on the size of internal radial clearance and applied external load. The larger external load will cause a greater number of active rolling elements to participate in the load transfer. On the other hand, at low values of external load, the number of active rolling elements will be smaller and will be in an interval $q=1\div3$. It implies that for the lower values of external load, the calculation with the factor of load k_{max3} will give more accurate results. This is also shown by the results presented in the previous subchapter. The factor k_{max3} corresponds to supporting on the three active rolling elements of the bearing.

Internal radial clearance negatively affects the number of active rolling elements, so larger values of internal clearance increase the zones in which the calculation with coefficient k_{max3} gives the best results. By reducing the value of the internal clearance, the value of this zone will be proportionally narrower. For example, for relatively small clearance of $e = 5 \mu\text{m}$ and bearing 6206 the length of the zone in which factor k_{max3} gives the best results reach about 10 N, and 40 N for bearing NU 2205. On the other side for clearance $e = 50 \mu\text{m}$ this zone increases to 200 N for bearing 6206, or 900 N for bearing NU 2205 EC.

On the other side, for larger values of external loads more accurate results are obtained with a calculation with factors k_{maxq} that correspond to a larger number of active rolling elements. The larger the value of the external load and the smaller the size of the internal gap, the calculation with load factor k_{maxq} gives more accurate results compared to the calculation recommended by Stribeck. This is especially pronounced in the bearings with rollers. For loads that allow supporting on the maximum possible number of rolling elements, the calculation using the load factor gives much more accurate results compared to Stribeck's method.

Absolutely accurate results are obtained for the boundary values of external load, which correspond to the boundary deflection. In this situation, the support system of the inner ring of the bearing switches from $q-1$ on q active rolling elements. Extremely high accuracy of the results is obtained also in the areas that are relatively close to the boundary values of the external load. Increasing the number of active rolling elements in the bearing increases the accuracy and width of high accuracy zones. The calculation results for the boundary loads in Tables 2-18 are highlighted in bold. Until the first bolded load the inner bearing ring will support according to the support system 1-2 [10, 20]. After this load, the third rolling element engages the ring of the bearing and becomes active. The next bolded column corresponds to the load when the inner ring begins to support four rolling elements, etc.

By analyzing the results shown in *Tables 2-18.* and the pictures *Fig.2-17.* it can be concluded that much greater precision of results can be acquired by the selection of load factor that will give the best accuracy for given bearing and applied external load. In doing so, it is good to keep the following recommendations which are mentioned in the analysis above:

- that the calculation gives the most accurate results in areas that are close to the boundary values of external load,
- that for relatively low loads, which are expected to rely on up to three active rolling elements, the best results are obtained by using the calculation coefficient k_{max3}
- for support system on the maximum possible number of active roller elements (z_s), the best results are obtained using the coefficient corresponding to that number (k_{maxzs}).

4. THE CALCULATION BY APPLYING A SELECTION OF COEFFICIENTS

In order to properly verify the above recommendations, *Figures 18-33* show the results of the calculation by applying the selection of coefficients. The previously discussed bearings 6206 and NU 2205 were again taken as an example. The load distribution factors were selected according to the above recommendations. Here again, the diagrams on the right show loads of the most loaded rolling element, while the left shows the comparison of the error according to the proposed method and the Stribeck method.

- Load of the most loaded rolling element of the radial ball bearing 6206 obtained by the selection of the load factors

Fig 18. Radial ball bearing 6206 with an internal radial clearance $e = 0 \mu\text{m}$

Fig 19. Radial ball bearing 6206 with an internal radial clearance $e = 5 \mu\text{m}$

Fig 20. Radial ball bearing 6206 with an internal radial clearance $e = 10 \mu\text{m}$

Fig 21. Radial ball bearing 6206 with an internal radial clearance $e = 15 \mu\text{m}$

Fig 22. Radial ball bearing 6206 with an internal radial clearance $e = 20 \mu\text{m}$

Fig 23. Radial ball bearing 6206 with an internal radial clearance $e = 30 \mu\text{m}$

Fig 24. Radial ball bearing 6206 with an internal radial clearance $e = 40 \mu\text{m}$

Fig 25. Radial ball bearing 6206 with an internal radial clearance $e = 50 \mu\text{m}$

