Wave Behaviors of Kundu–Mukherjee–Naskar Model Arising In Optical Fiber Communication Systems with Complex Structure

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Abstract

Rogue waves are very mysterious and extra ordinary waves. They appear suddenly even in a calm sea and are hard to be predicted. Although nonlinear Schrödinger equation (NLS) provides a perspective, it alone can neither detect rogue waves nor provide a complete solution to problems. Therefore, some approximations are still mandatory for both obtaining an exact solution and predicting rogue waves. Such as Kundu-Mukherjee-Naskar (KMN) model which allows obtaining lump-soliton solutions considered as rogue waves. In this study the functional variable method is utilized to obtain the analytical solutions of KMN model that corresponds to the propagation of soliton dynamics in optical fiber communication system.

Keywords: KMN model, functional variable method, Rogue waves.

1. Introduction

Wave motion is very predictable in basic level and can be explained very deterministic way. However, it becomes much more complicated with taking the nonlinear interactions into account. This type of systems called nonlinear or dynamical systems. Although, there has been some development in the mathematical field to understand the behavior of a nonlinear system we are still far from predicting the behavior of such systems deterministically. The first studies on this issue date back almost 150 years to Riemann and stokes. And the studies are continuing with increasing importance and interest [4-6].

The basic idea is concentrated on two types of wave behaviors which are hyperbolic [1] and dispersive. Hyperbolic wave behavior can be formulated mathematically in terms of hyperbolic partial differential equations. Klein-Gordon [13] is a prototype for hyperbolic wave equation. Dispersive waves cannot be characterized easily. Nonhyperbolic waves generally categorized as dispersive waves. However, classification is made on the type of solution rather than on the equations. Korteweg-DeVries equation [14] can be a good example for dispersive waves. There are many equations developed for determining the wave behavior of a dynamical system [7-24].

For a small intersection of a spatiotemporal system the dynamics can be assumed as linear. However, they must be evaluated in terms nonlinear dynamics due to significant modulation of the wave amplitude originated from cumulative nonlinear interactions. Nonlinear

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Schrödinger (NLS) equation is a very common equation which is providing a canonical design of involucre dynamics of a quasi-monochromatic planar wave propagating in a weakly nonlinear dispersive medium when dissipative effects are insignificant [25,26,27].

NLS is employed for many situational models such as propagation of a wave in a Kerr type [28,29] or non- Kerr type [2] medium. Most of them are not fully integrable which means exact solutions cannot be obtained directly. Only approximate numerical solutions with no stable solitons can be obtained [30,31]. Approximations cannot predict rogue waves which can be defined as "localized and isolated surface waves, apparently appear from nowhere, make a sudden hole in the sea just before attaining surprisingly high amplitude and disappear again without a trace" KMN [3]. They proposed a model to by extension of NLS to have an integrable form which allows lump-soliton can be considered as rogue wave model;

$$iq_t = d_1 q_{xx} - d_2 q_{yy} + 2iq\left(\sqrt{d_1}\, j^x - \sqrt{d_2}\, j^y\right),\; j^a \equiv qq_a^* - q^* q_a. \tag{1.1}$$

And then, they replaced the conventional amplitude-like nonlinear term with the a current-like nonlinear term which allows them to obtain a fully integrable form of NLS;

$$iq_t + q_{xy} + 2iq\left(qq_x^* - q^* q_x\right) = 0. \tag{1.2}$$

In this study the wave solutions of KMN model

$$iq_t + \alpha q_{xy} + i\beta q(qq_x^* - q^* q_x) = 0. \tag{1.3}$$

which describes the propagation of soliton dynamics in optical fiber communication system. Yıldırım [35] obtained dark, bright and singular solitons by using trial equation technique for KMN model. Rivzi et al. [36] used csch method, extended Tanh–Coth method and extended rational sinh-cosh method to get the exact solutions of KMN model. Talarposhti et al. [37] employed Exp-function method to yield the optical soliton solutions of considered KMN model.

This work is structured as follows: In Section 2, mathematical analysis of KMN model is given. In Section3, we demonstrate the structure of the functional variable method. In Section 4, we apply this method to find some wave solutions of the equation written above. In Section 5, we give the results and discussion, Section 6 gives the conclusion of the whole research.

