Induction Suspension of Conductive Microring and Its Gyroscopic Stabilization

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INDUCTION SUSPENSION OF CONDUCTIVE MICRORING and ITS GYROSCOPIC STABILIZATION

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Abstract. The main task of our work is determination of possible levitation of micro-ring with eddy current in magnetic field of down ring with set alternating current and determination of critical value of «ohmic» damping separated field of parameters, at which motions of suspension ring transit from divergent to meeting to steady-state equilibrium position. I. e. in this critical case the motion practically coincides with motions of conservative system. The possibility of gyroscopic stabilization of suspension ring taking into account initial set rotation is considered. Thereby it can serve as contactless micro-gyroscope.

Key words: induction levitation, critical damping, gyroscopic stabilization

Introduction. Induction suspension, as electrodynamical is founded on interaction of eddy current, induced in suspension ring, with alternating in time external magnetic field set by alternating current of down ring. In electrodynamic suspension eddy current is created in conductive body taking into account the motion of magnetic field sources, in induction suspension the sources are not mobile, and external magnetic field periodically changes. The large attention to induction suspension the groups of German scientists Jan Gerrit Korvink and Kirill Poletkin et al are rendered [1-3]. The theoretical justification of existence the critical damping at excitement of alternating magnetic field by contour with alternating current can be found in article K. Sh. Khodzhaev and S.D. Shatalov [4]. The case of closed current in work of Yu. G. Martynenko [5] is generalized on a case then the directing contour is powered by alternating EMF. The main moment, which is noted in work [4], is that at presence of slow motion in this electromechanical system (slow in relation to period of external action), the eddy currents in first approximation give conservative system and for investigation the equilibrium stability it's necessary the search of second approximation of eddy currents. Moreover, at the frequency of electromagnetic influence more than values inverting to attenuation time of clear electric system it appears that interaction of eddy currents with alternating magnetic field renders divergent influence, for damping of which it's necessary sufficiently large external «mechanical» damping. The example of this can be found in work of D.Yu. Skubov, D.S. Vavilov [6], in which equilibriums and motions of conductive pendulum in magnetic field of coil with alternative current are considered. In this work it's shown that at some parameters the upper position of pendulum may be stable and self-oscillations nearly down and upper position of pendulum are possible.

In our work levitation of conductive ring in magnetic field or coaxial moveless ring with alternating current situated in parallel plane is considered. Levitation arises for the account induction of eddy current in free ring. The main purpose of this work is determination the critical value of «ohmic» damping in influence from main parameter of system (ratio of amplitude magnetic and kinetic energy) separated the field of parameters on divergent and fading motion of upper wring (meeting to steady-state equilibrium position). For simplicity only vertical motion of upper ring is considered.

The motions of this electromechanical system are described by Lagrange-Maxwell equations [7]. Also, the possibility of gyroscopic stabilization of upper ring at small angles of deviations from vertical is considered.

1. Mathematical model

The energy of magnetic field consists of combination of energies of down ring powered by alternating current $I_0 cos\ \omega t$ and upper ring with eddy current. The expression of magnetic energy has a form

$$W = IM_{01}(y)I_0 \cos \omega t + \frac{1}{2}LI^2. \tag{1}$$

The energy of suggestive ring as known time function doesn't include in Lagrange-Maxwell equation. In equation (1) $L = R_0\mu_0\left(ln\frac{8R}{h+\delta} - 0.5\right) -$ inductance of linear conductor with rectangular section at low current frequency ($R_0 -$ median radius of ring, $h -$ high of section, $\delta -$ section width on radius), $M_{01} = \mu_0 R_0\left[\left(\frac{2}{\kappa} - \kappa\right)\mathrm{K}(\kappa) - \frac{2}{\kappa}\mathrm{E}(\kappa)\right] -$ mutual inductance of rings, $\kappa^2 = \frac{4R_0^2}{4R_0^2+y^2} -$ elliptic module, $y -$ distance in vertical between upper and down coaxial rings [7], $I -$ eddy current in suspended ring. It's supposed, that at small angles of deviation of upper ring from vertical eddy currents are induced in coaxial circles.

Mechanical potential gravitation energy and kinetic energy are written by formulas

$$\Pi = \widetilde{m}gy, \quad T = \frac{1}{2}\widetilde{m}\dot{y}^2, \tag{2}$$

where $\widetilde{m}$ −mass of ring. Electric dissipation function is

$$\Phi = \frac{1}{2}RI^2 \tag{3}$$

$R = \frac{\varrho 2\pi R_0}{ar} -$ ohmic resistance of ring.

Lagrange-Maxwell equations of described electromechanical system

$$\ddot{y} - \frac{dM_{01}}{dy}I_1 I_0 \cos \omega t + \widetilde{m}g = 0. \tag{4}$$

$$L\dot{I} + \frac{dM_{0n}}{dy}\dot{y}I_0 \cos \omega t - M_{01}(y)I_0\omega \sin \omega t + R_n I_n = 0 \tag{5}$$

At calculation of derivation $\frac{dM}{dy}$ the known relations are used

$$\frac{\partial \mathrm{K}}{\partial \kappa} = \frac{1}{\kappa}\left(\frac{E(\kappa)}{1-\kappa^2} - \mathrm{K}(\kappa)\right), \quad \frac{\partial E}{\partial \kappa} = \frac{1}{\kappa}(E(\kappa) - \mathrm{K}(\kappa)). \tag{6}$$

For dimensionless system of Lagrange-Maxwell equations the next relations and dimensionless time $\tau = \omega t$ are inputted

$$\xi = \frac{y}{2R_0}, \ i = \frac{I}{I_0}, \ r = \frac{R}{L_0\omega}, m_{01} = \frac{M_{01}}{L_0}, \tag{7}$$

Dimensionless mutual inductance and its derivations contained in system (6), (7) are written in view:

$$m_{01} = \beta\left[\left(\frac{2}{\kappa} - \kappa\right)\mathrm{K}(\kappa) - \frac{2}{\kappa}\mathrm{E}(\kappa)\right], \quad \kappa = \sqrt{\frac{1}{1+\xi^2}}, \beta = \frac{\mu_0 R_0}{L_0},$$

$$\frac{\mathrm{d}m_{01}}{\mathrm{d}\xi} = \frac{\partial m_{01}}{\partial\kappa}\frac{\partial\kappa}{\partial\xi} = -\beta\frac{1}{\kappa^2}\left[\frac{(2-\kappa^2)\mathrm{E}(\kappa)}{(1-\kappa)} - 2\mathrm{K}(\kappa)\right]\frac{\xi}{(1+\xi^2)^{3/2}} \tag{8}$$

Thus, we obtain next system dimensionless equations

$$i' + \frac{dm_{01}}{d\xi}\xi'\cos\tau - m_{01}(\xi)\sin\tau + ri = 0, \tag{9}$$

$$\xi'' - \varepsilon^2\alpha\frac{dm_{01}}{d\xi}i\cos\tau + \varepsilon^2 = 0. \tag{10}$$

$(\ )'$ designates dimensionless time derivative. Let's believe that ratio $\frac{g}{2R_0\omega^2} = \varepsilon^2$ and also ratio of energies $\frac{L_0 I_0^2}{4R_0^2\omega^2} = \varepsilon^2\alpha$, $\alpha \sim O(1)$ values of the second order of smallness, that coincides with technical applications in microscale (MEMS). System (10), (11) may be solved numerically directly, but in this case it's difficult to find border of parameters, which corresponds to transition to divergent of oscillation.

