

Expansion and Reduction of Soft Sets and their Applications in Decision Making

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Expansion and reduction of soft sets and their applications in decision making

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Abstract In this study, we first define expansion and reduction of the soft sets that are based on the linguistic modifiers. By using these new notions we then construct a decision making method called soft reduction method, which selects a set of optimum alternatives. We finally present an example which shows that the methods can be successfully applied to many problems containing uncertainties.

Keywords Soft sets · Linguistic modifiers · Expansion soft sets · Reduction soft sets · Soft reduction method · Soft decision making

1 Introduction

In many fields, such as economics, engineering, environment, involve data that contain uncertainties. To understand and manipulate the uncertainties, there are many approaches such as probability theory, fuzzy set theory [23], intuitionistic fuzzy sets [3], rough set theory [18], etc. Each of these theories have its own difficulties as pointed out in [17]. To address these difficulties, [17] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from difficulties.

The theory of fuzzy sets is the most suitable framework concerning uncertainties. A fuzzy set is described by its membership function, whose values are defined on the closed interval $[0, 1]$. The value of the membership function specifies the grade or degree to any element in a set. By contrast, the soft set theory, expresses vagueness, not by means of membership function, but employing a parametrized family of subsets of a set. The parametrization can be used with the help of words and sentences, real numbers, functions, mappings, and so on. The main advantage of soft set theory in data analysis is that it does not need any grade of membership as in the fuzzy set

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theory. Therefore, the theory of soft sets has advanced in a variety of ways and in many disciplines. [17] applied the soft sets to fields such as game theory, operations research, Riemann-integration, Perron integration, probability and so on.

After [17], different types of operations of soft sets have been defined. First operations of soft sets and their properties were given by [15]. [2,4,5,20,26] also made some contributions to the operations of soft sets. By using these operations, works on soft set theory and its applications have been progressing rapidly, some of which are [1,5–8,10,12–14,16,19,21,22].

In general, the applications of the soft set theory solve problems with the help of the rough sets or fuzzy soft sets. But, [5] introduced a soft decision making method, called *uni – int* decision making, which selects a set of optimum elements from the alternatives without using the rough sets and fuzzy soft sets. [11] intended to improve and further extend the *uni – int* decision making method. They defined choice value soft sets and by using these new notions they gave deeper insights into the principle of *uni – int* decision making. They then proposed several new soft decision making schemes. These methods could reduce a large set of alternatives to its subset of optimal objects according to the criteria given by decision makers. The *uni – int* decision making methods depend on the AND-product of soft sets and more than one decision maker. In this paper, we define a decision making method, called soft reduction method that reduces a large set of alternatives to its subset of optimal objects according to the criteria given by a decision maker and without using the AND-product of soft sets.

Before giving our new decision making method, we first define expansion and reduction of the soft sets that are based on the linguistic modifiers. [24,25] used the linguistic modifiers to express imprecise values of linguistic variables. We apply the linguistic modifiers to soft set theory to make applications of soft sets more suitable. By using the reduction of the soft we then construct a decision making method called soft reduction method that selects a set of optimum alternatives. We finally present an example which shows that the methods can be successfully applied to many problems containing uncertainties. This paper is a condensation of part of the PhD thesis of [9].

2 Soft Sets

In this section, we present basic definitions of soft sets and their operations. For more detail of the soft sets, we refer to the earlier studies [4,5,15,17].

Definition 1 [17] Let U and X be two non empty set and $P(U)$ is the power set of U . Then, a soft set f over U is a function defined by

$$f : X \rightarrow P(U)$$

where U refer to an initial universe and X is a set of parameters.

In other words, the soft set is a parametrized family of subsets of the set U . Every set $f(x)$, $x \in X$, from this family may be considered as the set of x -elements of the soft set f , or as the set of x -approximate elements of the soft set.

As an illustration, let us consider the following examples.

A soft set f describes the attractiveness of the houses which Mr.X is going to buy.

U - is the set of houses under consideration.

X - is the set of parameters. Each parameter is a word or a sentence.

$X = \{ \text{expensive; beautiful; wooden; cheap; in the green surroundings; modern; in good repair; in bad repair} \}$

In this case, to define a soft set means to point out *expensive* houses, *beautiful* houses, and so on.

It is worth noting that the sets $f(x)$ may be arbitrary. Some of them may be empty, some may have nonempty intersection.

A soft set over U can be represented by the set of ordered pairs

$$f = \{(x, f(x)) : x \in X\}$$

Note that the set of all soft sets defined from X to $P(U)$ will be denoted by S_X^U .

Definition 2 [5] Let $f \in S_X^U$. Then, f is an empty soft set, denoted by Φ_X , if $f(x) = \emptyset$ for all $x \in X$. f is a universal soft set, denoted by $f_{\tilde{X}}$, if $f(x) = U$ for all $x \in X$.

Definition 3 [5] Let $f, g \in S_X^U$. Then, f is a soft subset of g , denoted by $f \tilde{\subseteq} g$, if $f(x) \subseteq g(x)$ for all $x \in X$. f and g are soft equal, denoted by $f = g$, if and only if $f(x) = g(x)$ for all $x \in X$.

Definition 4 [5] Let $f, g \in S_X^U$. Then, union of f and g , denoted by $f \tilde{\cup} g$, if $(f \tilde{\cup} g)(x) = f(x) \cup g(x)$ for all $x \in X$. Intersection of f and g , denoted by $f \tilde{\cap} g$, if $(f \tilde{\cap} g)(x) = f(x) \cap g(x)$ for all $x \in X$. Complement of f , denoted by f^c , if $f^c(x) = U \setminus f(x)$ for all $x \in X$.

Definition 5 Let $f \in S_X^U$. Then, the set $\bar{f} = \{x : f(x) \neq \emptyset\}$ is called *support set* of f .

Proposition 1 Let $f, g, h \in S_X^U$. Then,

1. $f \tilde{\cup} g = g \tilde{\cup} f$, $f \tilde{\cap} g = g \tilde{\cap} f$,
2. $(f \tilde{\cup} g) \tilde{\cup} h = f \tilde{\cup} (g \tilde{\cup} h)$, $(f \tilde{\cap} g) \tilde{\cap} h = f \tilde{\cap} (g \tilde{\cap} h)$,
3. $f \tilde{\cup} (g \tilde{\cap} h) = (f \tilde{\cup} g) \tilde{\cap} (f \tilde{\cup} h)$, $f \tilde{\cap} (g \tilde{\cup} h) = (f \tilde{\cap} g) \tilde{\cup} (f \tilde{\cap} h)$,
4. $(f \tilde{\cup} g)^c = f^c \tilde{\cap} g^c$, $(f \tilde{\cap} g)^c = f^c \tilde{\cup} g^c$.

