Static friction coefficient model of joint surface based on the modified fractal model and experimental investigation

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Static friction coefficient model of joint surface based on the modified fractal model and experimental investigation

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Abstract:

It has been widely recognized that influences of static friction characteristics in a bolted joint on pre-tightening and service reliability. This study proposed a static friction coefficient model of joint surfaces based on the modified three-dimensional fractal model. A method of measuring the static friction coefficient of joint surfaces was investigated to verify the accuracy of model. The results showed that calculated value of the model and measured value of experiment are highly matched. Further, the
influential mechanisms of material, pressure and surface topography were investigated. The relative error between experimental results and model calculations are all less than 15%. This research provides a theoretical basis for subsequent research on the friction characteristics of joint surfaces and connection reliability of bolted joints.

Key words: joint surface, modified three-dimensional fractal model, static friction coefficient model, experiment verification

1. Introduction

Bolted joints have been widely used to connect the parts or components of computer numerical control (CNC) machine tools. However, the bolt preload relaxation and self-loosening of bolted joints subjected to external excitation are inevitable, which increase the risk of machine tool reliability degradation. The static friction coefficient and initial preload are main factors affecting reliability of bolted joints[1, 2]. Also, existing studies showed that friction coefficient is a key factor affecting initial preload. Therefore, it is an important way to improve the structural reliability by controlling static friction coefficient in design stage.

It is well known that 40%-60% of stiffness and 90% of damping in machine tool structure are caused by joint surface[3], so it is very important to study the friction characteristics of joint surface. The static friction coefficient is one of the indispensable parameters in friction characteristic research. Its concept was first given by Leonardo da Vinci[4] Out. Then, research on characteristics of static friction
coefficient of joint surface has attracted much attention.

Greenwood and Williamson[5] proposed a combined surface contact model (G-W model), assuming that the deformation of a single asperity satisfies Hertz contact theory and the height of the asperity obeys Gaussian random distribution. On the basis of classic G-W model, Chang[6] et al. established coefficient model of the metal joint by considering the tangential deformation resistance of the elastic node. Luo[7] et al. used the fractal geometry theory proposed by Majumdar and Bhushan[8] in 1991 to calculate static friction coefficient. Through the finite element analysis of the elastoplastic contact and adhesion state of a single asperity, Kogut[9] proposed an elastoplastic static friction model. You[10] established the statistical calculation model for static friction of joint surface by considering different deformation conditions of asperities, such as complete elasticity, elasto-plasticity and complete plasticity. Tian [11-14] et al. improved the fractal theory to establish an fractal model of normal load, friction force and static friction coefficient based on Luo[7]. Zhang[15-17] et al. established a fractal model of joint surface friction by considering the three deformation mechanisms of asperity's complete elasticity, elasto-plasticity and complete plasticity, also the influence of different factors on contact state of bonding surface was explored. On the basis of traditional M-B model, Li[18-19] et al. introduced the elastoplastic stage of asperities to establish a static friction coefficient model. Subsequently, this model has been revised by considering the domain expansion factor and the nonlinear relationship between relevant factors and static friction coefficient was obtained through numerical simulation. Recently, Zhang[20]
et al. proposed a scale-related three-dimensional fractal model of static friction of joint surface, which solved the inconsistency between existing fractal model or statistical model of static friction coefficient and test results.

Nowadays, most researches focus on establishing or revising the static friction coefficient model based on statistical theory or fractal theory. Then mathematical simulation analysis was performed to discuss the influence of fractal parameters and loads on the static friction coefficient. There is a lack of verification on the accuracy of static friction coefficient theoretical model, especially using experiment methods.

Regarding the measurement of static friction coefficient, although there are many types of friction and wear testing machines, but the measurement of static friction was few. At present, the measurement principle of static friction force can be summarized into two categories[21]: One category is to gradually increase the thrust, tension or torque until the upper and lower specimens have a relative displacement, the static friction coefficient are obtained by calculating measured force. The second type uses the inclined plane principle to gauge static friction coefficient, angle of flat plate is gradually increased until the upper and lower samples start to slide relatively. At this time the tangent value of angle is the maximum static friction coefficient.

In this paper, a static friction coefficient model of metal joint surface is derived based on the improved three-dimensional fractal theory while considering domain expansion factor and the elastoplastic stage of asperities. A method of measuring static friction coefficient of joint surface was proposed to verify the accuracy of model, then lots of experiments considering the influence of materials, pressures and surface
morphologies were carried out. Error analysis of the model was performed to explore the reasons for errors and best applicable working conditions of the model. The research can effectively predict the friction coefficient of the joint surface and provide a theoretical basis for bolt assembly and reliability research.

2. Materials and Methods

2.1 Materials

Iron sheet, aluminum sheet, copper sheet, titanium sheet, all provided by Dingyuan Metal Products Co. Ltd. SUS304 sheet with three different surfaces of ordinary industrial board, wire drawing, and mirror, provided by Zhongzhiyuan Stainless Steel Co. Ltd. The size of sheets are all 20mm*20mm*1mm.