2. Asymptotic transformation.

Let's $\xi' = \varepsilon\zeta$, then at including small «mechanical» damping with coefficient ν the equation (9), (10) rewritten in form

$$\begin{gathered}\xi' = \varepsilon\zeta,\\ \zeta' = \varepsilon\alpha\frac{dm_{01}}{d\xi}i\cos\tau - 2\varepsilon\nu\zeta - \varepsilon,\\ i' + \varepsilon\frac{dm_{01}}{d\xi}\zeta\cos\tau - m_{01}(\xi)\sin\tau + ri = 0.\end{gathered} \tag{11}$$

For this type of systems, for which currents are noncritical fast variables [4], substitution of first approximation for currents into two first «slow» equations with subsequent averaging gives conservative system. Therefore, it's necessary to find second approximation for current, believed that slow variables ξ, ζ are constants. From the structure of equation for current itself it can see that this equation has periodical solution.

Let's search second approximation for eddy current, used asymptotic decomposition

$$\begin{gathered}\xi = \xi_0 + \varepsilon\xi_1(\xi,\zeta,\tau) + \cdots,\\ \zeta = \zeta_0 + \varepsilon\zeta_1(\xi,\zeta,\tau) + \cdots,\\ i_n = i_{n0} + \varepsilon i_{n1}(\xi,\zeta,\tau) + \cdots,\end{gathered} \tag{12}$$

Stationary solution for current in first approximation, believed that $\xi_0, \zeta_0 = const$, is searched in view

$$i_0 = A_0\cos\tau + B_0\,\mathrm{sins}\,\tau. \tag{14}$$

Balanced terms contain $\sin\tau$ and $\cos\tau$, we obtain the algebraic system of equations

$$-A_0 + rB_0 = m_0 a, \qquad rA_0 + LB_0 = 0. \tag{15}$$

Its solution is

$$A_0 = -\frac{1}{r+r^{-1}}r^{-1}m_{01}, \quad B_0 = \frac{1}{r+r^{-1}}m_{01} \tag{16}$$

After substitution of found first approximation for eddy currents into expression for ponderomotive force and subsequent averaging we obtain

$$P_1 = \langle \varepsilon\alpha \frac{dm_{01}}{d\xi} i_0 \rangle = \frac{\varepsilon\alpha}{2} \frac{dm_{01}}{d\xi} A_0 = -\frac{\varepsilon\alpha}{2} \frac{dm_{01}}{d\xi} \frac{1}{1+r^2} m_{01} = -\varepsilon\alpha \frac{d\Lambda}{d\xi} \quad (17)$$

where $\Lambda = \frac{1}{4(1+r^2)} m_{01}^2$ − potential energy. Thus, averaging electromagnetic forces calculated in first approximation are potentiality and their potential is averaging value of eddy current magnetic energy Λ, found in same approximation. In first approximation we have conservativity system

$$\xi' = \varepsilon\zeta,$$
$$\zeta' = -\varepsilon\alpha \frac{d\Lambda}{d\xi} - \varepsilon. \quad (19)$$

Because mutual inductance m_{01} is decreasing function in vertical its derivation is negative and at determinate combination of parameters the contactless equilibrium position is possible. The periodic solution of averaging system is shown in Fig.1

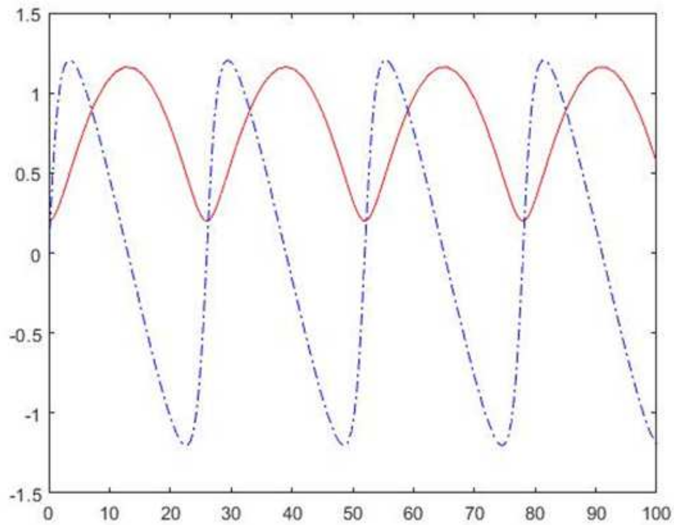

Fig.1 Periodical solution of system (19) $\alpha = 1.5, \beta = 2, \varepsilon = 0.1$

The motion of system (19) with introduced external dissipation $2\varepsilon n\zeta$ is shown in Fig. 2

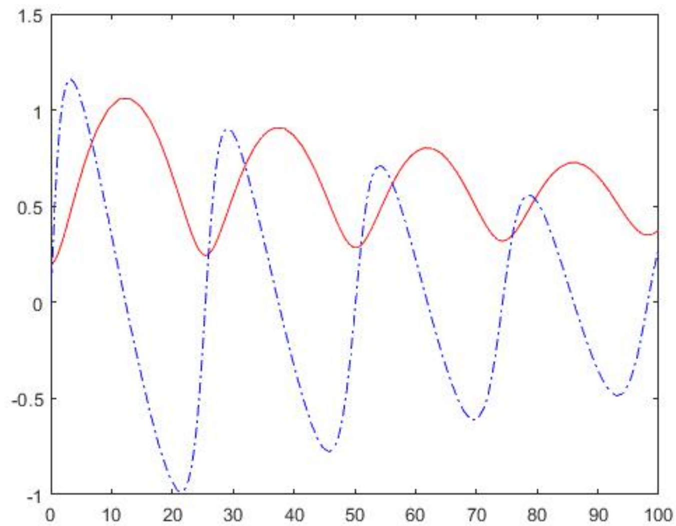

Fig. 2 $\alpha = 1.5, r = 0.1, \beta = 2, \varepsilon = 0.1, n = 0.1$

From Fig.3 it can see that motion aims to equilibrium position $\xi \approx 0.5$, corresponded to contactless equilibrium of upper ring.