3 Expansions and Reductions of Soft Sets

In this section, we give definition of linguistic expansion and reduction of the soft sets based on linguistic modifiers and investigate their basic properties.

Throughout this paper, $I_n = \{1, 2, \dots, n\}$ for all $n \in N$.

3.1 Linguistic Modifiers and Modified Sets

In this subsection, we first introduce the linguistic modifiers that were used by [24, 25] to express imprecise values of linguistic variables. We apply the linguistic modifiers to soft set theory to make applications of soft sets more suitable.

Definition 6 Linguistic modifiers are words like “not very”, “more or less”, “very”, “very very”, “extremely” which modify the meaning of parameters of a soft set. For example, “beautiful house” becomes “very beautiful house”.

In this case, if x is a parameter and m is a linguistic modifier, then modified parameter x by m is denoted by $m(x)$.

In general, unless otherwise specified, the linguistic modifiers will be shown by the symbols m_i for all $i \in I_n$.

Example 1 Let $x_1 =$ “cheap”, $x_2 =$ “long”, $x_3 =$ “black” be three parameters of a soft set and $m_1 =$ “not very”, $m_2 =$ “very”, $m_3 =$ “extremely” be three linguistic modifiers. Then, some modified parameters will be as follows;

$$\begin{aligned} m_1(x_1) &= \text{“not very cheap”} \\ m_1(x_3) &= \text{“not very black”} \\ m_2(x_3) &= \text{“very black”} \\ m_3(x_2) &= \text{“extremely long”} \end{aligned}$$

Definition 7 Let m_1, m_2, \dots, m_n be linguistic modifiers. Then,

$$M^n = \{m_i : i \in I_n\}$$

is called an n -level linguistic modifier set.

Definition 8 Let X be a set of parameters and M^n be an n -level linguistic modifiers set. Then, an n -level modified set of set of parameter x is a set defined by

$$M_x^n = \{m(x) : m \in M^n\}, \text{ for all } x \in X.$$

Example 2 Assume that we have a 4-level linguistic modifier set as

$$M^4 = \{\text{very very, very, extremly, slightly}\}$$

Then, for a parameter ”good”, the 4-level modified set of ”good” will be written by

$$M_{good}^4 = \{\text{very very good, very good, extremly good, slightly good}\}$$

Definition 9 Let X be a set of parameters and M^n be an n -level linguistic modifier set. Then, n -level modified set of X is a set defined by

$$M_X^n = \{m(x) : x \in X, m \in M^n\}$$

Note that if $M^n = \{m_j : j \in I_n\}$ and $X = \{x_i : i \in I_k\}$, then when no confusion arises we will briefly use x_{ij} instead of $m_j(x_i)$, that is

$$m_j(x_i) = x_{ij}$$

Proposition 2 Let X be a set of parameters, M_x^n be an n -level modified set of x for every $x \in X$ and M_X^n be an n -level modified set of X . Then,

$$M_X^n = \bigcup_{x \in X} M_x^n$$

PROOF. It is easy from Definition 9.

3.2 Expansion of Soft Sets

In this subsection, we define expansion of a soft set and investigate their related properties.

Definition 10 Let $f \in S_X^U$ and M_x^n be an n -level modified set of x for $x \in X$. Then, an n -level expansion of each element $(x, f(x))$ of f is a soft set over U is defined by

$$f_x : M_x^n \rightarrow P(U), \text{ for all } x \in X,$$

if following conditions hold;

- a) $f_x(m_i(x)) \cap f_x(m_j(x)) = \emptyset$ for all $i, j \in I_n, i \neq j$,
- b) $\bigcup_{i \in I_n} f_x(m_i(x)) = f(x)$.

It is clear that $f_x \in S_{M_x^n}^U$, for every $x \in X$.

Note that from now on, we may not use "n-level" if does not cause confusion.

Example 3 If $U = \{u_1, u_2, \dots, u_8\}$ is a set of houses and $X = \{x_1, x_2, \dots, x_6\}$ is a set of parameters where $x_i, (i = 1, 2, \dots, 6)$, stand for the parameters "expensive", "large", "beautiful", "wooden", "cheap" and "in green surroundings" respectively, then we can consider a soft set f to describe the attractiveness of houses as follows:

$$f = \{(x_1, \{u_1, u_2, u_4, u_7\}), (x_3, \{u_2, u_3, u_5, u_6\}), (x_5, \{u_3\}), (x_6, \{u_1, u_3, u_5, u_8\})\}$$

Let us consider an element $(x_1, \{u_1, u_2, u_4, u_7\})$ of f and 4-level linguistic modifier set

$$M^4 = \{m_1, m_2, m_3, m_4\}$$

where $m_j, (j = 1, 2, 3, 4)$, stand for the linguistic modifiers "less", "very", "very very" and "extremely" respectively. Then, 4-level modified set of $x_1 = \text{"expensive"}$ can be written by

$$M_{x_1}^4 = \{x_{11}, x_{12}, x_{13}, x_{14}\}$$

where $x_{11} = \text{"less expensive"}$, $x_{12} = \text{"very expensive"}$, $x_{13} = \text{"very very expensive"}$ and $x_{14} = \text{"extremely expensive"}$. Assume that

$$\begin{aligned} f_{x_1}(x_{11}) &= \{u_1, u_2\} \\ f_{x_1}(x_{12}) &= \emptyset \\ f_{x_1}(x_{13}) &= \emptyset \\ f_{x_1}(x_{14}) &= \{u_4, u_7\} \end{aligned}$$

then, a 4-level expansion of $(x_1, f(x_1))$ can be written by

$$f_{x_1} = \{(x_{11}, \{u_1, u_2\}), (x_{14}, \{u_4, u_7\})\}$$

Note 1 It is well known that construction of a soft set may depend on experts. Therefore, each element of a soft set may have different expansions. If it is necessary to use more than one expansion of an element of a soft set, then to avoid the confusion different expansions will be indicated by a, b, c, \dots as a superscript of the soft sets. In this case, the expansions of an element $(x, f(x))$ of soft set f can be denoted by $f_x^a, f_x^b, f_x^c, \dots$ and then they called a -expansion, b -expansion, c -expansion, ..., respectively.