2.2 Scanning of workpiece surface topography

The cleaned workpieces were measured on NANOVEA three-dimensional non-contact surface profiler. A 40um probe is used to scan the surface of workpiece. The scanning area is 2*2mm and step length is 20um. The scanning results are post-processed and a txt surface data file is exported.

2.3 Fractal parameter acquisition of workpiece surface

In this paper, the structure function method is adopted to solve the fractal parameters of workpiece surface. The surface topography data of workpiece scanned by three-dimensional surface topography instrument is imported into the pre-written
program (Matlab). Three-dimensional fractal dimension (D) and fractal scale length parameters (G) of workpiece surface can be calculated. This provides a data basis for subsequent calculation of static friction coefficient model.

2.4 Experimental measurement of static friction coefficient

A universal tribometer (UMT-3, BRUKER) was used to observe the tribological behavior of SUS304, aluminum, iron, copper and titanium. One plate is used to rub another plate with the same material and the reciprocating movement mode was chosen. The set movement speed was 10 mm/s and the initial normal load was 24 MPa. The temperature was controlled at 25°C during the sliding experiment, considering the average comfort temperature.

2.5 Technical route of this article

The surface parameters D and G of workpiece were firstly obtained through the structure function method, based on the surface topography data measured by three-dimensional profiler. Then, static friction coefficient was calculated by according to derivation formula (40). At the same time, the static friction coefficient was also measured by frictional experiments. At last, the accuracy of the theoretical model was verified by comparing the static friction coefficient results acquired by two ways. The experimental errors were further analyzed.
3. Results and Discussion

3.1 Theoretical model derivation

3.1.1 Revision of three-dimensional fractal contact theory

Mandelbrot[22, 23] studied the surface topography of the earth, he believed that the area distribution of contact points of asperities on metal surface is similar to the distribution of islands on the earth. Majumdar and Bhushan established the classic M-B fractal contact theory based on this research. The model uses two parameters to describe contour features, namely the fractal dimension D and the scale factor G. After that, more attractions were paid to improve the M-B contact model to increase its accuracy. Among them, in order to more accurately describe the relationship
between the actual contact area of the largest contact point and the actual contact area of the rough surface, Ji[24] et al. introduced a parameter to modify the contact point distribution function which is called the domain expansion factor. Since the MB fractal contact model ignores the elasto-plastic deformation stage of the asperity, Liou[25] proposed a critical area expression for three deformation stages, which makes the analysis of the contact mechanism of the joint surface more perfect. In this study, the domain expansion factor and the critical expression method of elastoplastic deformation are introduced into three-dimensional fractal theory of the joint surface. Then the static friction coefficient model is established based on this theory.

In this paper, based on the research of Wang and Liou, the domain expansion factor and the critical representation method of elastic-plastic deformation are introduced into three-dimensional fractal theory of joint surface at the same time, then normal and tangential contact loads between joint surfaces are deduced. Based on this theory, a static friction coefficient model of joint surface is established.

### 3.1.2 Normal contact load of joint surface

Asperities between joint surfaces have three deformation stages: elasticity, plasticity and elasto-plasticity. Through analysis of the contact area of the asperity deformation, the expression of the critical area between each stage is divided, which provides basis for normal contact load calculation of the joint surface.

The analysis starts from contact of asperities, then normal and tangential contact loads of joint surface are derived, finally static friction coefficient model of joint
Asperity contact analysis

The machined surface has fractal characteristics. Its characteristics of self-similarity between the local and the whole is mathematically continuous and undirected everywhere[26]. The rough surface profile is expressed as[27],

\[
z(x, y) = L \left( \frac{G}{L} \right)^{(D-2)} \left( \frac{\ln \gamma}{M} \right)^{\frac{1}{2}} \times \sum_{m=1}^{M} \sum_{n=0}^{n_0} \gamma^{-(D-2)n} \left\{ \cos \phi_{m,n} - \cos \left( \frac{2\pi\gamma^m}{L} \left( x^2 + y^2 \right)^{\frac{1}{2}} \cos \left( \tan^{-1} \left( \frac{y}{x} \right) - \frac{\pi m}{M} \right) + \phi_{m,n} \right) \right\}
\]

Where, \( D \) is fractal dimension. \( G \) is fractal scale parameter. \( L \) is fractal sample length. \( z \) is the height of rough surface profile. \( x, y \) are surface sampling length coordinates. \( \gamma \) is spectral density scaling ratio which is a constant greater than 1. \( M \) is the number of overlapping ridges on the structural surface. \( L_s \) is cross-sectional area diameter. \( \phi_{m,n} \) is the random phase. \( n_0, n_{\text{max}} \) are the sequences corresponding to the lowest and highest cutoff frequencies respectively, expressed as \( n_0 = \text{int}[\log(L/2\gamma)/\log \gamma], n_{\text{max}} = \text{int}[\log(L/L_s)/\log \gamma] \).