2. Mathematical Analysis

In order to get started, the following hypothesis is selected:

$$q(x,y,t) = P(\eta)\exp\left[i\,\Phi(x,y,t)\right], \tag{2.1}$$

where P(ξ) represents the amplitude portion and

$$\xi = \chi_1 x + \chi_2 y - \sigma t, \tag{2.2}$$

and the phase portion of the soliton is defined as

$$\Phi(x,y,t) = -\vartheta_1 x - \vartheta_2 y + \varpi t + \theta_0. \tag{2.3}$$

Here, ϑ_1 and ϑ_2 are the frequencies of the soliton in the *x*-and *y*-directions respectively while ϖ is the wave number of the soliton and finally θ_0 is the phase constant. Also, the parameters χ_1 and χ_2 in (2.2) represent the inverse width of the soliton along *x*-and *y*-directions respectively, while σ represents the velocity of the soliton. Inserting (2.1) along with (2.2) and (2.3) into (1.1) and decomposing into real and imaginary parts, the following pair of equations, respectively yield

$$\alpha\chi_1\chi_2 P'' - (\omega + \alpha\vartheta_1\vartheta_2)P - 2\beta\vartheta_1 P^3 = 0, \tag{2.4}$$

$$\sigma = -\alpha(\vartheta_1\chi_2 + \vartheta_2\chi_1). \tag{2.5}$$

Eq.(2.4) is transformed into the following one

$$P'' = \frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} P + \frac{2\beta\vartheta_1}{\alpha\chi_1\chi_2} P^3. \tag{2.6}$$

3. The functional variable method

This section presents the brief descriptions of the functional variable method [32,33,34]. Suppose that a the NLEE, say in two independent variables to x and t is given by

$$G(u, u_t, u_{xx}, \ldots,) = 0, \tag{3.1}$$

where G is a function of $u, u_t, u_{xx}, \ldots$ and the subscripts denote the partial derivatives of $u(x,t)$ with respect to x and t.

A transformation $u(x,t) = U(\eta), \eta = x - \sigma t$ converts the NLEE (3.1) to a nonlinear ODE

$$F(U, U_\eta, U_{\eta\eta}, \ldots,) = 0, \tag{3.2}$$

where F is a function of $U, U_\eta, U_{\eta\eta}, \ldots$ and its derivatives point out the ordinary derivatives with respect to η and where and σ is constant to be determine.

Then we make a transformation in which the unknown function U is considered as a functional variable in the form:

$$U_\eta = \Omega(U), \tag{3.3}$$

and some successive derivatives of U are

$$\begin{aligned} U_{\eta\eta} &= \frac{1}{2}(\Omega^2)', \\ U_{\eta\eta\eta} &= \frac{1}{2}(\Omega^2)''\sqrt{\Omega^2}, \\ U_{\eta\eta\eta\eta} &= \frac{1}{2}[(\Omega^2)'''\Omega^2 + (\Omega^2)''(\Omega^2)'], \\ &\vdots \end{aligned} \tag{3.4}$$

where "$'$" stands for $\frac{d}{dU}$.

The ODE (3.2) can be reduced in terms of U, F and its derivatives upon using the expressions of Eq. (3.4) into Eq. (3.2) gives

$$H(U, \Omega, \Omega', \Omega'', \ldots) = 0. \tag{2.5}$$

by integrating of Eq. (3.5), Eq. (3.5) can be written with respect to H, and it is found the appropriate solutions by using Eq. (3.3) for the investigated problem.

4. Solutions to the equation (1)

In this Section we obtain wave solutions of the KMN model by using the functional variable method described in Section 3.

Following Eq. (3.4), it is easy to deduce from Eq. (2.6) an expression for the function $\Omega(U)$

$$\frac{1}{2}(\Omega^2)' = \frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} P + \frac{2\beta\vartheta_1}{\alpha\chi_1\chi_2} P^3. \tag{4.1}$$

Integrating Eq. (4.1) and setting the constant of integration to Ξ yields

$$\Omega^2 = \frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} P^2 + \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} P^4 + \Xi, \tag{4.2}$$

or

$$\Omega = P_\eta = \pm\sqrt{\frac{\omega + \alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2} P^2 + \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2} P^4 + \Xi}. \tag{4.3}$$

Using Eqs. (2.1), (2.2) and (2.3), we obtain the following wave solutions of the KMN model