For example, let us consider Example 3, we can construct different expansions of $(x_1, f(x_1))$ as follows;

$$\begin{aligned} f_{x_1}^a &= \{(x_{11}, \{u_1, u_2\}), (x_{14}, \{u_4, u_7\})\} \\ f_{x_1}^b &= \{(x_{11}, \{u_4\}), (x_{13}, \{u_2\}), (x_{14}, \{u_1, u_7\})\} \\ f_{x_1}^c &= \{(x_{12}, \{u_1\}), (x_{13}, \{u_2\}), (x_{14}, \{u_7\})\} \end{aligned}$$

Definition 11 Let $f \in S_X^U$, $f_x \in S_{M_x^U}$ and \bar{f} be the support set of f . Then, an n -level expansion family of f is defined by

$$E_n(f) = \{f_x : x \in \bar{f}\}$$

Example 4 Let us consider Example 3 where f_{x_1} is given and assume that in a similar way 4-level expansion of $(x_3, f(x_3))$, $(x_5, f(x_5))$ and $(x_6, f(x_6))$ can be written respectively by

$$\begin{aligned} f_{x_3} &= \{(x_{32}, \{u_2, u_3\}), (x_{33}, \{u_5, u_6\})\} \\ f_{x_5} &= \{(x_{52}, \{u_3\})\} \\ f_{x_6} &= \{(x_{61}, \{u_1, u_5, u_8\}), (x_{64}, \{u_3\})\} \end{aligned}$$

Then, an 4-level expansion family of f can be written by

$$\begin{aligned} E_4(f) &= \{f_{x_1}, f_{x_3}, f_{x_5}, f_{x_6}\} \\ &= \left\{ \{(x_{11}, \{u_1, u_2\}), (x_{14}, \{u_4, u_7\})\}, \{(x_{32}, \{u_2, u_3\}), (x_{33}, \{u_5, u_6\})\}, \right. \\ &\quad \left. \{(x_{52}, \{u_3\})\}, \{(x_{61}, \{u_1, u_5, u_8\}), (x_{64}, \{u_3\})\} \right\} \end{aligned}$$

Definition 12 Let $f, g \in S_X^U$ and $f_x, g_x \in S_{M_x^U}$ for $x \in X$. Then, the expansion of each element of the union, intersection and complement of soft sets are defined, respectively, by

$$\begin{aligned} (f \tilde{\cup} g)_x(m(x)) &= f_x(m(x)) \cup g_x(m(x)), \quad x \in X, \\ (f \tilde{\cap} g)_x(m(x)) &= f_x(m(x)) \cap g_x(m(x)), \quad x \in X, \\ (f^c)_x(m(x)) &= f_x^c(m(x)), \quad x \in X. \end{aligned}$$

Example 5 Let us consider Example 3 where the 4-level expansion of $(x_1, f(x_1))$ is given as $f_{x_1} = \{(x_{11}, \{u_1, u_2\}), (x_{14}, \{u_4, u_7\})\}$ and assume that another soft set g is given as follows

$$g = \{(x_1, \{u_1, u_3, u_5, u_7\}), (x_2, \{u_2, u_5\}), (x_3, \{u_5, u_7, u_8\}), (x_5, \{u_3, u_4, u_6\})\}$$

If 4-level expansion of $(x_1, g(x_1))$ is given as

$$g_{x_1} = \{(x_{11}, \{u_5\}), (x_{13}, \{u_1, u_3\}), (x_{14}, \{u_7\})\}$$

then, we can construct $(f \tilde{\cup} g)_{x_1}$. We know that

$$(f \tilde{\cup} g)_{x_1}(m(x_1)) = f_{x_1}(m(x_1)) \cup g_{x_1}(m(x_1)), \text{ for all } m(x_1) \in M_{x_1}^4$$

which gives

$$\begin{aligned} (f \tilde{\cup} g)_{x_1}(x_{11}) &= f_{x_1}(x_{11}) \cup g_{x_1}(x_{11}) = \{u_1, u_2\} \cup \{u_5\} = \{u_1, u_2, u_5\} \\ (f \tilde{\cup} g)_{x_1}(x_{12}) &= f_{x_1}(x_{12}) \cup g_{x_1}(x_{12}) = \emptyset \cup \emptyset = \emptyset \\ (f \tilde{\cup} g)_{x_1}(x_{13}) &= f_{x_1}(x_{13}) \cup g_{x_1}(x_{13}) = \emptyset \cup \{u_1, u_3\} = \{u_1, u_3\} \\ (f \tilde{\cup} g)_{x_1}(x_{14}) &= f_{x_1}(x_{14}) \cup g_{x_1}(x_{14}) = \{u_4, u_7\} \cup \{u_7\} = \{u_4, u_7\} \end{aligned}$$

Hence, we get

$$(f\tilde{\cup}g)_{x_1} = \{(x_{11}, \{u_1, u_2, u_5\}), (x_{13}, \{u_1, u_3\}), (x_{14}, \{u_4, u_7\})\}$$

By the similar way we get

$$(f\tilde{\cap}g)_{x_1} = \{(x_{14}, \{u_7\})\}$$

and

$$(f^{\tilde{c}})_{x_1} = \{(x_{11}, \{u_3, u_4, u_5, u_6, u_7, u_8\}), (x_{14}, \{u_1, u_2, u_3, u_5, u_6, u_8\})\}$$

Theorem 1 Let $f, g \in S_X^U$ and $f_x, g_x \in S_{M_x^n}^U$ for $x \in X$. Then, for every $x \in X$, followings hold;

1. $(f\tilde{\cup}g)_x = f_x\tilde{\cup}g_x$
2. $(f\tilde{\cap}g)_x = f_x\tilde{\cap}g_x$
3. $(f^{\tilde{c}})_x = (f_x)^{\tilde{c}}$