The system of equations for components of current in second approximation is

$$(i_1' + \frac{di_0}{d\xi}\zeta) + ri_1 = -\frac{dm_{01}}{d\xi}\zeta \cos\tau, \tag{20}$$

Its periodic solution we search in view

$$i_1 = A_1 \cos\tau + B_1 \sin\tau \tag{21}$$

Balanced by $\sin\tau$ and $\cos\tau$ we have algebraic system of two equations

$$A_1 - rB_1 = 0,$$

$$B_1 + rA_1 = -\frac{\partial A_0}{\partial\xi}\zeta - \frac{\partial m_{01}}{\partial\xi}\zeta = \frac{1-r}{1+r^2}\frac{\partial m_{01}}{\partial\xi}\zeta \tag{22}$$

The solution of this system is written in form

$$A_1 = \frac{r(1-r)}{(1+r^2)^2}\frac{\partial m_{01}}{\partial\xi}\zeta, \quad B_1 = \frac{(1-r)}{(1+r^2)^2}\frac{\partial m_{01}}{\partial\xi}\zeta \tag{23}$$

Then average ponderomotive force in second approximation will be equal

$$P_2 = < \varepsilon\alpha\frac{dm_{01}}{d\xi} i_1 \cos\tau > = \varepsilon\alpha\frac{1}{2}\frac{dm_{01}}{d\xi}A_1 = \frac{\varepsilon\alpha}{2}\frac{dm_{01}}{d\xi}\frac{r(1-r)}{(1+r^2)^2}\frac{dm_{01}}{\partial\xi}\zeta \tag{24}$$

After substitution mechanical damping of second order with coefficient $2\varepsilon\nu$ we obtain the system of equations of mechanical motions in second approximation

$$\xi' = \varepsilon\zeta,$$

$$\zeta' = -\frac{\varepsilon\alpha}{4(1+r^2)}\frac{dm_{01}}{d\xi}m_{01} + \frac{\varepsilon\alpha}{2}\frac{dm_{01}}{d\xi}\frac{r(1-r)}{(1+r^2)^2}\frac{dm_{01}}{\partial\xi}\zeta - 2\varepsilon\nu\zeta - \varepsilon, \tag{25}$$

Hence it follows, that at $r > 1$, i. e. when frequency of external field (current in suggestive ring) is smaller than the value, invert to attenuation time of electromagnetic process in contactless hanging ring, averaging ponderomotive forces in second approximation play the dissipation role. And, contra versa, at $r < 1$ can be «divergent», that is determinates by their influence from position coordinates ξ.

Numerically it can be identified the relation of parameters, at which the border of relation of parameters corresponded to difference of divergent and attenuating motions takes place.

In Fig.3 it's shown the motion convergent to equilibrium position at $r = 1.2$

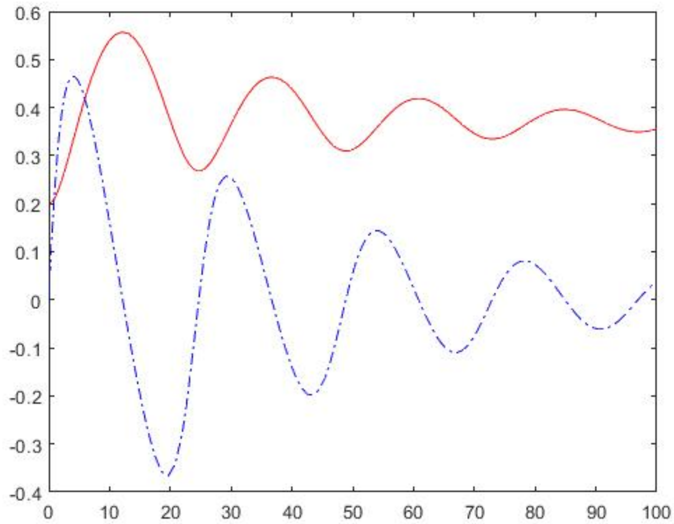

Fig.3 $\alpha = 1.5, r = 1.2, b = 2, \varepsilon = 0.1, \nu = 0.1$

In Fig.4 it is shown divergent motions at $r = 0.8$ for averaged system

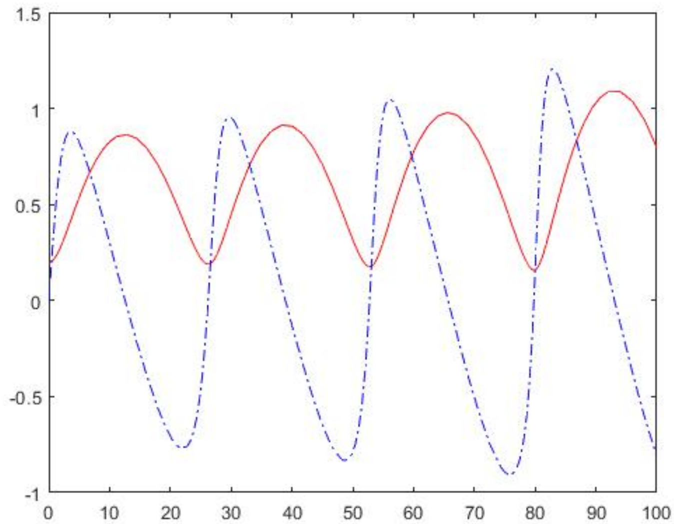

Рис.4 $\alpha = 1.5, r = 0.85, b = 2, \varepsilon = 0.1, \nu = 0.1$

Let's try to find the critical value r, separating convergent and divergent motions. Critical case practically non-fading oscillation in averaging system is shown in Fig.5.

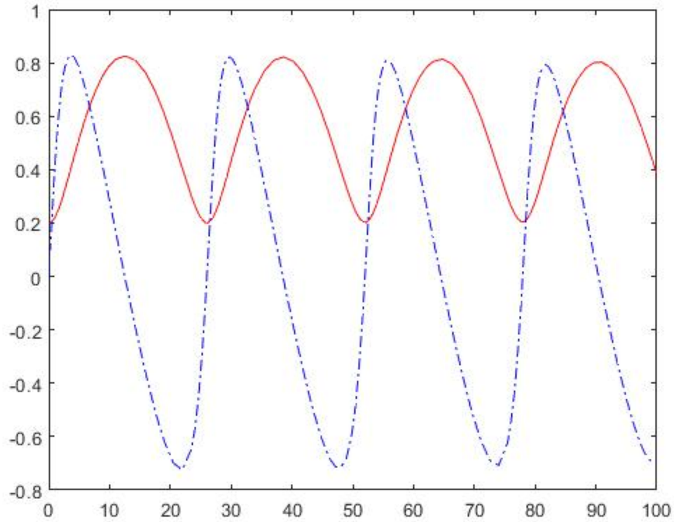

Fig.5 $\alpha = 1.5, r = 0.885, b = 2, \varepsilon = 0.1, \nu = 0.1$

Periodical motions, which is shown in Fig.5, by the character is similar to motion of conservative system is shown in Fig.1 but another smaller amplitude of oscillation.