If $\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}>0,\ \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}<0$ and $\Xi=0$, we obtain the following bright soliton solutions

$$q_1^{\pm}(x,y,t)=\pm\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}}\operatorname{sech}\left(\sqrt{\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}}\left(\chi_1x+\chi_2y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right)\times\exp\left[i\left(-\vartheta_1x-\vartheta_2y+\varpi t+\theta_0\right)\right]. \tag{4.4}$$

If $\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}>0,\ \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}<0$ and $\Xi=0$, we obtain the following singular soliton solutions

$$q_2^{\pm}(x,y,t)=\pm\sqrt{\frac{\omega+\alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}}\operatorname{csch}\left(\sqrt{\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}}\left(\chi_1x+\chi_2y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right)\times\exp\left[i\left(-\vartheta_1x-\vartheta_2y+\varpi t+\theta_0\right)\right]. \tag{4.5}$$

If $\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}<0,\ \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}>0$ and $\Xi=0$, we obtain the following periodic wave solutions

$$q_3^{\pm}(x,y,t)=\pm\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}}\sec\left(\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}}\left(\chi_1x+\chi_2y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right)\times\exp\left[i\left(-\vartheta_1x-\vartheta_2y+\varpi t+\theta_0\right)\right], \tag{4.6}$$

$$q_4^{\pm}(x,y,t)=\pm\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{\beta\vartheta_1}}\csc\left(\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}}\left(\chi_1x+\chi_2y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right)\times\exp\left[i\left(-\vartheta_1x-\vartheta_2y+\varpi t+\theta_0\right)\right]. \tag{4.7}$$

If $\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}<0,\frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}>0$ and $\Xi=-\frac{(\omega+\alpha\vartheta_1\vartheta_2)^2}{4\alpha\beta\vartheta_1\chi_1\chi_2}$, we obtain the following dark soliton solution

$$q_5^{\pm}(x,y,t)=\pm\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}}\tanh\left(\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}\left(\chi_1x+\chi_2y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right)\times\exp\left[i\left(-\vartheta_1x-\vartheta_2y+\varpi t+\theta_0\right)\right]. \tag{4.8}$$

If $\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}<0,\ \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}>0$ and $\Xi=-\frac{(\omega+\alpha\vartheta_1\vartheta_2)^2}{4\alpha\beta\vartheta_1\chi_1\chi_2}$, we obtain the following singular dark soliton solutions

$$q_6^{\pm}(x,y,t)=\pm\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}}\coth\left(\sqrt{-\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}\left(\chi_1x+\chi_2y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right)\times\exp\left[i\left(-\vartheta_1x-\vartheta_2y+\varpi t+\theta_0\right)\right]. \tag{4.9}$$

If $\frac{\omega+\alpha\vartheta_1\vartheta_2}{\alpha\chi_1\chi_2}>0,\ \frac{\beta\vartheta_1}{\alpha\chi_1\chi_2}>0$ and $\Xi=-\frac{(\omega+\alpha\vartheta_1\vartheta_2)^2}{4\alpha\beta\vartheta_1\chi_1\chi_2}$, we obtain the following periodic wave solutions

$$q_7^{\pm}(x,y,t)=\pm\sqrt{\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}}\tan\left(\sqrt{\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}\left(\chi_1x+\chi_2y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right)\times\exp\left[i\left(-\vartheta_1x-\vartheta_2y+\varpi t+\theta_0\right)\right], \tag{4.10}$$

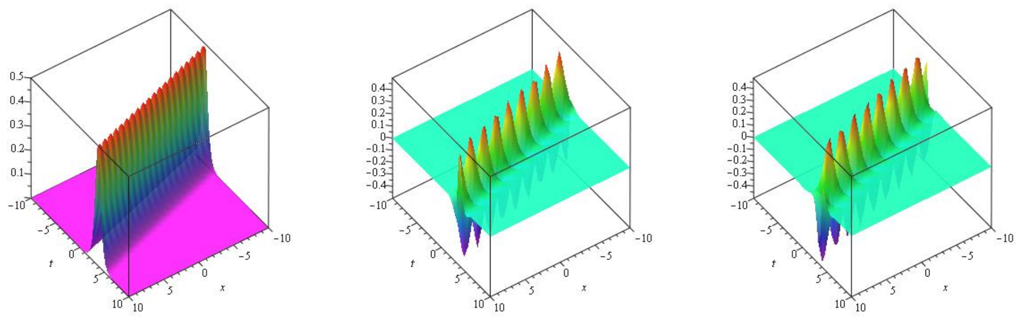
$$q_8^{\pm}(x,y,t)=\pm\sqrt{\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\beta\vartheta_1}}\cot\left(\sqrt{\frac{\omega+\alpha\vartheta_1\vartheta_2}{2\alpha\chi_1\chi_2}}\left(\chi_1 x+\chi_2 y+(\alpha(\vartheta_1\eta_2+\vartheta_2\eta_1))t\right)\right) \times\exp\left[i\left(-\vartheta_1 x-\vartheta_2 y+\varpi t+\theta_0\right)\right]. \tag{4.11}$$

