Exponential or Pareto: towards an evolution of equal opportunity

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Pareto distribution and exponential distribution are related by the generalized Pareto distribution[1-3], which has been proposed to describe the income structure of the total population[4-5]. The underlying mechanism for driving the Pareto distribution has been known as the Matthew effect of income accumulation[6-10]. Today, the Pareto distribution has been observed universally in the richest class[11-14] (1%~3% of populations); however, this distribution could dominate a larger proportion of populations when the investigation dated back to Renaissance Europe[15], Hungarian medieval society[16], and ancient Egypt[17]. By contrast, the underlying mechanism for driving exponential distribution is due to the equal opportunity of market competition[18-20], which radically differs from the Matthew effect. Here, we empirically find that, during the last 40 years, the income structure of different market-economy countries uniformly exhibits a two-class pattern, in which a larger proportion of populations is evolving to an exponential distribution, while the Pareto distribution is squeezed into a fairly small proportion. In particular, we empirically show how the income structure of China evolved to an exponential distribution after the market-oriented economic reformation. The finding of a larger proportion of populations evolving to an exponential income distribution may reveal a potential trend of human civilization towards equal opportunity.

Through time, roughly speaking, we humans have explored four social structures, from hunter-gatherer societies, slave-based societies, feudal hierarchical societies, to various types and levels of democratic societies. At the early stage of human society, humans' behaviors were dominated by "The law of the jungle", which causes the Matthew effect in material allocations. Nowadays, it has been known that the Matthew effect leads to the Pareto distribution of income[6-10]. At the end of the 19th century, the economist Vilfredo Pareto discovered that[21] the rich part of populations in several European countries, kingdoms and cities for the 15th-19th centuries obeyed a Pareto distribution. Later, Pareto's finding was further supported by investigating the income structure of market-economy countries in the 20th and 21st centuries[11-14, 22]. In particular, there had

been evidence showing that the income structure of an ancient Egypt city in the 14th century BC followed a Pareto distribution[17]. Because slaves have no property right of personal income, this evidence implies that slave-based societies might undergo a Pareto distribution of the total population. As the time came to the end of the 20th century, it had been found that the income structure of market-economy countries around the world uniformly exhibited a two-class pattern[19-20, 23-33]: the great majority of populations obeyed an exponential distribution and only the remaining (richest) part followed a Pareto distribution. Empirical evidence had covered more than 66 countries[19], ranging from Europe to Latin America, North America, and Asia. Recently, Blanchet et al[4-5] proposed to use the generalized Pareto distribution to describe the income structure of the total population, where the generalized Pareto distribution is defined as follows[1-3]:

$$F_\xi(t \geq x) = \left(1 + \xi \frac{x-\mu}{\theta}\right)^{-1/\xi}, \quad (1)$$

where $\xi > 0$, x denotes the income level, and $F_\xi(t \geq x)$ denotes the fraction of population with the income higher than x.

The generalized Pareto distribution (1) is a fairly general family which includes the Pareto distribution and the exponential distribution as two special cases[1-3]. To see this, by ordering $\mu = \theta/\xi$, equation (1) yields the Pareto distribution:

$$F_\xi(t \geq x) = (x/\mu)^{-1/\xi}, \quad (2)$$

where the Pareto exponent is denoted by $1/\xi$, which measures the degree of income inequality[14], that is, a larger Pareto exponent is associated with a lower income inequality. Intuitively, a lower income inequality should correspond to a more equal-opportunity society. Therefore, we anticipate that, as the Pareto exponent $1/\xi \to \infty$ (or $\xi \to 0$), the generalized Pareto distribution (1) yields a distribution being close to equal opportunity:

$$F_0(t \geq x) = \lim_{\xi \to 0} \left(1 + \xi \frac{x-\mu}{\theta}\right)^{-1/\xi} = e^{-\frac{x-\mu}{\theta}}, \quad (3)$$

which is an exponential distribution.