3. Gyroscopic stabilization

Let's consider the motion of levitating ring near from vertical taking into account gyroscopic stabilization. The position of ring axe is determined with help gimbal angle α and β, which in difference from Euler angles allow you to conduct linearization of motion equations at small deviation from vertical. The projections of vector of angular velocity on cartesian axes, connected with suspension ring (axe OZ is directed on the ring axis) are equal

$$\omega_x = \dot{\alpha}\, cos\beta,$$

$$\omega_y = \dot{\beta}, \qquad (25)$$

$$\omega_z = \dot{\alpha}\, sin\beta + \dot{\varphi}.$$

Here $\dot{\varphi}$ − angular velocity of ring rotation relatively of its axis.
Kinetic energy o ring

$$T = \frac{1}{2}\left(J_x\omega_x^2 + J_y\omega_y^2 + J_z^2\right) + \frac{1}{2}\tilde{m}\dot{z}^2 \qquad (26)$$

Designated inertial moments $J_x = J_y = A,\ J_z = 2A$ and used eq. (19), let's written (20) in a view

$$T = \frac{1}{2}\left(A\left(\dot{\alpha}^2 cos^2\beta + \dot{\beta}^2\right) + 2A(\dot{\alpha}\, sin\beta + \dot{\varphi})^2\right) + \frac{1}{2}\tilde{m}\dot{z}^2 \qquad (21)$$

Potential energy

$$\Pi = mgz\, cos\alpha\, cos\beta + mgz, \qquad (22)$$

here z − lifting height of ring center. In case of consideration a levitation of ring variable z is alternative. Generally speaking, mutual inductance of rings depends from angles of inclination, that itself is a separate task.

The coordinate φ is a cyclic coordinate, corresponding momentum is a constant (integral of motion)

$$2A(\dot{\alpha}\, sin\beta + \dot{\varphi}) \doteq p_{\varphi 0} = 2A\omega_{z0} \qquad (23)$$

Availability of cyclic integral allows you to exclude cyclic velocity and write the equations of motion in form of Routh equations

$$R = (T - p_{\varphi 0}\dot{\varphi})|\ \dot{\varphi}{=}\omega_{z0} - \dot{\alpha}sin\beta + \frac{1}{2}\tilde{m}\dot{z}^2,$$

$$R = \frac{1}{2}A\left(\dot{\alpha}^2cos^2\beta + \dot{\beta}^2\right) + p_{\varphi 0}\dot{\alpha}\ sin\beta + \frac{1}{2}\tilde{m}\dot{z}^2const. \tag{24}$$

In a task about small angular oscillations the Routh function and potential energy are simplified

$$R = \frac{1}{2}A\left(\dot{\alpha}^2 + \dot{\beta}^2\right) + p_{\varphi 0}\dot{\alpha}\ \beta + \frac{1}{2}\tilde{m}\dot{z}^2 + const$$

$$\Pi = -\frac{1}{2}mgz(\alpha^2 + \beta^2) + mgz + const \tag{25}$$

Routh equations for positional coordinates α, β have a form

$$A\ddot{\alpha} + p_{\varphi 0}\dot{\beta} - mgyz = 0,$$

$$A\ddot{\beta} - p_{\varphi 0}\dot{\alpha} - mgyz = 0. \tag{26}$$

We reduce the equation to dimensionless form, by entering dimensionless time $\tau = \omega t$

$$\alpha'' + 2\nu\beta' - \lambda\alpha\xi = 0,$$

$$\beta'' - 2\nu\alpha' - \lambda\beta\xi = 0, \qquad \xi = {}^{z}/_{2R_0}, \tag{27}$$

where $\nu = \frac{p_{\varphi 0}}{A\omega} = \frac{\omega_{z0}}{\omega} -$ ratio of rotation frequency and frequency of current. In result we have set of differential equations, interconnected via lift heigh of levitation ring

$$\xi' = \varepsilon\zeta,$$

$$\zeta' = \varepsilon\alpha\frac{dm}{d\xi}i_1cos\tau - \varepsilon - 2\varepsilon n\zeta, \tag{12}$$

$$i' + ri = m(\xi)sin\tau - \varepsilon\frac{dm}{d\xi}\zeta\cos\tau$$

$$\alpha'' + 2\nu\beta' - \lambda\alpha\xi = 0,$$

$$\beta'' - 2\nu\alpha' - \lambda\beta\xi = 0,$$

Their numerical solution was carried out by the MATLAB program. Motion of ξ, ζ is shown in Fig.8. At the same time continuous vertical oscillation occur. And there is a continuous rotation relatively to the vertical, as it shown in Fig.9

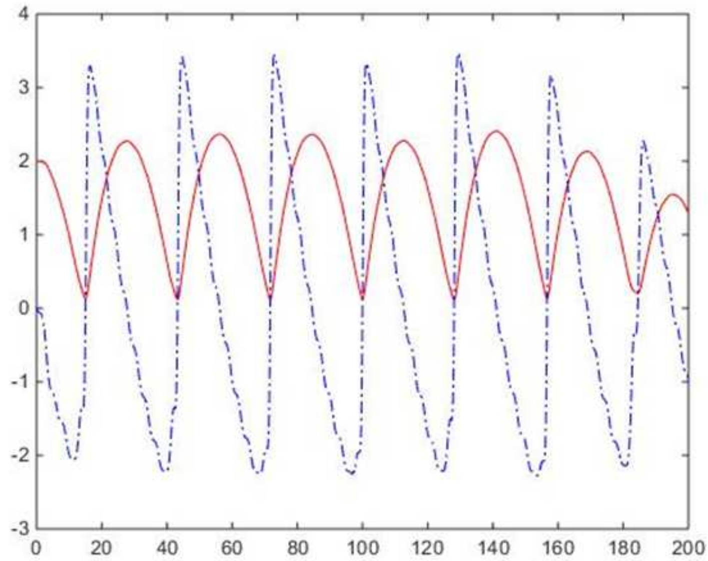

Fig.8 Displacement and speed of vertical at parameters

$\alpha = 1.5, r = 0.2, b = 0.2, n = 0.2, \varepsilon = 0.1, \nu = 10, \lambda = 1$

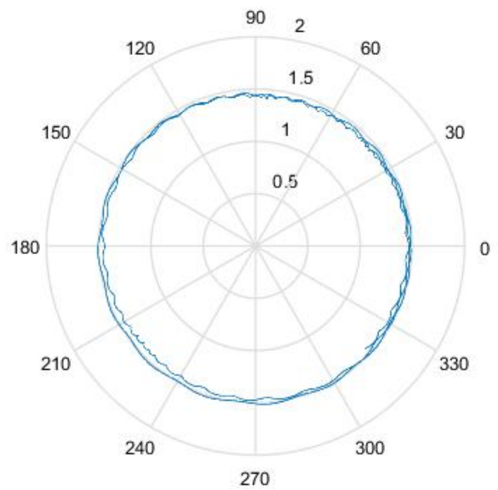

Fig.9 $\alpha = 1.5, r = 1.5, b = 2, n = 0.1, \varepsilon = 0.1, \nu = 5, \lambda = 1$

Conclusion. At parameters appropriate technical application (MEMS) the levitation of micro-ring was considered. Critical value of «ohmic» damping separated field of parameters, at which motions of suspension ring transit from divergent to meeting to steady-state equilibrium position was found. I. e. in this critical case the motion practically coincides with motions of conservative system. It was shown the possibility of gyroscopic stabilization of micro-ring at small angles of deviation.

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Conflict of interest. The authors declare that they have no conflict of interest.