4. Results and discussion

Figure 1 shows the graphs obtained from the space-time mapping of the solution q_1. It can be seen from the figure that the waves have a spatiotemporally extended homoclinic breather wave structure. In this respect, it can be concluded that this q_1 solution can be useful in examining the dynamic behavior of rogue waves. It can also be seen that breather waves extend periodically along with time while extending at a certain angle with the X-axis spatially.

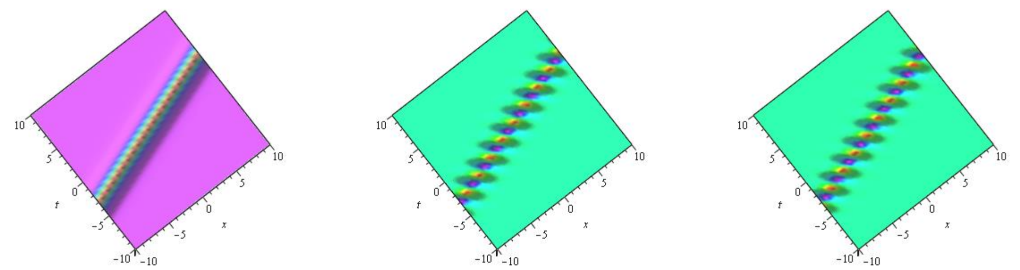
Figure 2 shows the graphs obtained from the space-time mapping of the solution q_4. Interestingly, although this solution seems to be the solution of heteroclinic waves at first glance, a careful look reveals the difference of the situation. This solution shows the existence of periodically extended homoclinic waves both spatially and temporally.

Figure 2 shows the graphs obtained from the space-time mapping of the solution q_5. From this solution, the existence of singular waves extended in time can be seen.



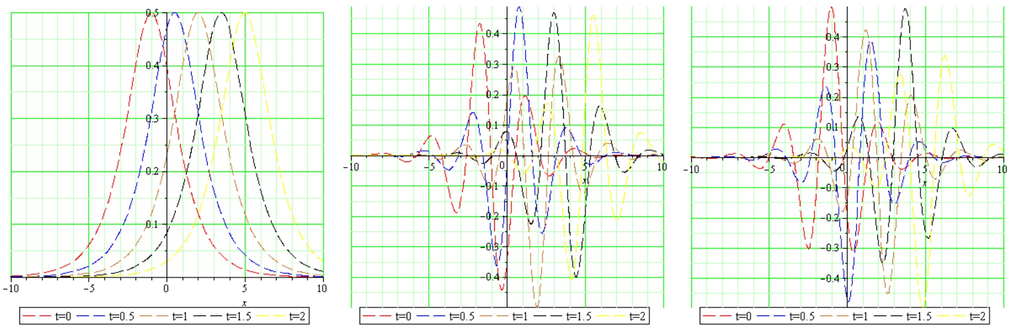
$|q_1^+|$ $\mathrm{Re}(q_1^+)$ $\mathrm{Im}(q_1^+)$

(a)