PROOF. 1. Assume that X is a set of parameters and M_x^n be an n -level modified set of x for $x \in X$. $f, g \in S_X^U$ and $f_x, g_x \in S_{M_x^n}^U$ for $x \in X$ are given. Then, we can construct an n -level expansion of $(x, f(x))$ and $(x, g(x))$ for $x \in X$, respectively,

$$\begin{aligned} f_x &= \{(m(x), f_x(m(x))) : m(x) \in M_x^n\}, \\ g_x &= \{(m(x), g_x(m(x))) : m(x) \in M_x^n\}. \end{aligned}$$

We can write the union of f_x and g_x from Definition 4 as;

$$f_x\tilde{\cup}g_x = \{(m(x), f_x(m(x)) \cup g_x(m(x))) : m(x) \in M_x^n\}$$

Now, we have to obtain an expansion of $(x, (f\tilde{\cup}g)(x))$ for every $x \in X$,

$$(f\tilde{\cup}g)_x = \{(m(x), (f\tilde{\cup}g)_x(m(x))) : m(x) \in M_x^n\}$$

from Definition 12 we get the respected result,

$$(f\tilde{\cup}g)_x = \{(m(x), f_x(m(x)) \cup g_x(m(x))) : m(x) \in M_x^n\}$$

The proofs of 2. and 3. can be made by the similar way.

Definition 13 Let $f \in S_X^U$, $f_x \in S_{M_x^n}^U$ and M_x^n be an n -level modified set of X . Then, an n -level expansion of f is a soft set over U is defined by

$$f_X : M_x^n \longrightarrow P(U), \quad f_X(m(x)) = f_x(m(x))$$

This expansion is denoted by

$$f \prec f_X$$

It is clear that $f_X \in S_{M_x^n}^U$.

From now on, n -level expansions of soft sets f, g, h, \dots will be denoted by f_X, g_X, h_X, \dots , respectively.

Proposition 3 Let $f_x \in S_{M_x^U}^U$ and $f_X \in S_{M_X^U}^U$. Then,

$$f_X = \bigcup_{x \in X} f_x$$

PROOF. It is easy from Definition 13.

Example 6 Let us consider Example 3 and 4 where $X = \{x_1, x_2, \dots, x_6\}$ and $f_{x_1}, f_{x_3}, f_{x_5}, f_{x_6}$ are given as a 4-level expansion of $(x_1, f(x_1)), (x_3, f(x_3)), (x_5, f(x_5)), (x_6, f(x_6))$, respectively. We know that 4-level expansion of $(x_2, f(x_2))$ and $(x_4, f(x_4))$ are the empty soft set. Then,

$$\begin{aligned} f_X(m_1(x_1)) &= f_{x_1}(m_1(x_1)) = f_{x_1}(x_{11}) = \{u_1, u_2\}, \\ f_X(m_4(x_1)) &= f_{x_1}(m_4(x_1)) = f_{x_1}(x_{14}) = \{u_4, u_7\}, \\ f_X(m_2(x_3)) &= f_{x_3}(m_2(x_3)) = f_{x_3}(x_{32}) = \{u_2, u_3\}, \\ f_X(m_3(x_3)) &= f_{x_3}(m_3(x_3)) = f_{x_3}(x_{33}) = \{u_5, u_6\}, \\ f_X(m_2(x_5)) &= f_{x_5}(m_2(x_5)) = f_{x_5}(x_{52}) = \{u_3\}, \\ f_X(m_1(x_6)) &= f_{x_6}(m_1(x_6)) = f_{x_6}(x_{61}) = \{u_1, u_5, u_8\}, \\ f_X(m_4(x_6)) &= f_{x_6}(m_4(x_6)) = f_{x_6}(x_{64}) = \{u_3\}, \end{aligned}$$

and it is known that

$$f_X(m_j(x_i)) = f_{x_i}(m_j(x_i)) = f_{x_i}(x_{ij}) = \emptyset \text{ for the other } i \text{ and } j.$$

Hence, a 4-level expansion of f can be written by

$$f_X = \{(x_{11}, \{u_1, u_2\}), (x_{14}, \{u_4, u_7\}), (x_{32}, \{u_2, u_3\}), (x_{33}, \{u_5, u_6\}), (x_{52}, \{u_3\}), (x_{61}, \{u_1, u_5, u_8\}), (x_{64}, \{u_3\})\}$$

It can also be obtained easily by using following Proposition 3 as

$$f_X = f_{x_1} \cup f_{x_3} \cup f_{x_5} \cup f_{x_6}$$

Theorem 2 Let $f, g \in S_X^U$ and $f_X, g_X \in S_{M_X^U}^U$. If $f \prec f_X$ and $g \prec g_X$, then followings hold;

1. $(f \tilde{\cup} g) \prec (f_X \tilde{\cup} g_X)$
2. $(f \tilde{\cap} g) \prec (f_X \tilde{\cap} g_X)$
3. $f^{\tilde{c}} \prec (f_X)^{\tilde{c}}$

PROOF. 1. It is known that $(f \tilde{\cup} g) \prec (f \tilde{\cup} g)_X$. Now, it is enough to show that $(f \tilde{\cup} g)_X = (f_X \tilde{\cup} g_X)$. Hence,

$$\begin{aligned} (f \tilde{\cup} g)_X &= \bigcup_{x \in X} (f \tilde{\cup} g)_x, && \text{by Proposition 3,} \\ &= \bigcup_{x \in X} (f_x \tilde{\cup} g_x), && \text{by Theorem 1,} \\ &= (\bigcup_{x \in X} f_x) \tilde{\cup} (\bigcup_{x \in X} g_x), && \text{by Proposition 1-2,} \\ &= f_X \tilde{\cup} g_X, && \text{by Proposition 3.} \end{aligned}$$

The proofs of 2. and 3. can be made by the similar way.

Corollary 1 Let $f, g \in S_X^U$ and $f_X, g_X \in S_{M_X^U}^U$. Then,

1. $(f \tilde{\cup} g)^{\tilde{c}} \prec (f_X)^{\tilde{c}} \tilde{\cup} (g_X)^{\tilde{c}}$
2. $(f \tilde{\cap} g)^{\tilde{c}} \prec (f_X)^{\tilde{c}} \tilde{\cap} (g_X)^{\tilde{c}}$

PROOF. They are clear from Theorem 2.

3.3 Reduction of Soft Sets

In this subsection, we define reduction of soft sets and investigate their related properties.