Reference

1. Kirill V. Poletkin, Asa Asadallanbaik, Ronald Kampmann and Jan G. Korvink. A Review. Levitating Micro-Actuators// Actuators 2018, 7, 17
2. Kirill V. Poletkin, Jan G. Korvink. Modeling a Pull-In Instability in Micro Mashined Hibrid Contactless Suspension// Actuators 2018, 7, 11
3. Kirill V. Poletkin, Zhgiu Lu, Ulrika Wallraba, Vlad Badilita. Journal of Microelectromechanical System. October. 2015
4. K.Sh. Hodzhaev, S.D. Shatalov. Slow motions of conductive bodies in magnetic field // Proced. AS USSR. Mechanic of solid bodies, 1981, No.2, p.175-182.
5. Martynenko Yu. G. The motion of solid body in electric and magnetic fields. – M.: Nauka, 1988
6. D.Yu. Skubov, D.S. Vavilov. Dynamics the conductivity solid bodies in high-frequency alternating magnetic field// Acta Mechanic, V.224, No.12, December 2013
7. Kalantarov P.L. Zeitlin L.A. Calculation of inductances. Reference book. Leningrad. Energoatomizdat, 1986
8. Skubov D.Yu., Khodzhaev K.Sh. Non-Linear Electromechanics, Springer-Verlag, Berlin Heidelberg, 2008, P.395.