The function for a single asperity on a metal surface can be expressed as,

\[
z_0(x) = G^{(D-2)} \left( \ln \gamma \right)^{\frac{1}{2}} \left( 2r \right)^{(3-D)} \cos \phi_{1,n_0} - \cos \left( \frac{\pi x}{r} - \cos \phi_{1,n_0} \right)
\]

When the asperities are in contact with each other, it can be simplified as shown in Figure 2. In this model, the contact between the two surfaces in contact is reduced to a rigid planar with a rough surface, wherein the roughened surface profile of fractal
parameters described by the W-M function. Asperity deformation of the roughened surface is divided into three stages, namely elastic deformation, plastic deformation and elastic-plastic deformation. Its equivalent elastic modulus $E$ is expressed as,

$$E = \frac{1}{\frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2}}$$  \hspace{1cm} (3)

Where, $v_1, v_2$ are the Poisson's ratio and $E_1, E_2$ are elastic modulus of the two contact materials.

![Figure 2 Asperity contact model](image)

The normal deformation of an asperity on a rough surface can be expressed as

$$\delta = 2G^{D-2}(\ln \gamma)^{\frac{1}{2}}(2r')^{3-D}$$  \hspace{1cm} (4)

The theoretical contact area $a$ of a single asperity $a'$ is

$$a'=\pi r'^2 = 2\pi R\delta$$  \hspace{1cm} (5)

Then curvature $R$ of a single asperity can be expressed as

$$R = \frac{a'^{(D-1)/2}}{2^{(5-D)}\pi^{(D-3)/2}G^{(D-2)}(\ln \gamma)^{1/2}}$$  \hspace{1cm} (6)

Critical normal deformation and cross-sectional area[28] between the elastic
deformation and elastic-plastic deformation of asperity can be expressed as

\[ \delta_i = \left( \frac{\pi kH}{2E} \right)^2 R \]

(7)

\[ a'_i = \left[ \frac{2^{1-2D} G^{2(D-2)} (\ln \gamma)E^2}{\pi^{(4-D)(kH)^2}} \right]^{1-D-2} \]

(8)

Where, \( H = 2.8Y, k = 0.454 + 0.41\nu \).

According to the research of Liou and others on the theory of micro-contact model[26], critical normal deformation and cross-sectional area of elasto-plastic deformation and plastic deformation of asperity are

\[ \delta_2 = 76.4\delta_1 = 76.4\left( \frac{\pi kH}{2E} \right)^2 R \]

(9)

\[ a'_2 = \frac{1}{76.4^{1/(D-2)}} a'_1 \]

(10)

Micro size distribution function of the point of contact by introducing domain spreading factor[24] may be expressed as

\[ n\left( a' \right) = \frac{D-1}{2} \psi^{(3-D)/2} a'^{(D-1)/2} a'^{-(D-1)/2} \]

(11)

Where, \( a'_i \) is the truncated area of the largest elastic micro-contact.

In the formula, the domain expansion factor can be solved by the transcendental equation

\[ \psi^{(3-D)/2} \left( 1 + \psi^{(1-D)/2} \right)^{(D-3)/(D-1)} = \frac{3-D}{D-1} \]

(12)

**Real Contact Area of Joint Surface**

The degree of deformation of asperities is distinguished by the size of
cross-sectional area of asperities. For a single asperity, when complete elastic deformation occurs, the relationship between real contact area and cross-sectional area satisfies \( a = a' / 2 \), range of \( a' \) is \( a_i < a' < a_i \), so the real contact area of the elastic deformation area on the joint surface is,

\[
A_e = \int_{a_i}^{a'} \frac{an(a')}{a} da'
= \frac{D-1}{2(3-D)} \psi^{(3-D)/2} a_i^{(3-D)/2} \left( a_i^{(3-D)/2} - a'_i^{(3-D)/2} \right)
\]  

(13)

when \( a' \) is range of \( 0 < a' < a_i \), asperities undergo complete plastic deformation, the relationship between real contact area and cross-sectional area satisfies \( a = a' \), the total plastic contact area can be expressed as,

\[
A_p = \int_{a_i}^{a'} \frac{an(a')}{a} da'
= \frac{D-1}{3-D} \psi^{(3-D)/2} a_i^{(3-D)/2} a_i^{(3-D)/2}
\]  

(14)

When the asperity is in the elastoplastic stage, the relationship between its real contact area and the theoretical cross-sectional area[27] is,

\[
a = a' / 2 \cdot \left( \delta / \delta_i \right)^{0.6} = a'^{(11/5 - 3D/5)} / H_1
\]  

(15)

Where, \( H_1 \) is parameters related to fractal parameters and material properties.

\[
H_1 = \frac{\pi^{(12/5 - 3D/5)} (kH)^{6/5}}{2^{(28/5 - 6D/5)} G^{(6D/5 - 12D/5)} (\ln \gamma)^{3/5}} E^{6/5}
\]  

(16)

At this time \( a' \) is range of \( a' < a_2 \), then the real contact area of joint surface in elastic-plastic deformation region is expressed as,

\[
A_{ep} = \int_{a_2}^{a_i} \frac{an(a')}{a} da'
= \begin{cases} 
\frac{(D-1)}{2H_1(2.7 - 1.1D)} \psi^{(3-D)/2} a_i^{(3-D)/2} \left( a_i^{(2.7 - 1.1D)} - a'_i(2.7 - 1.1D) \right) & D \neq 27/11 \\
\frac{(D-1)}{2H_1} \psi^{(3-D)/2} a_i^{(3-D)/2} \ln \left( \frac{a'_i}{a_2} \right) & D = 27/11 
\end{cases}
\]  

(17)
Calculation of Normal Load of Joint Surface

In practical application of fractal theory, normal load of the joint surface determines the real contact area and dynamic characteristic parameters. Therefore, the accurate modeling of the normal contact load of joint surface is very important.