Definition 14 Let U be a universal set. Then, for $n \in \mathbb{N}$, n -level choice set is a set defined by

$$C_n = \{c_i^n : i \in I_n\}$$

where $c_i^n = (e_{i1}, e_{i2}, \dots, e_{in})$ is an n -tuple such that for $j \in I_n$

$$e_{ij} = \begin{cases} U, & i = j \\ \emptyset, & i \neq j \end{cases}$$

Example 7 Assume that U is a universal set. Let us write 4-level choice set

$$\begin{aligned} C_4 &= \{c_1^4, c_2^4, c_3^4, c_4^4\} \\ &= \{(c_{11}, c_{12}, c_{13}, c_{14}), (c_{21}, c_{22}, c_{23}, c_{24}), (c_{31}, c_{32}, c_{33}, c_{34}), (c_{41}, c_{42}, c_{43}, c_{44})\} \\ &= \{(U, \emptyset, \emptyset, \emptyset), (\emptyset, U, \emptyset, \emptyset), (\emptyset, \emptyset, U, \emptyset), (\emptyset, \emptyset, \emptyset, U)\} \end{aligned}$$

By the similar way, 1, 2, 3, ..., n -level choice sets are written respectively by

$$\begin{aligned} C_1 &= \{(U)\} \\ C_2 &= \{(U, \emptyset), (\emptyset, U)\} \\ C_3 &= \{(U, \emptyset, \emptyset), (\emptyset, U, \emptyset), (\emptyset, \emptyset, U)\} \\ &\vdots \\ C_n &= \{(U, \emptyset, \emptyset, \dots, \emptyset), (\emptyset, U, \emptyset, \dots, \emptyset), \dots, (\emptyset, \emptyset, \dots, \emptyset, U)\} \end{aligned}$$

Definition 15 Let X be a set of parameters, C_n be the n -level choice set. Then, an n -level choice function is defined by

$$\alpha^n : X \rightarrow C_n, \quad \alpha^n(x) = c_k^n$$

where c_k^n is one of $c_1^n, c_2^n, \dots, c_n^n$ chosen by a decision maker according to the situation of $x \in X$. The $\alpha^n(x)$ is called n -level choice value of $x \in X$.

Example 8 Let us consider $X = \{x_1, x_2, x_3, x_4, x_5, x_6\}$ as a set of parameters and C_4 as a 4-level choice set. Then, 4-level choice values for x_i , $i \in I_6$, may be chosen respectively by

$$\begin{aligned} \alpha^4(x_1) &= c_1^4 = (c_{11}, c_{12}, c_{13}, c_{14}) = (U, \emptyset, \emptyset, \emptyset) \\ \alpha^4(x_2) &= c_4^4 = (c_{41}, c_{42}, c_{43}, c_{44}) = (\emptyset, \emptyset, \emptyset, U) \\ \alpha^4(x_3) &= c_2^4 = (c_{21}, c_{22}, c_{23}, c_{24}) = (\emptyset, U, \emptyset, \emptyset) \\ \alpha^4(x_4) &= c_1^4 = (c_{11}, c_{12}, c_{13}, c_{14}) = (U, \emptyset, \emptyset, \emptyset) \\ \alpha^4(x_5) &= c_2^4 = (c_{21}, c_{22}, c_{23}, c_{24}) = (\emptyset, U, \emptyset, \emptyset) \\ \alpha^4(x_6) &= c_4^4 = (c_{41}, c_{42}, c_{43}, c_{44}) = (\emptyset, \emptyset, \emptyset, U) \end{aligned}$$

Definition 16 Let $f \in S_X^U$, $f_x \in S_{M_x^U}$, $E_n(f)$ be an n -level expansion family of f , $P(f_x)$ be the powerset of f_x and α^n be an n -level choice function. Then, a reduction function of f , for $x \in X$, is defined by

$$\alpha_x^n : E_n(f) \rightarrow P(f_x),$$

$$\alpha_x^n(f_x) = \{(m_j(x), e_j(\alpha^n(x)) \cap f_x(m_j(x))) : j \in I_n\}$$

Where, the value $\alpha_x^n(f_x)$ is called α^n -reduction of $(x, f(x))$ for $x \in X$.

It is clear that $|\alpha_x^n(f_x)| = 1$ for $x \in X$.

Example 9 Let us consider $E_n(f)$ which is given in Example 4 and α^4 which is given in Example 8. Since the expansions are 4-level, we have to use the 4-level choice set C_4 . Then, α^4 -reduction of $(x_1, f(x_1))$ is computed by

$$\begin{aligned} \alpha_{x_1}^4(f_{x_1}) &= \{(m_j(x_1), e_j(\alpha^4(x_1)) \cap f_{x_1}(m_j(x_1))) : j \in I_4\} \\ &= \{(x_{1j}, e_j(\alpha^4(x_1)) \cap f_{x_1}(x_{1j})) : j \in I_4\}, \text{ since } m_j(x_i) = x_{ij} \\ &= \{(x_{1j}, e_j(c_1^4) \cap f_{x_1}(x_{1j})) : j \in I_4\}, \text{ since } \alpha^4(x_1) = c_1^4 \\ &= \{(x_{1j}, c_{1j} \cap f_{x_1}(x_{1j})) : j \in I_4\}, \text{ since } e_j(c_1^4) = c_{1j} \\ &= \{(x_{11}, c_{11} \cap f_{x_1}(x_{11})), (x_{12}, c_{12} \cap f_{x_1}(x_{12})), (x_{13}, c_{13} \cap f_{x_1}(x_{13})), \\ &\quad (x_{14}, c_{14} \cap f_{x_1}(x_{14}))\} \\ &= \{(x_{11}, U \cap f_{x_1}(x_{11})), (x_{12}, \emptyset \cap f_{x_1}(x_{12})), (x_{13}, \emptyset \cap f_{x_1}(x_{13})), \\ &\quad (x_{14}, \emptyset \cap f_{x_1}(x_{14}))\} \\ &= \{(x_{11}, f_{x_1}(x_{11}))\} \\ &= \{(x_{11}, \{u_1, u_2\})\} \end{aligned}$$

and also by the similar way, α^4 -reduction of $(x_i, f(x_i))$, $i = 3, 5, 6$, are computed respectively by

$$\begin{aligned} \alpha_{x_3}^4(f_{x_3}) &= \{(x_{32}, \{u_2, u_3\})\} \\ \alpha_{x_5}^4(f_{x_5}) &= \{(x_{52}, \{u_3\})\} \\ \alpha_{x_6}^4(f_{x_6}) &= \{(x_{64}, \{u_3\})\} \end{aligned}$$