Figures

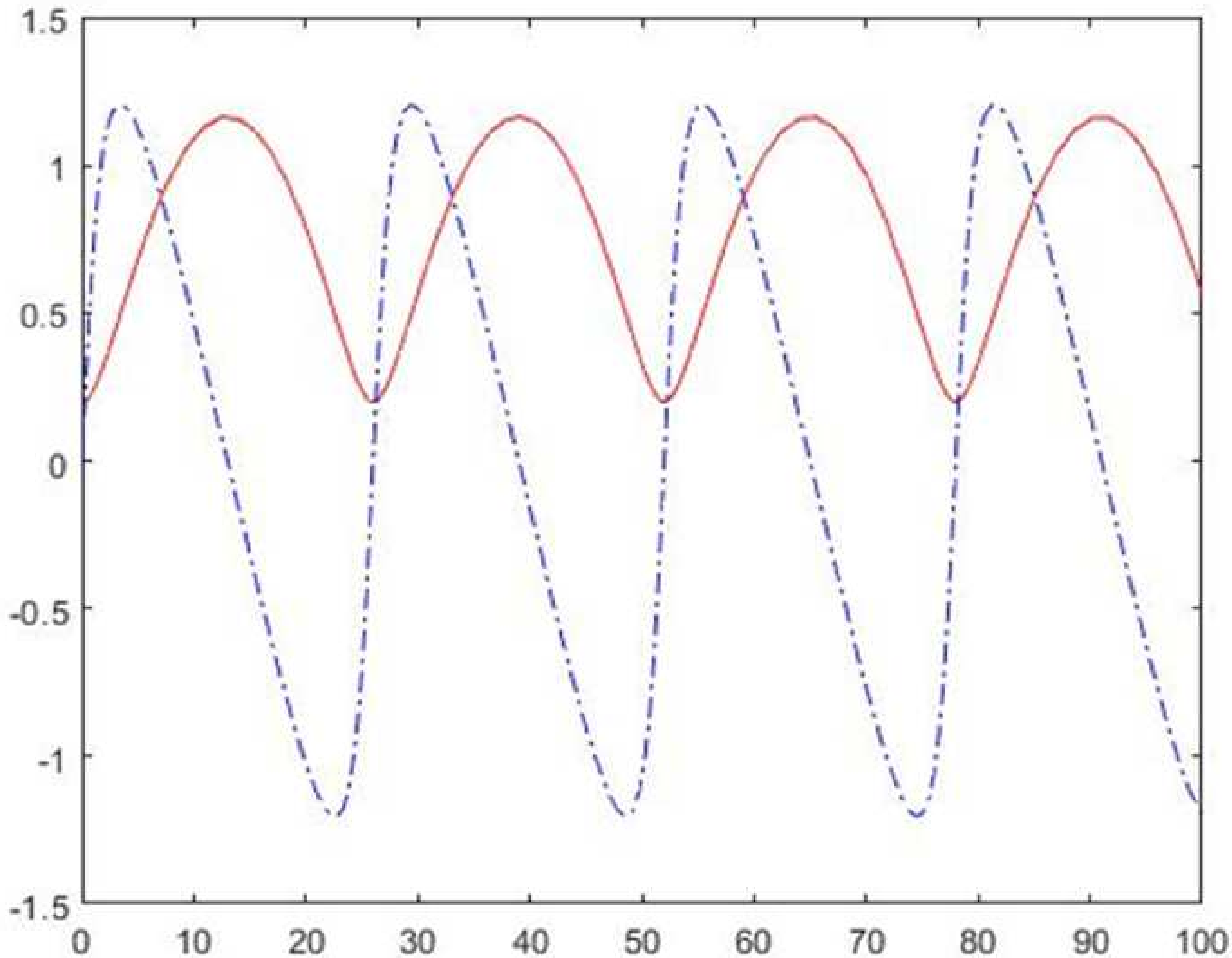

Figure 1

Periodical solution of system (19) $\alpha=1.5,\beta=2,\varepsilon=0.1$

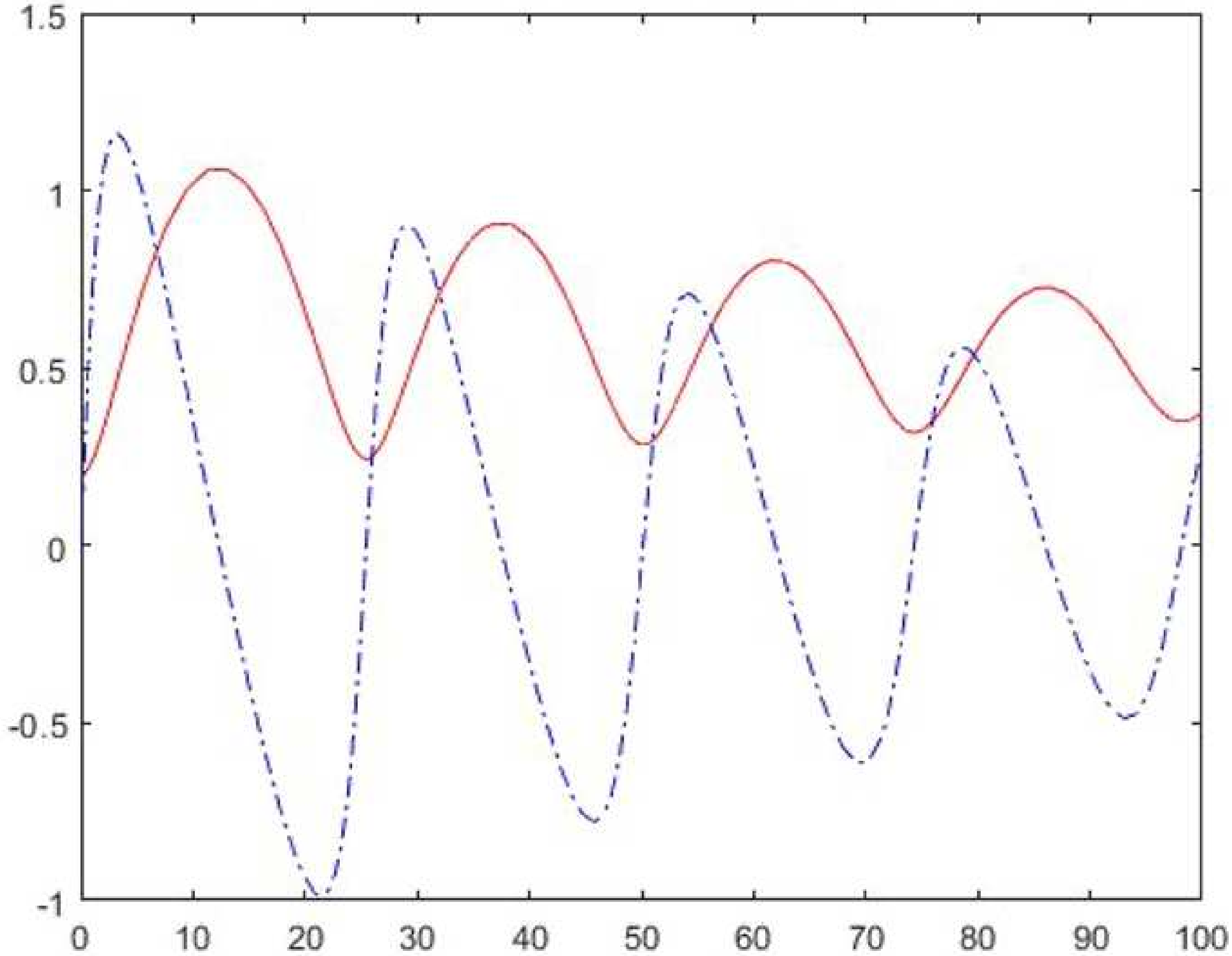

Figure 2

The motion of system (19) with introduced external dissipation $2\varepsilon n\zeta$ is shown. $\alpha=1.5, r=0.1, \beta=2, \varepsilon=0.1, n=0.1$

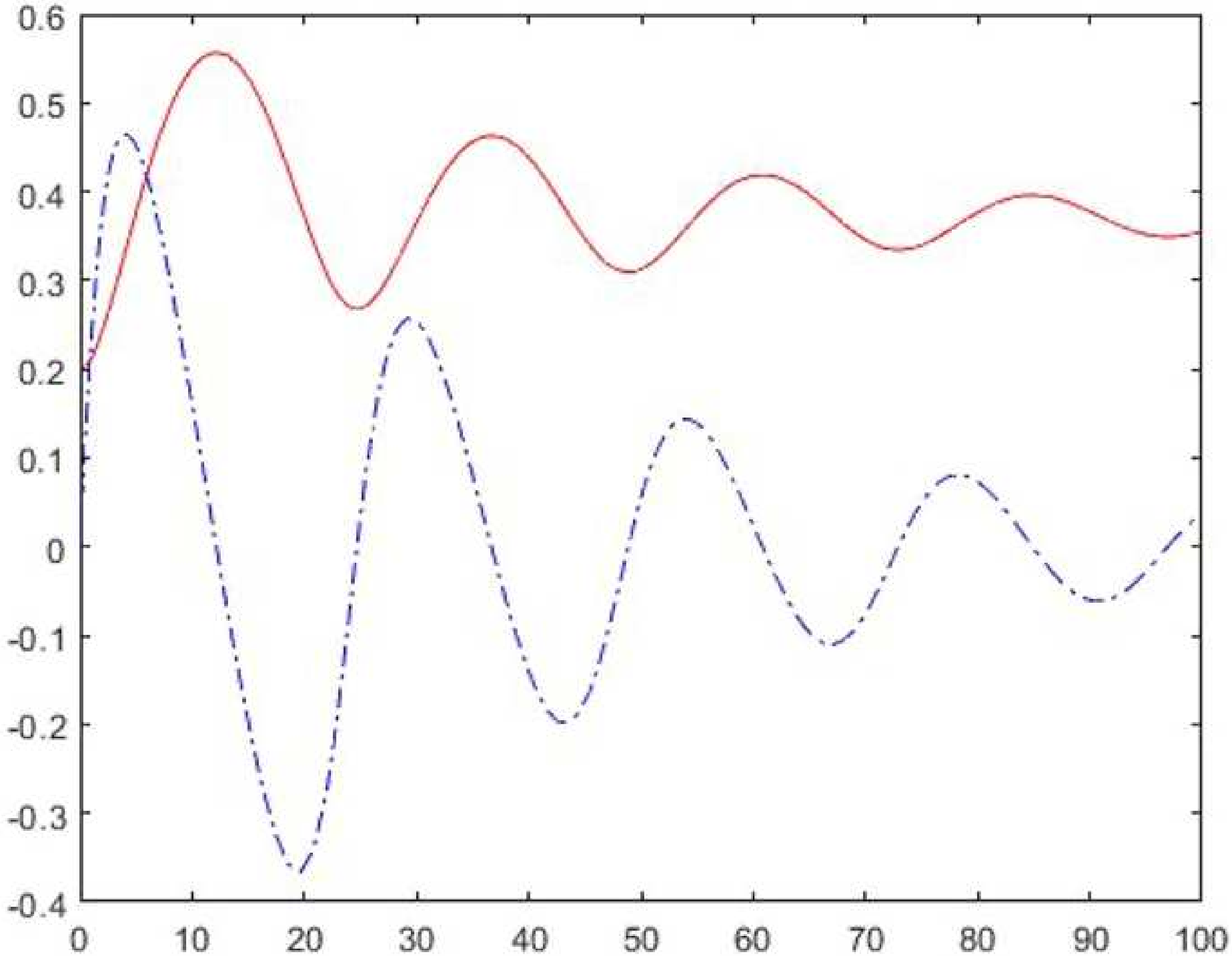

Figure 3

the motion convergent to equilibrium position at r=1.2. α=1.5,r=1.2,b=2,ε=0.1,v=0.1