Combined with Hertz theory[29], normal load of asperity in the elastic stage can be expressed as,

\[ f_e = 4ER^{1/2} \delta^{3/2} / 3 \]  \hspace{1cm} (18)

Then the total elastic load of the joint surface can be calculated by the integral of the critical cross-sectional area.

\[ P_e = \int_{a_0}^{a_i} f_e n(a') da' \]

\[ = \frac{2^{(1-2D)/2}}{3\pi^{(4-D)/2}} \frac{D - 1}{5 - 2D} (\ln \gamma)^{1/2} G^{(D-2)} E \]

\[ \times \psi^{3/2} \alpha_i^{(D-1)/2} \left( a_i^{1/2} (5 - 2D)^{1/2} - a_i^{(5-2D)/2} \right) \]

\[ D \neq 2.5 \]

\[ 2\pi^{-3/4} (\ln \gamma)^{1/2} G^{1/2} E \psi^{1/4} \alpha_i^{-3/4} \ln \left( \frac{a_i}{\alpha_i} \right) \]

\[ D = 2.5 \]

When asperities in plastic deformation, normal load can be expressed as

\[ f_p = Ha' \]  \hspace{1cm} (20)

Then the total elastic load of the joint surface can be solved by the integral of the critical cross-sectional area.

\[ P_p = \int_{a_0}^{a_i} f_p n(a') da' = \frac{H(D - 1)a_i^{(D-1)/2}}{3 - D} \psi^{(3-D)/2} \alpha_i^{(3-D)/2} \]

\[ (21) \]

When asperity in elastoplastic deformation, normal load can be known from the literature[25] is
\[ f_{ep} = \frac{2}{3} kH \pi R \delta \left( \frac{\delta}{\delta_1} \right)^3 = H_2 a^{(1.76 - 0.38D)} \]  \hspace{1cm} (22)

Where, \( H_2 \) is coefficients related to material properties and fractal parameters of the joint surface.

\[ H_2 = \frac{2^{(4.18 - 0.76D)} (kH)^{0.24} E^{-0.76} G^{0.76(D - 2)} (\ln \gamma)^{0.38}}{3^{(1.52 - 0.38D)}} \]  \hspace{1cm} (23)

Then total normal load in the elastoplastic stage of asperities is calculated by definite integral.

\[
P_{ep} = \int_{a_1}^{a_2} f_{ep} \pi a' \\, da'
\]

\[
= \left[ \frac{H_2 (D - 1) \psi^{(3-D)/2} \frac{a_1^{(113/50 - 22D/25)}}{2(113/50 - 22D/25)} - a_2^{(113/50 - 22D/25)}}{69/88 H_2 \psi^{19/88} a_1^{69/88} \ln \left( \frac{a_1}{a_2} \right)} \right] \quad D \neq \frac{113}{44} \hspace{1cm} (24)
\]

Finally, total normal contact load of the joint surface can be obtained by adding the normal loads of the asperity in the three deformation stages.

\[ P = P_e + P_{ep} + P_p \]  \hspace{1cm} (25)

3.1.3 Tangential contact load of joint surface

When the asperity generates plastic deformation under the normal load, plastic flow will occur. It lost the ability to withstand tangential loads. Therefore, When calculating the tangential load of the asperity, there is no need to consider the plastic stage[30].

According to Hamilton hypothesis[31], assuming \( Q \div P_e > 0.3 \), the three principal stresses of the contacting surface are respectively:
\[ \sigma_1 = \frac{(1-2\nu)P_e}{2\pi r^2} + \frac{3Q}{4r^2} \left( \frac{\nu}{4} + 1 \right) \]  
(26)

\[ \sigma_2 = 0 \]  
(27)

\[ \sigma_3 = \frac{9\nu Q}{16r^2} - \frac{(1-2\nu)P_e}{2\pi r^2} \]  
(28)

Known from Tresca's yield condition[32]:

\[ \sigma_1 - \sigma_3 = \sigma_y \]  
(29)

Substitute available,

\[ Q = \frac{8r^2}{6-3\nu} \sigma_y + \frac{8(2\nu-1)}{\pi(6-3\nu)} P_e = \frac{8\sigma_y}{\pi(6-3\nu)} a + \frac{8(2\nu-1)}{\pi(6-3\nu)} P_e \]  
(30)

When only elastic deformation is considered, the tangential force can be expressed as,

\[ Q_e = \int_{a_1}^{a_i} Q_e n(a') da' \]

\[ = \frac{8\sigma_y}{\pi(6-3\nu)} \int_{a_1}^{a_i} an(a') da' + \frac{8(2\nu-1)}{\pi(6-3\nu)} \int_{a_1}^{a_i} P_e n(a') da' \]  
(31)