Proposition 4 Let $f_x, g_x \in S_{M_x^U}$ and α_x^n be a reduction function for $x \in X$. Then, for $x \in X$,

1. $\alpha_x^n(f_x \widetilde{\cup} g_x) = \alpha_x^n(f_x) \widetilde{\cup} \alpha_x^n(g_x)$
2. $\alpha_x^n(f_x \widetilde{\cap} g_x) = \alpha_x^n(f_x) \widetilde{\cap} \alpha_x^n(g_x)$
3. $\alpha_x^n(f_x^{\widetilde{c}}) = (\alpha_x^n(f_x))^{\widetilde{c}}$

PROOF. 1. Since $f_x \widetilde{\cup} g_x = \{(m(x), f_x(m(x)) \cup g_x(m(x)) : m(x) \in M_x^n\}$, by using Definition 16 we can write

$$\alpha_x^n(f_x \widetilde{\cup} g_x) = \{(m_j(x), e_j(\alpha^n(x)) \cap (f_x(m_j(x)) \cup g_x(m_j(x)))) : j \in I_n\} \quad (1)$$

Since

$$\begin{aligned} \alpha_x^n(f_x) &= \{(m_j(x), e_j(\alpha^n(x)) \cap f_x(m_j(x))) : j \in I_n\}, \\ \alpha_x^n(g_x) &= \{(m_j(x), e_j(\alpha^n(x)) \cap g_x(m_j(x))) : j \in I_n\} \end{aligned}$$

by using definition of union of soft sets we can write

$$\alpha_x^n(f_x) \tilde{\cup} \alpha_x^n(g_x) = \{(m_j(x), (e_j(\alpha^n(x)) \cap f_x(m_j(x))) \cup (e_j(\alpha^n(x)) \cap g_x(m_j(x)))) : j \in I_n\} \quad (2)$$

From (1) and (2), we get that $\alpha_x^n(f_x \tilde{\cup} g_x) = \alpha_x^n(f_x) \tilde{\cup} \alpha_x^n(g_x)$

The proofs of 2. and 3. can be made by the similar way.

Definition 17 Let $f \in S_X^U$ and $\alpha_x^n(f_x)$ be an α^n -reduction of $(x, f(x))$ for $x \in X$. Then, an α^n -reduction of soft set f , denoted by f_{α^n} , is defined as

$$f_{\alpha^n} = \{\alpha_x^n(f_x) : x \in X\}$$

This soft set reduction will be written by

$$f \triangleright f_{\alpha^n}$$

Note that if it is necessary to use different reduction of a soft set with a different level, then to avoid the confusion the reductions and levels will be generally indicated by $\alpha^n, \beta^m, \gamma^k$ and so on.

Example 10 Let us consider Example 9. Then, an α^4 -reduction of f can be written by

$$f_{\alpha^4} = \{\{(x_{11}, \{u_1, u_2\})\}, \{(x_{32}, \{u_2, u_3\})\}, \{(x_{52}, \{u_3\})\}, \{(x_{64}, \{u_3\})\}\}$$

Proposition 5 Let $f \in S_X^U$, $\alpha_x^n(f_x)$ be an α^n -reduction of $(x, f(x))$ for $x \in X$ and f_{α^n} be an α^n -reduction of soft set f . Then,

$$f_{\alpha^n} = \bigcup_{x \in X} \alpha_x^n(f_x)$$

PROOF. It is easy from Definition 17.

Theorem 3 Let $f, g \in S_X^U$. If $f \triangleright f_{\alpha^n}$ and $g \triangleright g_{\alpha^n}$, then followings hold;

1. $(f \tilde{\cup} g) \triangleright (f_{\alpha^n} \tilde{\cup} g_{\alpha^n})$
2. $(f \tilde{\cap} g) \triangleright (f_{\alpha^n} \tilde{\cap} g_{\alpha^n})$
3. $f^{\tilde{c}} \triangleright (f_{\alpha^n})^{\tilde{c}}$

PROOF. 1. It is known that $(f \tilde{\cup} g) \triangleright (f \tilde{\cup} g)_{\alpha^n}$. Now, it is enough to show that $(f \tilde{\cup} g)_{\alpha^n} = f_{\alpha^n} \tilde{\cup} g_{\alpha^n}$. Hence,

$$\begin{aligned} (f \tilde{\cup} g)_{\alpha^n} &= \bigcup_{x \in X} \alpha_x^n((f \tilde{\cup} g)_x), && \text{by Definition 17,} \\ &= \bigcup_{x \in X} \alpha_x^n(f_x \tilde{\cup} g_x), && \text{by Theorem 1,} \\ &= \bigcup_{x \in X} (\alpha_x^n(f_x) \tilde{\cup} \alpha_x^n(g_x)), && \text{by Poroposition 4,} \\ &= (\bigcup_{x \in X} \alpha_x^n(f_x)) \tilde{\cup} (\bigcup_{x \in X} \alpha_x^n(g_x)), && \text{by Poroposition 1-2,} \\ &= f_{\alpha^n} \tilde{\cup} g_{\alpha^n}, && \text{by Definition 17.} \end{aligned}$$

The proofs of 2. and 3. can be made by the similar way.

Corollary 2 Let $f, g \in S_X^U$ and $f_{\alpha^n}, g_{\alpha^n} \in S_{M_X^n}^U$. Then,

1. $(f \tilde{\cup} g)^{\tilde{c}} \triangleright f_{\alpha^n}^{\tilde{c}} \tilde{\cup} g_{\alpha^n}^{\tilde{c}}$
2. $(f \tilde{\cap} g)^{\tilde{c}} \triangleright f_{\alpha^n}^{\tilde{c}} \tilde{\cap} g_{\alpha^n}^{\tilde{c}}$

PROOF. They are clear from Theorem 3.

4 Soft Reduction Method

In this section, we define a decision making method called soft reduction decision making method (in short, soft reduction method). The aim of this method is to obtain a set of optimum alternatives by an α^n -reduction of a soft set.