Theoretically, it has been shown that[18-20] the exponential income distribution (3) emerges spontaneously in an idealized market economy with equal-opportunity, which is a decentralized self-organizing economic network, just as described by the Arrow-Debreu general equilibrium model[34]. Here, we briefly introduce such an economic network. To this end, we consider an "N-agent non-cooperative game", where there are N agents, each of whom operates a firm, and each agent's status is equal. In this economic network, all of these firms pursue maximum profit, and all of these agents exchange with each other to obtain maximum satisfaction. Arrow and Debreu have proved that[34], for each agent, if there is free of cost for information transmission and transaction, then all of these agents will reach consensuses on resource allocation (the so-called "general equilibrium"). Tao further proves that[18, 35], if the property right of each firm is endowed to a different agent, then these consensuses lead to the exponential income distribution (3). However, information transmission and transaction would take an additional cost, which undermines the prerequisite of validating Arrow and Debreu's proof. Therefore, to guarantee the presence of these consensuses, neoclassical economists[36] have to assume an imaginary "Walrasian auctioneer", who has an advanced technology to eliminate the cost for information transmission and transaction, to selflessly assist all agents in reaching exchanges without any transaction cost. Unfortunately, when an agent has the potential to become a Walrasian auctioneer, motivated by selfishness it may instead take advantage of technology to acquire more incomes, undermining equal opportunity of market competition. For example, the internet business platforms (such as Amazon and Alibaba) could have the potential to become Walrasian auctioneers; however, all of them eventually become oligopolies benefiting from the Matthew effect. This explains why the Pareto distribution is observed in the top income class in a market economy. Although the internet technology did not eliminate beneficiaries of the Matthew effect, with rise of information technology, today the "block-chain" is further proposed to play the role of a Walrasian auctioneer. This implies that there seems to be an intrinsic motivation[20] for driving a human society to achieve an equal-opportunity economic network. Therefore, it may

be conjectured that a larger proportion of populations in a market-economy country is evolving towards an exponential income distribution.

In this paper, we employ the household income data from four representative market-economy countries to verify this conjecture. The four countries include three typical developed economy (the United Kingdom, the United States, and Canada) and one typical developing economy (China). In particular, because China is a special sample that has undergone the transition from a planned economy to a market economy, it is more interesting to check if there is a transition towards an exponential income distribution after the market-oriented economic reformation. Here, we first demonstrate that it is more reasonable to analyze the household income data by using the cumulative distribution function, rather than the density distribution function. In particular, we show that when one employs the density distribution function to analyze the household income data, the results may be inconsistent. In Figure 1, we use the household income data from the United Kingdom in 2010 to show this inconsistency: *The shape of the density distribution of income may depend on the bin width of the histogram*. For example, when the bin width is denoted by 4000 GBP, Figure 1a shows a right-skewed density distribution with a peak, which is close to a Log-normal distribution or a Gamma distribution. Unfortunately, when the bin width is increased to 8000 GBP, as shown in Figure 1b, the shape of density distribution begins to deviate from a Log-normal distribution or a Gamma distribution. In particular, when the bin width is increased to 12000 GBP, the shape of density distribution turns to a monotonic function, see Figure 1c. This means that, for the same data, density distribution may lead to inconsistent graphs. However, when one uses the cumulative distribution function to describe the household income data, there are no such inconsistencies. Figure 1d shows that the great majority of populations in the United Kingdom obeys the exponential income distribution (3), which is a cumulative distribution function. Henceforth, we always employ the cumulative distribution function to analyze the household income data.

[**Insert Figure 1 here**]

Furthermore, Figure 2 shows that the income structures of China (in 2015), the United Kingdom (in 2017), Canada (in 2018) uniformly exhibit a two-class pattern in which the great majority of populations obeys the exponential income distribution (3) and only the remaining (richest) part follows a Pareto distribution. Figure 2 also shows that the great majority of populations in the United States (in 2019) follows the exponential income distribution (3). The dataset was gathered from the most recent years. The empirical observation supports our conjecture that a larger proportion of populations in a market-economy country is evolving to an exponential income distribution.

[**Insert Figure 2 here**]

Here, we particularly employ the household income data of China from 1978 to 2000 to demonstrate that the exponential income distribution (3) emerges spontaneously in a market economy, as argued by Tao[18]. This period of time covers the main stages of China's market-oriented economic reformation. In Figure 3, we empirically show that the income structure of China gradually evolved to an exact exponential distribution after the 1990s. By contrast, the income structure of China around the 1990s only approximately obeyed the exponential distribution, while the income structures in 1978 and 1980 radically differed from the exponential distribution. This is strong evidence that the exponential income distribution (3) emerges spontaneously in a market economy.

[**Insert Figure 3 here**]

The evolution of human society seems to reveal a trend from hunter-gatherer societies, slave-based societies, feudal hierarchical societies, to various types and levels of democratic societies. Along with historical advancement, human societies are getting

close to an equal-opportunity economic network. This is reflected as that a larger proportion of populations is evolving to an exponential income distribution, while the Pareto distribution is squeezed into a fairly small proportion. By contrast, the Pareto distribution dominated a larger proportion of populations in the period of advocating the law of the jungle (e.g., slave-based societies), which causes the Matthew effect in material allocations. Today, with the continuous rise of digital interconnection, instantaneous communications, media, and blog-based echo chambers, our societies have been promoted to a more democratic and equal-opportunity structure. This structure may be further improved by the rise of artificial intelligence and block-chain technologies.