Substitute \( a = a'/2 \), eq(13)and eq(19) into the calculation to get,

\[ Q_y = \frac{8\sigma_y}{\pi(6-3\nu)} W + \frac{8(2\nu-1)}{\pi(6-3\nu)} X \]  
(32)

Where,

\[ W = \frac{D-1}{2(3-D)} \psi^{(3-D)/2} a_i^{(D-1)/2} \left( a_i^{(3-D)/2} - a_i^{(3-D)/2} \right) \]  
(33)

\[ X = \begin{cases} 
(D-1)^2 (1-2D) / (5-2D) 3\pi^{(3-D)/2} \ln \gamma^{1/2} G^{(D-2)} & D \neq 2.5 \\
 2\pi^{3/4} (\ln \gamma)^{1/2} G^{1/2} E \psi^{1/4} a_i^{1/4} \ln \left( \frac{a_i}{a_1} \right) & D = 2.5 
\end{cases} \]  
(34)

When considering elastoplastic deformation, the tangential force is expressed as,
\[
Q_{ep} = \int_{a_2}^{a_1} Q_{ep} n(a') da'
= \frac{8\sigma_y}{\pi(6-3\nu)} \int_{a_2}^{a_1} an(a') da' + \frac{8(2\nu-1)}{\pi(6-3\nu)} \int_{a_2}^{a_1} P_{ep} n(a') da'
\] (35)

Substitute eq(17) and eq(24) into the calculation to get

\[
Q_{ep} = \frac{8\sigma_y}{\pi(6-3\nu)} U + \frac{8(2\nu-1)}{\pi(6-3\nu)} V
\] (36)

Where,

\[
U = \begin{cases}
\frac{(D-1)}{2H_1(2.7-1.1D)} \psi^{(3-D)/2} a_i^{(D-1)/2} & D \neq 27/11 \\
\times \left( a_1^{(27/10-11D/10)} - a_2^{(27/10-11D/10)} \right) & D = 27/11
\end{cases}
\] (37)

\[
V = \begin{cases}
\frac{H_2(D-1)\psi^{(3-D)/2} a_i^{(D-1)/2}}{2(113/50-22D/25)} & D \neq 113/44 \\
\times \left( a_1^{(113/50-22D/25)} - a_2^{(113/50-22D/25)} \right) & D = 113/44
\end{cases}
\] (38)

Finally, the tangential contact load of the joint surface can be obtained as

\[
Q = Q_n + Q_{ep}
\] (39)

### 3.1.4 Static friction coefficient model of joint surface

Assuming that normal load of the joint surface is represented by P, when the joint surface has a relative sliding tendency, asperities in elastic and elasto-plastic stage will yield. At this time, static friction force can be used as the total tangential load Q to represent. At last, static friction coefficient of joint surface can be expressed as
3.2 Experimental verification

In this study, a method for measuring the static friction coefficient of the joint surface is investigated to verify the accuracy of the model. The UMT-3 with designed fixture is used. The upper and lower test pieces are respectively fixed to the upper sensor module and the lower reciprocating module. As shown in Figure 3, the reciprocating mode is used to measure the static friction coefficient. The friction coefficient at the moment of the reciprocating mode movement is recognized as the static friction coefficient of the joint surface.

![Figure 3 Schematic diagram of static friction coefficient measurement](image)

3.2.1 Experimental verification of static friction coefficient model using different materials

In order to explore the accuracy and effectiveness of static friction coefficient model, experiments are carried out using workpieces of different metal materials. The
SUS304, aluminum, iron, copper, and titanium workpieces used for experiment were firstly ultrasonic cleaned for 15 minutes. Then, the pretreated samples were place onto the NANOVEA three-dimensional non-contact surface profilometer to scan the surface topography. The scanning area is 2*2mm and the scan step length is 20um. The scanning results are post-processed to export the surface topography data file in txt format. Then fractal parameters D and G of the workpiece surface were calculated using the structure function method through the Matlab program. The fractal parameters average value of the upper and lower contact surfaces was obtained. The results of different materials are shown in the table 1(a) to 1(e). The five different metal materials characteristics are shown in Table 2.

Through method described in Chapter 2, the surface topography of workpiece is scanned by a three-dimensional topography instrument, and then the surface fractal parameters can be calculated by structure function method. In static friction coefficient model, the parameters that need to be input are fractal parameters D and G of workpiece surface, the elastic modulus of material and the Poisson's ratio, and when normal load of 10KN is input, the tangential load can be reversed, so the static friction coefficient of the combined surface can be calculated. At the same time, the static friction coefficient of five different materials were experimental tested through UMT-3 with the normal load of 10KN. Eight experiments were carried out for each material, the two sets of data with the largest deviation were removed and the remaining six sets of static friction coefficient measurement data were recorded. The experimental and theoretical values of static friction coefficient are shown in Figure
4(a) to 4(e). Error of static friction coefficient between model calculation value and experimental measurement value is shown in Table 3.