Definition 18 Let $f \in S_X^U$. Then,

$$uni(f) = \bigcup_{x \in X} f(x)$$

is a set called *union set* of soft set f .

Definition 19 Let $f \in S_X^U$. Then,

$$int(f) = \bigcap_{x \in X} f(x)$$

is a set called *intersection set* of soft set f .

Example 11 Let $f = \{(x_1, \{u_1, u_2, u_4\}), (x_3, \{u_2, u_5, u_6\}), (x_5, \{u_2, u_6\})\}$ be a soft set over $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$. Then the union set of f is written as

$$uni(f) = \{u_1, u_2, u_4, u_5, u_6\}$$

and the intersection set of f is written as

$$int(f) = \{u_2\}$$

Proposition 6 Let $f, g \in S_X^U$. Then,

1. $int(f \tilde{\cup} g) = int(f) \tilde{\cup} int(g)$
2. $int(f \tilde{\cap} g) = int(f) \tilde{\cap} int(g)$
3. $uni(f \tilde{\cup} g) = uni(f) \tilde{\cup} uni(g)$
4. $uni(f \tilde{\cap} g) = uni(f) \tilde{\cap} uni(g)$

PROOF. 1. For all $x \in X$,

$$\begin{aligned} int(f \tilde{\cup} g) &= \bigcap_{x \in X} (f \cup g)(x) \\ &= \bigcap_{x \in X} (f(x) \cup g(x)) \\ &= \left(\bigcap_{x \in X} f(x) \right) \cup \left(\bigcap_{x \in X} g(x) \right) \\ &= int(f) \tilde{\cup} int(g) \end{aligned}$$

The other proofs can be made similarly.

Proposition 7 Let $f \in S_X^U$. Then,

1. $int(f^c) = \bigcap_{x \in X} (U - f(x))$
2. $uni(f^c) = \bigcup_{x \in X} (U - f(x))$

PROOF. 1. For all $x \in X$,

$$\text{int}(f^c) = \bigcap_{x \in X} (f^c)(x) = \bigcap_{x \in X} (f^c(x)) = \bigcap_{x \in X} (U - f(x))$$

The proof of 2. can be made by a similar way.

Proposition 8 Let $f \in S_X^U$. Then,

1. $\text{uni}(f) \subseteq U$
2. $\text{int}(f) \subseteq U$
3. $\text{int}(f) \subseteq \text{uni}(f)$

PROOF. They are clear from Definition 18 and 19.

Definition 20 Let $f \in S_X^U$ and f_{α^n} be an α^n -reduction of f . Then, the union set of f_{α^n} , denoted R , is called a reduced set of the alternatives. That is,

$$R = \text{uni}(f_{\alpha^n})$$

Proposition 9 Let $f \in S_X^U$ and $f_{\alpha^n} \in S_{M_X^U}$. Then,

1. $R \subseteq U$
2. $R \subseteq \text{uni}(f)$
3. $\text{int}(f_{\alpha^n}) \subseteq R$

PROOF. They are clear from Definition 18, 19 and 20

4.1 Algorithm of Soft Reduction Method

In this subsection, we construct an algorithm of soft reduction method.

Assume that U is a set of alternatives and X is a set of parameters are given. Based on above definitions and results, we can construct an algorithm of soft reduction method as follows:

- Step 1.* Input a soft set $f \in S_X^U$.
- Step 2.* Input an n -level linguistic modifier set M^n .
- Step 3.* Compute M_x^n for each $x \in X$.
- Step 4.* Compute f_x for each $x \in X$ to obtain $E_4(f)$.
- Step 5.* Input an n -level choice set C_n .
- Step 6.* Compute $\alpha^n(x)$ for each $x \in X$.
- Step 7.* Compute $\alpha_x^n(f_x)$ for each $x \in X$.
- Step 8.* Compute f_{α^n} .
- Step 9.* Find the reduced set R .

4.2 An Application of Soft Reduction Method

In this subsection, to illustrate the basic idea of the soft reduction method, let us consider the following example:

Assume that a company received 20 applications for a position. There is a decision maker who wants to interview some of the suitable candidates instead of all of the candidates. Therefore, by using the soft reduction method the number of candidates are reduced to a suitable subset of candidates.

Let $U = \{u_1, u_2, \dots, u_{20}\}$ be the set of applicants which may be characterized by a set of parameters $X = \{x_1, x_2, \dots, x_7\}$. For $i = 1, 2, \dots, 7$, the parameters x_i stand for “work experience”, “computer knowledge”, “friendship”, “young age”, “higher education”, “foreign language” and “good health” respectively.

Now, by using the algorithm of soft reduction method we can solve this problem step by step as follows:

Step 1: The decision maker constructs a soft set f over U according to the CV of applicants,

$$f = \left\{ (x_1, \{u_1, u_2, u_3, u_5, u_8, u_9, u_{10}, u_{11}, u_{14}, u_{17}, u_{20}\}), (x_2, U), \right. \\ (x_3, \{u_1, u_3, u_4, u_6, u_8, u_9, u_{12}, u_{13}, u_{16}, u_{17}, u_{18}, u_{20}\}), \\ (x_4, \{u_2, u_4, u_6, u_8, u_9, u_{10}, u_{14}, u_{18}\}), \\ (x_5, \{u_4, u_7, u_8, u_9, u_{10}, u_{13}, u_{14}, u_{17}\}), (x_6, U), \\ \left. (x_7, \{u_2, u_3, u_4, u_6, u_8, u_{10}, u_{11}, u_{12}, u_{14}, u_{16}, u_{19}\}) \right\}$$

Step 2: The decision maker considers a 4-level linguistic modifier set as

$$M^4 = \{m_1, m_2, m_3, m_4\}$$

where, for $i \in I_4$, m_i stand for “very”, “more or less”, “not so” and “quite” respectively.