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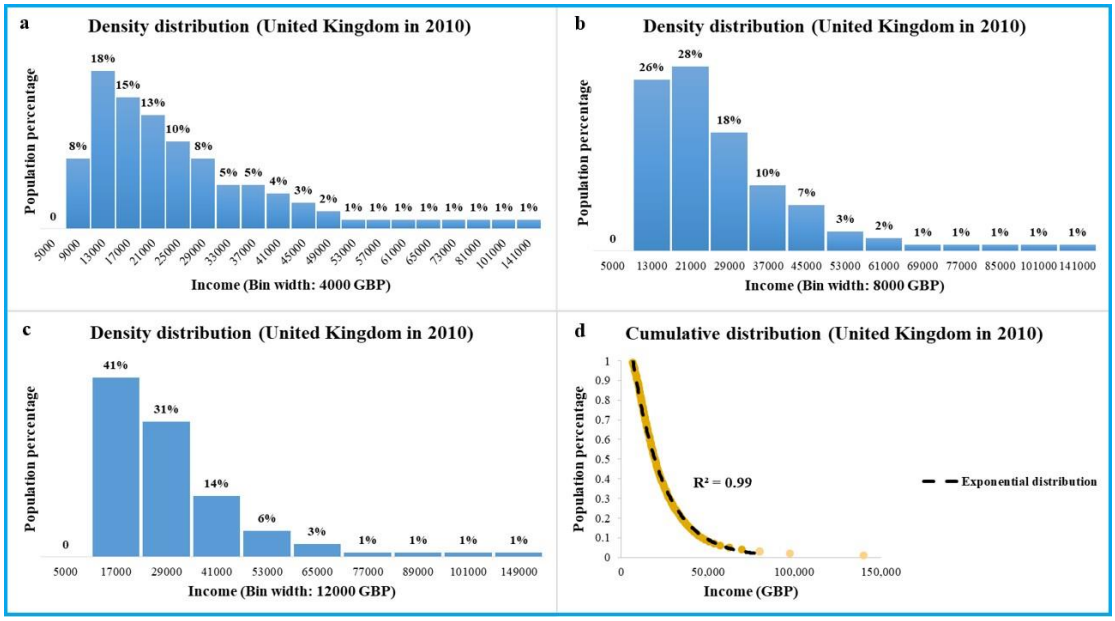

Figure 1. | Density distribution may lead to inconsistent graphs[1]. a: When the bin width is denoted by 4000 GBP, the histogram of density distribution is a right-skewed graph with a peak, which is close to a Log-normal distribution or a Gamma distribution. **b**: When the bin width is increased to 8000 GBP, the histogram of density distribution begins to deviate from a Log-normal distribution or a Gamma distribution. **c**: When the bin width is increased to 12000 GBP, the histogram of density distribution turns to a monotonic function. **d**: The cumulative distribution of income obeys an exponential function.