Table 1(a) Fractal parameters D and G of the workpiece surface (SUS304)

<table>
<thead>
<tr>
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<th>2</th>
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<th>4</th>
<th>5</th>
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<td>D</td>
<td>2.7586</td>
<td>2.7719</td>
<td>2.7639</td>
<td>2.7563</td>
<td>2.7589</td>
<td>2.7766</td>
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<tr>
<td>G</td>
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<td>2.9706e-08</td>
<td>2.5926e-08</td>
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</tbody>
</table>

Table 1(b) Fractal parameters D and G of the workpiece surface (Al)

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<td>D</td>
<td>2.8057</td>
<td>2.7720</td>
<td>2.6852</td>
<td>2.7059</td>
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<td>G</td>
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<td>1.6253e-08</td>
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<td>2.6895e-08</td>
<td>2.6410e-08</td>
</tr>
</tbody>
</table>

Table 1(c) Fractal parameters D and G of the workpiece surface (Fe)

<table>
<thead>
<tr>
<th>Times</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.7268</td>
<td>2.7713</td>
<td>2.7363</td>
<td>2.7911</td>
<td>2.7526</td>
<td>2.7455</td>
</tr>
<tr>
<td>G</td>
<td>2.2644e-09</td>
<td>1.7530e-09</td>
<td>2.5693e-09</td>
<td>1.9623e-09</td>
<td>2.3462e-09</td>
<td>2.2581e-09</td>
</tr>
</tbody>
</table>

Table 1(d) Fractal parameters D and G of the workpiece surface (Cu)

<table>
<thead>
<tr>
<th>Times</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.7408</td>
<td>2.7550</td>
<td>2.7623</td>
<td>2.7542</td>
<td>2.7658</td>
<td>2.7469</td>
</tr>
<tr>
<td>G</td>
<td>3.4416e-09</td>
<td>1.6790e-09</td>
<td>3.6525e-09</td>
<td>2.9360e-09</td>
<td>2.5481e-09</td>
<td>3.1261e-09</td>
</tr>
</tbody>
</table>

Table 1(e) Fractal parameters D and G of the workpiece surface (Ti)

<table>
<thead>
<tr>
<th>Times</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.8630</td>
<td>2.7779</td>
<td>2.7845</td>
<td>2.7156</td>
<td>2.7896</td>
<td>2.7562</td>
</tr>
<tr>
<td>G</td>
<td>2.4844e-09</td>
<td>2.1506e-09</td>
<td>2.5963e-09</td>
<td>2.4852e-09</td>
<td>2.3159e-09</td>
<td>2.6245e-09</td>
</tr>
</tbody>
</table>
Table 2 Five metal material properties

<table>
<thead>
<tr>
<th>materials</th>
<th>SUS304</th>
<th>Al</th>
<th>Fe</th>
<th>Cu</th>
<th>Ti</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic Modulus (GPa)</td>
<td>190</td>
<td>71.7</td>
<td>100</td>
<td>119</td>
<td>102.04</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.305</td>
<td>0.330</td>
<td>0.211</td>
<td>0.326</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 4 Comparison of the static friction coefficient of the joint surface of different materials (a:SUS304, b:Al, c:Fe, d:Cu, e:Ti)
Table 3 The error of static friction coefficient between model calculation value and experimental measurement value of different materials

<table>
<thead>
<tr>
<th>Materials</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Average Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>SUS304</td>
<td>11.013%</td>
<td>9.767%</td>
<td>10.390%</td>
<td>8.051%</td>
<td>10.798%</td>
<td>9.955%</td>
<td>9.996%</td>
</tr>
<tr>
<td>Fe</td>
<td>9.417%</td>
<td>9.647%</td>
<td>10.805%</td>
<td>12.326%</td>
<td>9.485%</td>
<td>9.198%</td>
<td>10.146%</td>
</tr>
<tr>
<td>Cu</td>
<td>10.042%</td>
<td>12.200%</td>
<td>9.495%</td>
<td>10.571%</td>
<td>9.960%</td>
<td>11.927%</td>
<td>10.699%</td>
</tr>
<tr>
<td>Ti</td>
<td>11.054%</td>
<td>13.287%</td>
<td>11.458%</td>
<td>10.412%</td>
<td>11.083%</td>
<td>14.218%</td>
<td>11.919%</td>
</tr>
</tbody>
</table>

The experimental results show that the average values of static friction coefficient of SUS304, aluminum, iron, copper and titanium are 0.224, 0.254, 0.474, 0.474, 0.406, respectively. The calculated values using the static friction coefficient model are corresponding 0.240, 0.275, 0.508, 0.524 and 0.454. The error of static friction coefficient between model calculation value and experimental measurement value of the five materials are 9.996%, 13.531%, 10.146%, 10.699%, 11.919%, respectively. All errors are less than 13.6%.