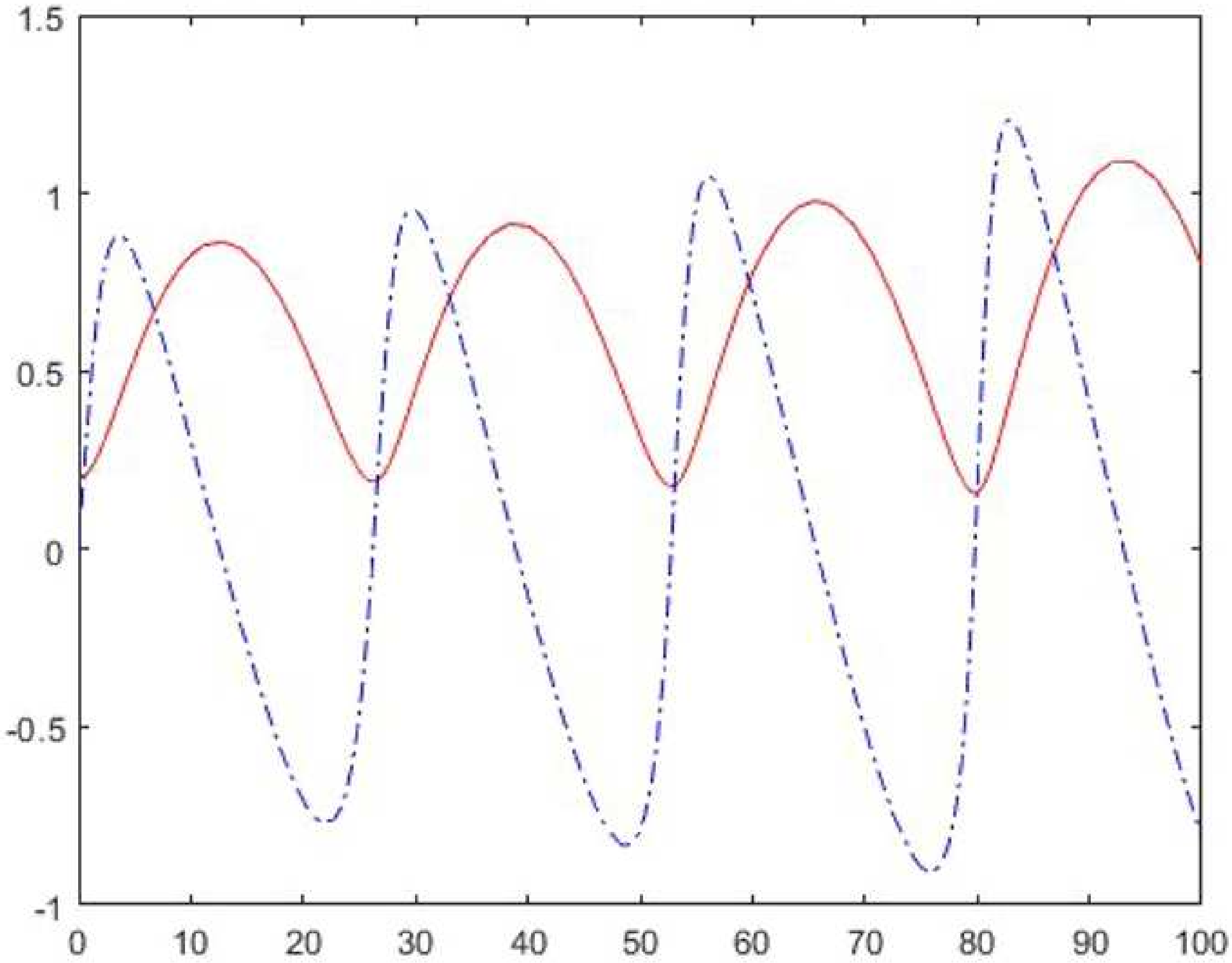

Figure 4

divergent motions at r=0.8 for averaged system. $\alpha=1.5, r=0.85, b=2, \varepsilon=0.1, v=0.1$

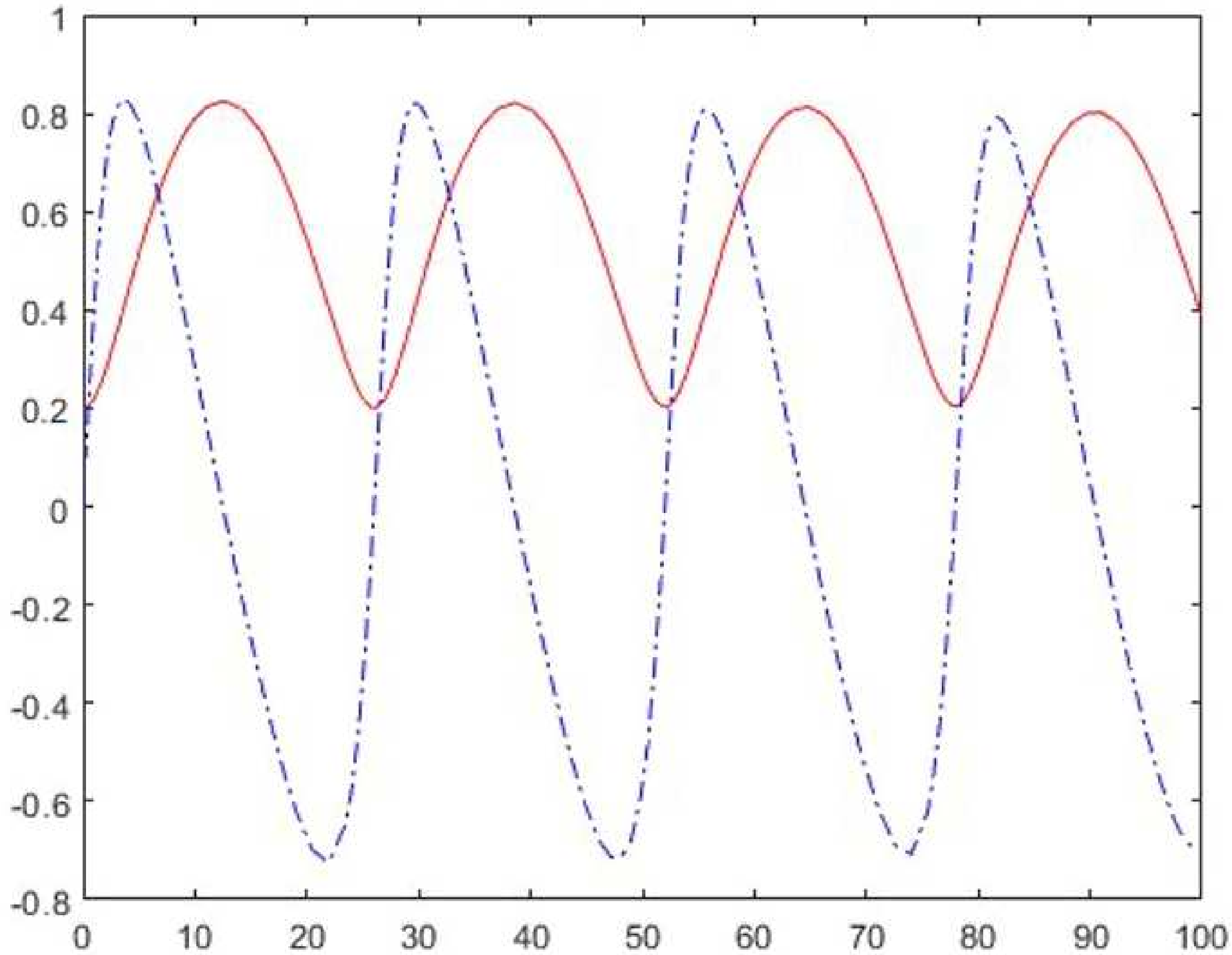

Figure 5

Critical case practically non-fading oscillation in averaging system. α=1.5,r=0.885,b=2,ε=0.1,ν=0.1