- Load of the most loaded rolling element of the radial roller bearing NU 2205 EC obtained by the selection of the load factors

Fig 26. Radial roller bearing NU 2205 EC with internal radial clearance $e = 0 \mu\text{m}$

Fig 27. Radial roller bearing NU 2205 EC with internal radial clearance $e = 5 \mu\text{m}$

Fig 28. Radial roller bearing NU 2205 EC with internal radial clearance $e = 10 \mu\text{m}$

Fig 29. Radial roller bearing NU 2205 EC with internal radial clearance $e = 15 \mu\text{m}$

Fig 30. Radial roller bearing NU 2205 EC with internal radial clearance $e = 20 \mu\text{m}$

Fig 31. Radial roller bearing NU 2205 EC with internal radial clearance $e = 30 \mu\text{m}$

Fig 32. Radial roller bearing NU 2205 EC with internal radial clearance $e = 40 \mu\text{m}$

Fig 33. Radial roller bearing NU 2205 EC with internal radial clearance $e = 50 \mu\text{m}$

The calculation results presented in Fig.18-33. show a much greater accuracy of the method proposed in this paper than the method of Stribeck.

This is especially pronounced at the roller bearing NU 2205. At this bearing, Stribeck's expressions show higher accuracy only at bearing with zeroth internal clearance, at high external loads (over 30 000 N), as well as in narrow zones in which inner ring support according to *the supports system 4-5* (on four rolling elements for BSE support system, and five rolling elements for BSO support system). In other cases at bearing NU 2205, calculation by the proposed method gives much more accurate results, compared to Stribeck's numbers.

On the dash-dot line, we can notice a higher number of peaks. These peaks correspond to zones with a different support system of the inner ring. In zones where a new rolling element engages with rings, the error of calculation decreases to zero. By further increase of load, it comes to some maximum value, so again it is lowered to zero in the new boundary zone when new rolling elements engage the rings. According, peaks on the dash-dot line are occurring in zones that are farthest from the zones of boundary bearing deflection. Also, we can notice that the height of peaks on the dot-dash lines decreases with the increase of external load.

The precision of the proposed method for the ball 6206 bearing is somewhat less. However, for the normal values of clearance (5-20 μm) [32], which are the most commonly encountered at practice, the zone of precision of results for the proposed method is much wider compared to Stribeck's method. From Fig. 21-25. it can be seen that Stribeck's method gives more precise results for loads of maximum 2000 N, which is the case at bearing with a clearance of 20 μm . In this zone, the *support system 3-4* occurs. Above this zone, and up until to the static rating of the bearing, which is 11200 N, the calculation by application of the recommended method gives more accurate results. For the bearings in which internal clearance is higher than 40 μm , Stribeck's method is more recommendable. These clearances fall under class C5 according to ISO 5753-1 [19].

5. CONCLUSION

The aim of the research presented in this paper is the verification of a new model for calculating the load carried by the most loaded rolling element in rolling bearings with internal radial clearance. The proposed model was verified on the example of the two different bearing types: the radial ball bearing 6206 and the roller bearing NU 2205 EC. By analysis of obtained results, it can be concluded the following:

- For the boundary values of the external load, which correspond to the boundary deflection of the bearing, the calculation by application of load factors gives absolutely accurate results.
- The results show extremely high accuracy in zones that are relatively close to the boundary values of the external load. The higher the number of active rolling elements, the accuracy of results is greater.
- Also, the width of the high precision zones increases when increasing the number of active rolling elements in the bearing.
- The proposed calculation provides more accurate results in the bearings with a larger total number of rolling elements because in these bearings it is easier to achieve a higher number of active rolling elements.

The proposed model was compared to a calculation based on Stribeck's numbers. Both calculations are based on simple mathematical operations, do not require the usage of computers and they are very useful for the fast calculations and fieldwork.

As opposed to Stribeck, the model proposed in this paper takes into account the influence of internal clearance size on the load distribution within the bearing. For the loads that allow support on the maximum possible number of rolling elements, the calculation using the load factor gives much more accurate results compared to Stribeck's method.

In relation to that, the high values of internal radial clearance unfavourably influence the accuracy of results according to the proposed method. For the bearings in which the size of clearance corresponds to class C5, Stribeck's method gives more accurate results for some load ranges. Stribeck's method also gives more accurate results for the bearings with zero clearance.