Based on these results, the average error of SUS304 material is smallest, while aluminum material shows the largest average error. The reason is that the elastic modulus of stainless steel is large. The ability to resist deformation is strong. Most of asperities are in the stage of elastic and elasto-plastic. However, the elastic modulus of aluminum is small. Its ability to resist deformation is relatively weak. Most of the asperities occurred plastic deformation. In the calculation model, it is assumed that there is no tangential force in the stage of the asperity, therefore, the more asperity with plastic deformation, the greater error in the calculated value of static friction coefficient.
3.2.2 Experimental verification of static friction coefficient model with different normal load

According to the results above, the error of static friction coefficient between model calculation value and experimental measurement value of SUS304 material is the smallest. In addition, SUS304 material is widely used because of the good corrosion resistance, heat resistance and hot workability. It is used to conduct experiments to verify the static friction coefficient model with different pressure gradients. The fractal parameters of the workpiece surface are obtained as shown in Table 4.

The static friction coefficient measurement experiment was carried out. The experimental procedure is the same as that in section 4.2.1. The normal load applied in the experiment was 5, 10, 15, 20, 25KN. Experimental and theoretical values of static friction coefficient are shown in Figure 5(a) to 5(e). Error of static friction coefficient between model calculated value and experimental measured value under different pressures are shown in Table 5.

<table>
<thead>
<tr>
<th>Times</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.7498</td>
<td>2.74255</td>
<td>2.78185</td>
<td>2.78105</td>
<td>2.75465</td>
<td>2.7809</td>
</tr>
<tr>
<td>G</td>
<td>7.67535E-08</td>
<td>3.73855E-08</td>
<td>6.80185E-08</td>
<td>4.93685E-08</td>
<td>4.42595E-08</td>
<td>5.6071E-08</td>
</tr>
</tbody>
</table>
Figure 5 Comparison of the static friction coefficient of the joint surface under different pressures (a: 5KN, b: 10KN, c: 15KN, d: 20KN, e: 25KN)

Table 5 The error of static friction coefficient between model calculated value and experimental measured value under different pressures

<table>
<thead>
<tr>
<th>Load</th>
<th>Times</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>Average Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>5KN</td>
<td>1</td>
<td>9.292%</td>
<td>8.559%</td>
<td>8.145%</td>
<td>9.129%</td>
<td>8.502%</td>
<td>9.502%</td>
<td>8.855%</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>9.426%</td>
<td>9.690%</td>
<td>10.569%</td>
<td>8.421%</td>
<td>10.227%</td>
<td>9.665%</td>
<td>9.667%</td>
</tr>
<tr>
<td>10KN</td>
<td>4</td>
<td>12.548%</td>
<td>11.808%</td>
<td>12.355%</td>
<td>10.135%</td>
<td>13.455%</td>
<td>11.307%</td>
<td>11.935%</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>11.828%</td>
<td>13.356%</td>
<td>14.336%</td>
<td>11.897%</td>
<td>13.131%</td>
<td>13.621%</td>
<td>13.028%</td>
</tr>
</tbody>
</table>
The experimental results show that the average value of the static friction coefficient are 0.230, 0.246, 0.261, 0.275, 0.294 successively under the normal load of 5KN, 10KN, 15KN, 20KN, 25KN. The calculated values based on the static friction coefficient model are 0.236, 0.261, 0.278, 0.307, 0.333, respectively. The error of static friction coefficient between model calculation value and experimental measurement value of different loads are 8.855%, 9.376%, 9.667%, 11.935%, 13.028%, respectively. All errors are less than 13.1%.

The static friction coefficient will increase with the increased normal load, error between experimental value and calculated value displays the same law, the higher load, the larger error. Reason is that increased normal load will increase the contact area of the asperities, which does some help to the friction coefficient increase. Meanwhile, the normal force increase will cause more plastic deformation of the asperity. The reason is same as in 3.2.1, the more asperity with plastic deformation, the greater error in the calculated value of the static friction coefficient.

3.2.3 Experimental verification of static friction coefficient model considering different surface morphologies

In the industry, different surface treatment for SUS304, such as mirror surface, wire drawing, sandblasting, and coloring, are performed, according to the various application requirements. The experiments of SUS304 workpieces were carried out to verify the accuracy of static friction coefficient model influenced by different surface morphologies. The treatments of ordinary cold rolling, wire drawing and mirror
surface for the SUS304 samples were performed. After finishing the surface treatment, it is cleaned and air-dried. The three pretreated surface morphologies were shown in Figure 6(a) to 6(c), and the surface roughness is 4.434, 2.397, and 0.171 respectively. The surface fractal parameters of each group are shown in Table 6(a) to 6(c).

The static friction coefficient experiment was carried out. The normal load applied in the experiment was 10KN. The experimental and theoretical values of the static friction coefficient are shown in Figure 7(a) to 7(c). The error of static friction coefficient between model calculation value and experimental measurement value of different surface treatment is shown in Table 7.