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Figure 6

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Figure 7

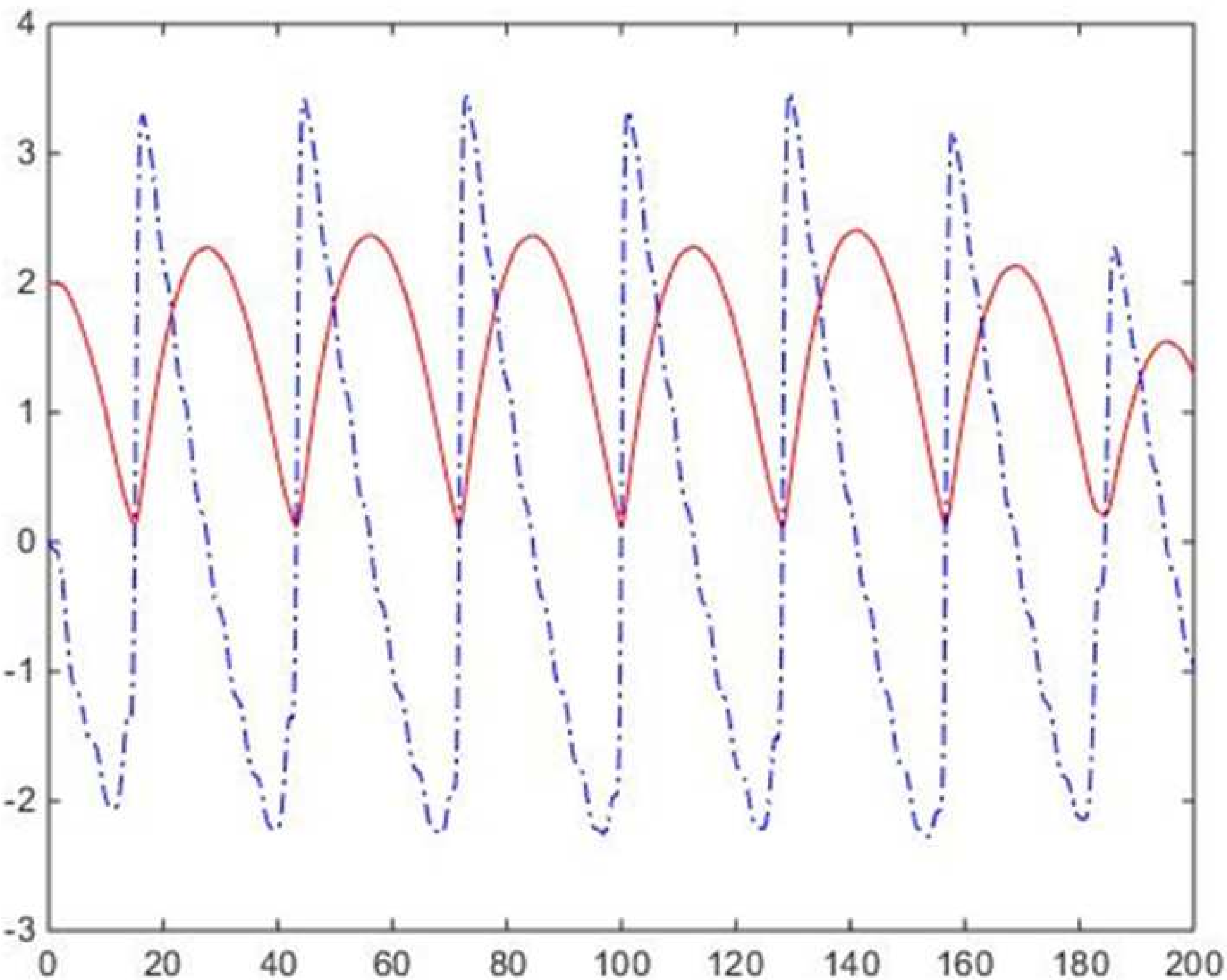

Figure 8

Motion of ξ,ζ. Displacement and speed of vertical at parameters. $\alpha=1.5, r=0.2, b=0.2, n=0.2, \varepsilon=0.1, v=10, \lambda=1$

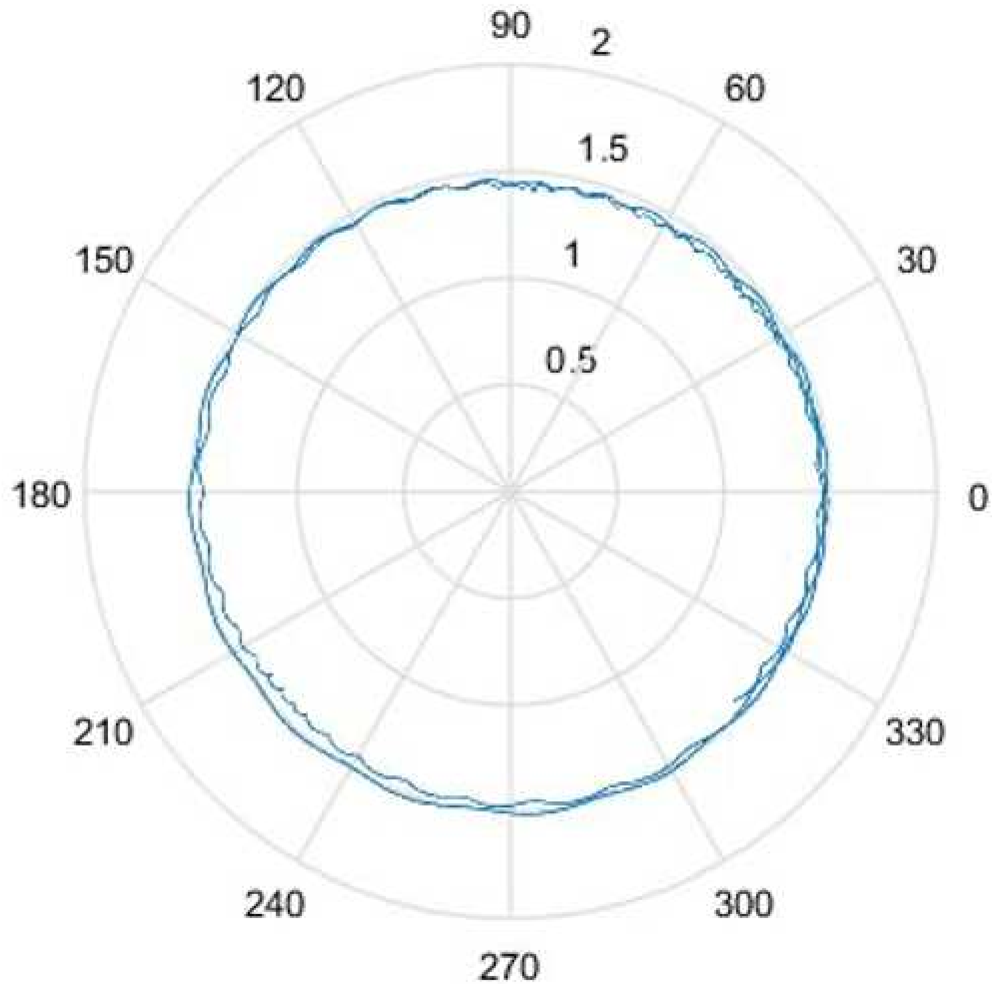

Figure 9

continuous rotation relatively to the vertical. $\alpha=1.5, r=1.5, b=2, n=0.1, \varepsilon=0.1, v=5, \lambda=1$