At other classes of clearance, the method proposed in this paper provides the significantly more accurate results. This is especially true for the bearings with normal values of the inner clearance. The bearings with this class of clearance are the most frequently encountered in practice.

At the bearings with a relatively large number of rolling elements (more than 12), the proposed method gives more accurate results for all classes of clearance.

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Availability of data and materials

All data generated or analysed during this study are included in this published article.

Competing interests

The author declare that they have no competing interests.

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APPENDIX A: THE LOAD FACTOR OF THE MOST LOADED ROLLING ELEMENT

Tables A1 and A2 show the values of the load factor of the most loaded rolling element for the radial ball bearing and roller bearing, respectively. The detailed procedure for the determination the values of the load factor of the most loaded rolling element is described in [4]. The values of the load factor are given in relation to the different number of active rolling elements and a total number of rolling elements (z).

Table A1. Load factor (k_{maxq}) of the most loaded rolling element for the radial ball bearing

z	z _{sn}	z _{sp}	γ	The number of rolling elements at loaded zone (q)							
				3	4	5	6	7	8	9	10
3	1	2	120,00								
4	1	2	90,00								
5	3	2	72,00	1.0000							
6	3	2	60,00	1.0000							
7	3	4	51,43	1.0000	0.5550						
8	3	4	45,00	1.0000	0.5412						
9	5	4	40,00	1.0000	0.5321	0.5182					
10	5	4	36,00	1.0000	0.5257	0.5010					
11	5	6	32,73	1.0000	0.5211	0.4887	0.3892				
12	5	6	30,00	1.0000	0.5176	0.4796	0.3780				
13	7	6	27,69	1.0000	0.5150	0.4727	0.3694	0.3510			
14	7	6	25,71	1.0000	0.5129	0.4673	0.3628	0.3412			
15	7	8	24,00	1.0000	0.5112	0.4630	0.3575	0.3335	0.2913		
16	7	8	22,50	1.0000	0.5098	0.4595	0.3533	0.3273	0.2842		
17	9	8	21,18	1.0000	0.5087	0.4566	0.3498	0.3222	0.2783	0.2655	
18	9	8	20,00	1.0000	0.5077	0.4542	0.3469	0.3180	0.2735	0.2594	
19	9	10	18,95	1.0000	0.5069	0.4522	0.3444	0.3145	0.2695	0.2542	0.2314
20	9	10	18,00	1.0000	0.5062	0.4505	0.3424	0.3115	0.2661	0.2499	0.2266

Table A2. Load factor (k_{maxq}) of the most loaded rolling element for the roller bearing

z	z _{sn}	z _{sp}	γ	The number of rolling elements at loaded zone (q)							
				3	4	5	6	7	8	9	10
3	1	2	120,00								
4	1	2	90,00								
5	3	2	72,00	1.0000							
6	3	2	60,00	1.0000							
7	3	4	51,43	1.0000	0.5550						
8	3	4	45,00	1.0000	0.5412						
9	5	4	40,00	1.0000	0.5321	0.4858					
10	5	4	36,00	1.0000	0.5257	0.4696					
11	5	6	32,73	1.0000	0.5211	0.4580	0.3705				
12	5	6	30,00	1.0000	0.5176	0.4495	0.3592				
13	7	6	27,69	1.0000	0.5150	0.4430	0.3506	0.3276			
14	7	6	25,71	1.0000	0.5129	0.4379	0.3440	0.3178			
15	7	8	24,00	1.0000	0.5112	0.4338	0.3387	0.3102	0.2744		
16	7	8	22,50	1.0000	0.5098	0.4306	0.3345	0.3041	0.2670		
17	9	8	21,18	1.0000	0.5087	0.4279	0.3310	0.2991	0.2611	0.2476	
18	9	8	20,00	1.0000	0.5077	0.4256	0.3281	0.2950	0.2562	0.2414	
19	9	10	18,95	1.0000	0.5069	0.4237	0.3257	0.2916	0.2522	0.2363	0.2171
20	9	10	18,00	1.0000	0.5062	0.4221	0.3236	0.2887	0.2488	0.2320	0.2122