Step 3: For all $i \in I_7$, 4-level linguistic modified set of x_i can be obtained respectively as

$$M_{x_1}^4 = \{x_{11}, x_{12}, x_{13}, x_{14}\}, M_{x_2}^4 = \{x_{21}, x_{22}, x_{23}, x_{24}\}, M_{x_3}^4 = \{x_{31}, x_{32}, x_{33}, x_{34}\} \\ M_{x_4}^4 = \{x_{41}, x_{42}, x_{43}, x_{44}\}, M_{x_5}^4 = \{x_{51}, x_{52}, x_{53}, x_{54}\}, M_{x_6}^4 = \{x_{61}, x_{62}, x_{63}, x_{64}\} \\ M_{x_7}^4 = \{x_{71}, x_{72}, x_{73}, x_{74}\}$$

Step 4: Assume that the decision maker can construct 4-level expansion set of $(x_i, f(x_i))$, f_{x_i} , for $i \in I_7$, respectively as,

$$f_{x_1} = \{(x_{11}, \{u_3, u_5, u_8, u_9\}), (x_{12}, \{u_1, u_2, u_{10}\}), (x_{13}, \{u_{11}, u_{20}\}), \\ (x_{14}, \{u_{14}, u_{17}\})\} \\ f_{x_2} = \{(x_{21}, \{u_3, u_5, u_8, u_9\}), (x_{22}, \{u_1, u_4, u_6, u_{10}, u_{12}, u_{15}, u_{18}\}), \\ (x_{23}, \{u_2, u_7, u_{11}, u_{13}, u_{14}\}), (x_{24}, \{u_6, u_{16}, u_{17}, u_{19}, u_{20}\})\} \\ f_{x_3} = \{(x_{31}, \{u_4, u_{16}, u_{17}\}), (x_{32}, \{u_1, u_{12}, u_{18}, u_{19}\}), (x_{33}, \{u_3, u_5, u_{13}\}), \\ (x_{34}, \{u_8, u_9\})\} \\ f_{x_4} = \{(x_{41}, \{u_8\}), (x_{42}, \{u_4, u_6, u_{14}\}), (x_{43}, \{u_2, u_{10}\}), (x_{44}, \{u_9, u_{18}\})\} \\ f_{x_5} = \{(x_{51}, \{u_4, u_9\}), (x_{52}, \{u_{14}, u_{17}\}), (x_{53}, \{u_7, u_{10}, u_{13}\})\}$$

$$f_{x_6} = \{(x_{61}, \{u_3, u_9, u_{10}, u_{15}, u_{17}, u_{18}\}), (x_{62}, \{u_1, u_2, u_3, u_{11}, u_{12}, u_{13}\}), \\ (x_{63}, \{u_5, u_8, u_{16}\}), (x_{64}, \{u_6, u_7, u_{19}, u_{20}\})\}$$

$$f_{x_7} = \{(x_{71}, \{u_{10}\}), (x_{72}, \{u_2, u_3\}), (x_{73}, \{u_4, u_5, u_8\}), \\ (x_{74}, \{u_{11}, u_{12}, u_{14}, u_{16}, u_{19}\})\}$$

Now, an 4-level expansion family of f can be written by

$$E_4(f) = \{f_{x_1}, f_{x_2}, f_{x_3}, f_{x_4}, f_{x_5}, f_{x_6}, f_{x_7}\}$$

Step 5: The decision maker inputs a 4-level choice set C_4 as

$$C_4 = \{(U, \emptyset, \emptyset, \emptyset), (\emptyset, U, \emptyset, \emptyset), (\emptyset, \emptyset, U, \emptyset), (\emptyset, \emptyset, \emptyset, U)\}$$

Step 6: According to the 4-level choice set a choice values $\alpha^4(x_i)$ can be computed for each $x_i, i \in I_7$, respectively,

$$\alpha^4(x_1) = (U, \emptyset, \emptyset, \emptyset), \alpha^4(x_2) = (U, \emptyset, \emptyset, \emptyset), \alpha^4(x_3) = (\emptyset, \emptyset, \emptyset, U), \\ \alpha^4(x_4) = (U, \emptyset, \emptyset, \emptyset), \alpha^4(x_5) = (U, \emptyset, \emptyset, \emptyset), \alpha^4(x_6) = (\emptyset, \emptyset, U, \emptyset), \\ \alpha^4(x_7) = (\emptyset, \emptyset, U, \emptyset)$$

Step 7: By using the reduction function $\alpha_{x_i}^4, \alpha_{x_i}^4(f_{x_i})$ can be computed for each $x_i, i \in I_7$, respectively,

$$\alpha_{x_1}^4(f_{x_1}) = \{(x_{11}, \{u_3, u_5, u_8, u_9\})\} \\ \alpha_{x_2}^4(f_{x_2}) = \{(x_{21}, \{u_3, u_5, u_8, u_9\})\} \\ \alpha_{x_3}^4(f_{x_3}) = \{(x_{34}, \{u_8, u_9\})\} \\ \alpha_{x_4}^4(f_{x_4}) = \{(x_{41}, \{u_8\})\} \\ \alpha_{x_5}^4(f_{x_5}) = \{(x_{51}, \{u_4, u_9\})\} \\ \alpha_{x_6}^4(f_{x_6}) = \{(x_{63}, \{u_5, u_8, u_{16}\})\} \\ \alpha_{x_7}^4(f_{x_7}) = \{(x_{73}, \{u_4, u_5, u_8\})\}$$

Step 8: α^n -reduction of f can be computed by

$$f_{\alpha^4} = \{\{(x_{11}, \{u_3, u_5, u_8, u_9\})\}, \{(x_{21}, \{u_3, u_5, u_8, u_9\})\}, \{(x_{34}, \{u_8, u_9\})\}, \\ \{(x_{41}, \{u_8\})\}, \{(x_{51}, \{u_4, u_9\})\}, \{(x_{63}, \{u_5, u_8, u_{16}\})\}, \\ \{(x_{73}, \{u_4, u_5, u_8\})\}\}, \{(x_{41}, \{u_8\})\},$$

Step 9: Finally, the reduced set R can be computed by

$$R = uni(f_{\alpha^4}) = \{u_3, u_4, u_5, u_8, u_9, u_{16}\}$$

which is a suitable subset of the set of alternatives U . In this problem, 20 applications is reduced to 6 applicants by the soft reduction method. So, decision maker interviews only 6 applicants instead of 20.

5 Conclusion

In this work, as a new notion on soft set theory, we first defined expansion and reduction of the soft sets based on linguistic modifiers. Using the expansion and reduction we then gave soft reduction method. The aim of this method is to obtain a subset of the set of alternatives through a decision maker. With this method, it is possible to reduce the number of alternatives significantly. We finally presented an example which shows that the method can be successfully applied to many problems containing uncertainties.

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