![Figure 6 Three different surface topography patterns](image)

Figure 6 Three different surface topography patterns(a: cold rolling treatment, b: drawing treatment, c: mirror treatment)
Table 6(a) Fractal parameters D and G of the workpiece surface (cold rolling treatment)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.6598</td>
<td>2.66195</td>
<td>2.66405</td>
<td>2.69095</td>
<td>2.69375</td>
<td>2.6541</td>
</tr>
<tr>
<td>G</td>
<td>9.02595E-0</td>
<td>8.80645E-0</td>
<td>8.94135E-0</td>
<td>9.38995E-0</td>
<td>8.38985E-0</td>
<td>8.50985E-0</td>
</tr>
</tbody>
</table>

Table 6(b) Fractal parameters D and G of the workpiece surface (drawing treatment)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.7793</td>
<td>2.7533</td>
<td>2.2826</td>
<td>2.7668</td>
<td>2.76845</td>
<td>2.76055</td>
</tr>
<tr>
<td>G</td>
<td>2.60465E-08</td>
<td>2.2618E-08</td>
<td>3.15035E-08</td>
<td>2.8507E-08</td>
<td>2.61755E-08</td>
<td>4.5427E-08</td>
</tr>
</tbody>
</table>

Table 6(c) Fractal parameters D and G of the workpiece surface (mirror treatment)

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>2.5669</td>
<td>2.52755</td>
<td>2.5611</td>
<td>2.5807</td>
<td>2.57205</td>
<td>2.5493</td>
</tr>
<tr>
<td>G</td>
<td>2.1738E-09</td>
<td>2.9042E-09</td>
<td>8.89935E-09</td>
<td>1.3096E-09</td>
<td>1.62495E-09</td>
<td>5.89455E-09</td>
</tr>
</tbody>
</table>

Figure 7 Comparison of the static friction coefficient of the joint surface of different surface treatment (a: cold rolling treatment, b: drawing treatment, c: mirror treatment)
The experimental results show that the average value of the static friction coefficient is 0.333, 0.245, 0.199 when the surface is not treated, wire drawing treatment, mirror treatment. The calculated values based on the static friction coefficient model are 0.330, 0.274, 0.227. The error of static friction coefficient between model calculation value and experimental measurement value of the three workpiece surfaces are 11.646%, 9.857%, 14.122%. All errors are less than 14.2%.

The Mirror treatment of the workpiece surface significantly reduces the static friction coefficient, but it increases the error of static friction coefficient between model calculation value and experimental measurement value. The reason is that the mirror treatment reduces the surface roughness of the workpiece, so that the surface asperity becomes smooth. At this time, the asperity distribution assumption in the fractal theory is consistent with the assumption of the asperity distribution. The wire drawing treatment reduces the static friction coefficient little, but it can reduce the theoretical model error. The reason is that although the wire drawing treatment does not reduce the surface roughness, but the workpiece surface is very uniformly and the morphology is more regular. Meanwhile, in the fractal theory, the hypothesis of the asperity distribution is more consistent with the actual contact surface, so the error will be reduced.
In a word, the experiments with different materials, different pressure gradients and different surface morphologies verified the accuracy of the proposed static friction coefficient model. Experiments on different materials show that the model error is less than 13.6%. The material properties of the workpiece will affect the model error, with the smaller elastic modulus, the larger error. Experiments with different pressure gradients show that the model error is less than 13.1%. The static friction coefficient increases with the increased normal load, the smaller normal load, the higher accuracy of the static friction coefficient model. Experiments with different surface morphologies show that the model error is less than 14.2%, the smoother surface of the workpiece (the smaller the roughness), the greater the error. The static friction coefficient model proposed in this study is stability enough for the analysis under different materials, loads, and surface topography.

4. Conclusion

In this study, based on the modified three-dimensional fractal theory, the high precision static friction coefficient model of the joint surface considering the domain expansion factor and the elastoplastic deformation stage of the asperity was established. A novel experimental method for measuring static friction coefficient is proposed to verify the accuracy of model. It was obtained that the error of static friction coefficient between model calculation value and experimental measurement value influenced by the material, pressure, and surface topography of the workpiece. All errors are less than 15%. The static friction coefficient model of joint surface is
accurate and reliable. Also the experimental verification method proposed is effective and feasible. By analyzing the error of static friction coefficient between model calculation value and experimental measurement value, it can be concluded that the material with a smaller elastic modulus will have a larger error. When the normal load increases, asperities in plastic deformation stage will increase, which lead to larger errors in calculation of the static friction coefficient. The uniform distribution of the asperities on the surface of the workpiece and the regular morphology can make the hypothesis of asperities in the fractal theory more consistent with the actual contact surface. Therefore, the calculation error of the theoretical model can be reduced. This study provides a theoretical basis for predicting the static friction coefficient of joint surfaces and is of great significance to the reliability research of bolted structures.

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**DECLARATIONS**

**Ethical Approval**
Not applicable.

**Consent to Participate**
Not applicable.

**Consent to Publish**
Not applicable.

**Authors Contributions**
Caixia Zhang and Ying Li are responsible for providing overall research ideas, Xiang Li, Jinlin He and Zhifeng Liu are responsible for the establishment of the theoretical
model, Yanhong Cheng and Ying Li are responsible for experimental data analysis.

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**Conflicts of interest**

There are no conflicts of interest.

**Availability of data and materials**

Not applicable.

**References:**