[1] Data resource: https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax

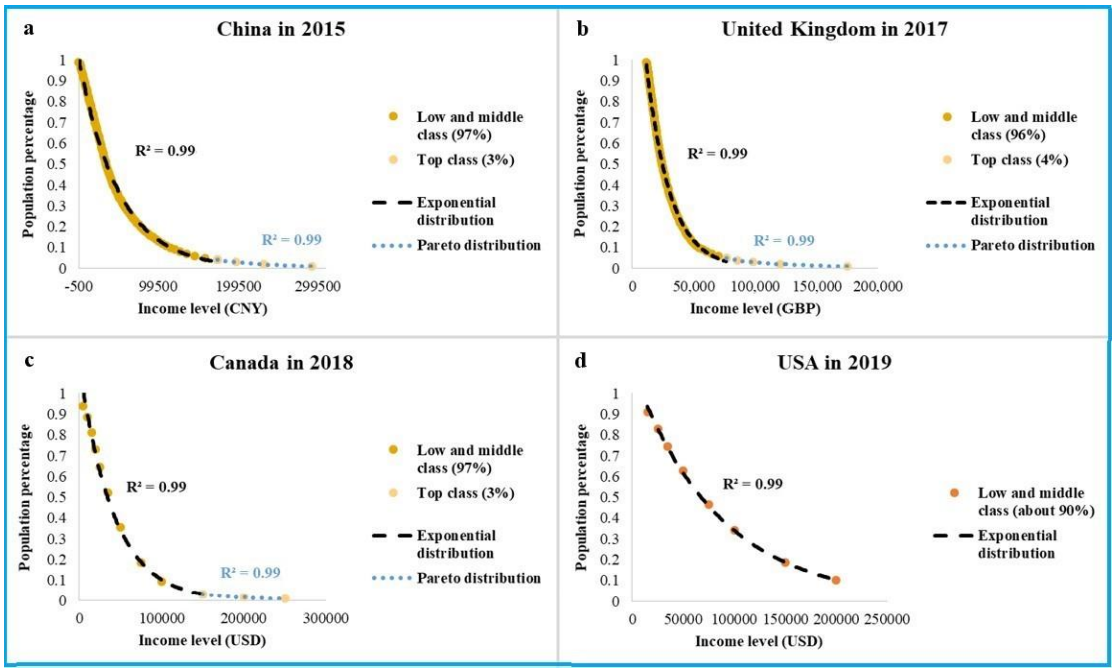

Figure 2. | A larger proportion of populations is evolving to an exponential income distribution. **a**: The low and middle income classes (97% of populations) in China obeys an exponential income distribution[2]. **b**: The low and middle income classes (96% of populations) in the United Kingdom obeys an exponential income distribution[3]. **c**: The low and middle income classes (97% of populations) in Canada obeys an exponential income distribution[4]. **d**: The low and middle income classes (more than 90% of populations) in the United States obeys an exponential income distribution[5].

[2] Data resource for China: http://wid.world/data/

[3] Data resource for the United Kingdom: https://www.gov.uk/government/statistics/percentile-points-from-1-to-99-for-total-income-before-and-after-tax

[4] Data resource for Canada: https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1110000801

[5] Data resource for the United States: https://www.statista.com/statistics/203183/percentage-distribution-of-household-income-in-the-us/

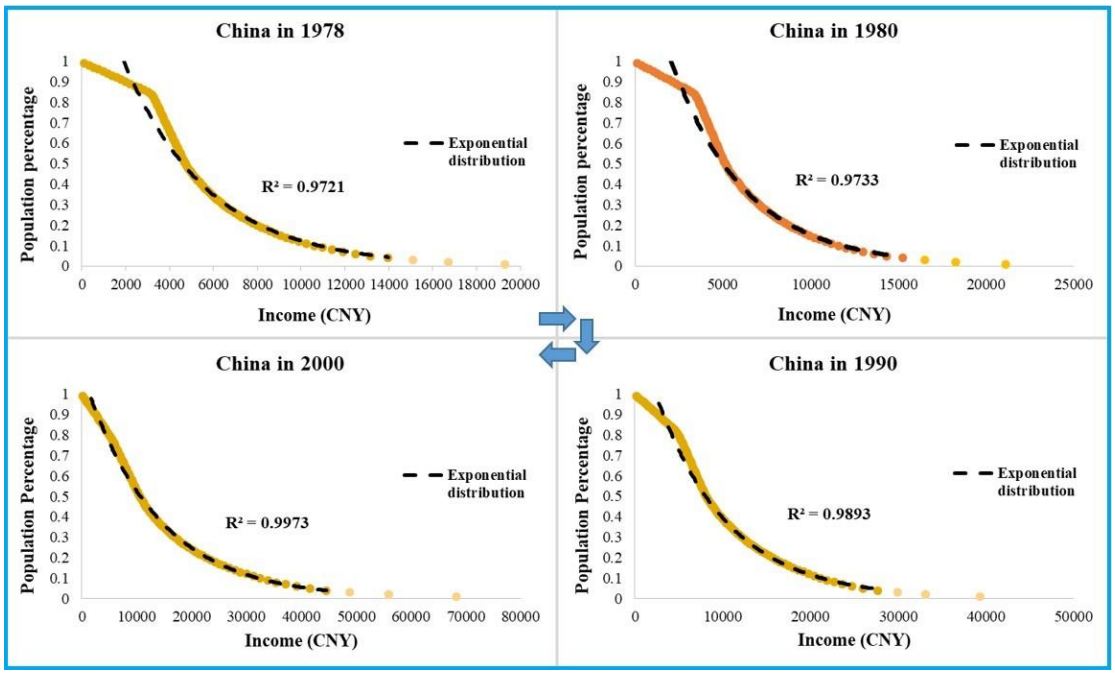

Figure 3. | The income structure of China gradually evolved to an exponential distribution after the market-oriented economic reformation[6].

[6] Data resource for China: http://wid.world/data/