$|q_1^+|$ $\mathrm{Re}(q_1^+)$ $\mathrm{Im}(q_1^+)$

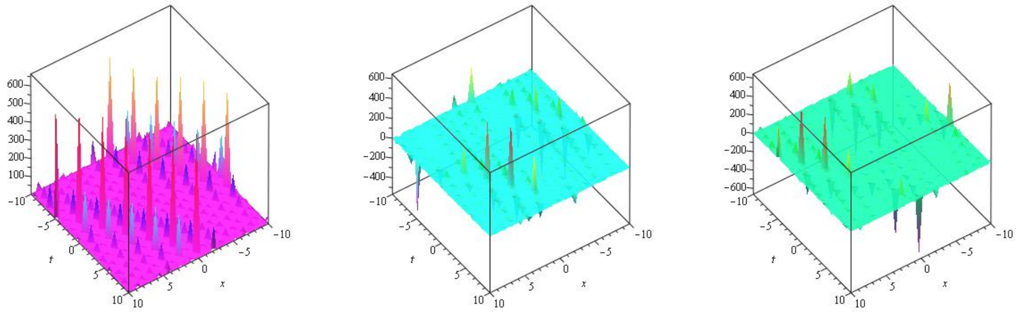
(b)



$|q_1^+|$ $\mathrm{Re}(q_1^+)$ $\mathrm{Im}(q_1^+)$

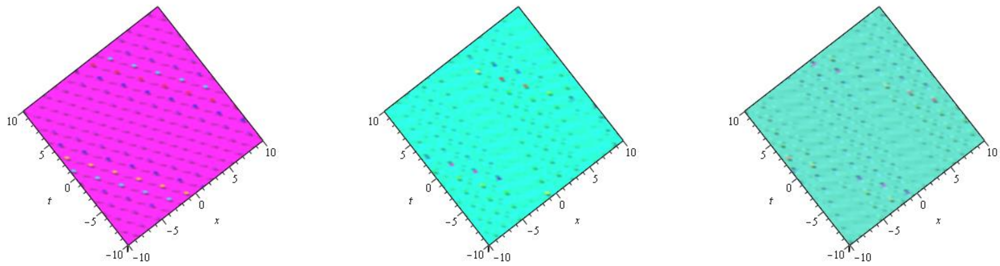
(c)

Fig 1. (a) 3D-plot for q_1^+ **(b)** the contour plot for q_1^+ **(c)** 2D-plot for q_1^+ at $t = 0$, $t = 0.5$, $t = 1$, $t=1.5$, $t=2$ .respectively, when $\omega = 3,\ \alpha = 1,\ \vartheta_1 = -2,\ \vartheta_2 = 1,\ \beta = 2,\ \chi_1 = 1,\ \chi_2 = 1,\ \eta_1 = 1,\ \eta_2 = 2,\ \varpi = 2,\ \theta_0 = 1$ and $y = 0.5$.



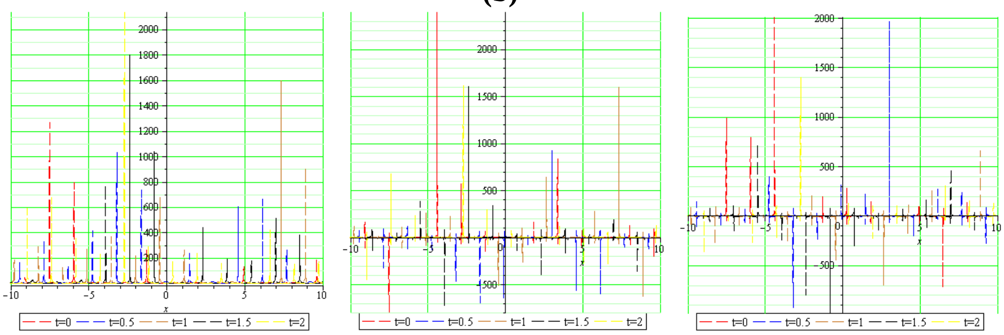
$|q_4^-|$ $\mathrm{Re}(q_4^-)$ $\mathrm{Im}(q_4^-)$

(a)



$|q_4^-|$ $\mathrm{Re}(q_4^-)$ $\mathrm{Im}(q_4^-)$

(b)



$|q_4^-|$ $\mathrm{Re}(q_4^-)$ $\mathrm{Im}(q_4^-)$

(c)

Fig 2. (a) 3D-plot for q_4^- **(b)** the contour plot for q_4^- **(c)** 2D-plot for q_4^- at t = 0, t = 0.5, t = 1, t=1.5, t=2 .respectively, when $\omega = -4,\ \alpha = 1,\ \vartheta_1 = 0.5,\ \vartheta_2 = 1,\ \beta = 2,\ \chi_1 = 1.75,\ \chi_2 = 1.5,\ \eta_1 = 1,\ \eta_2 = 0.5,\ \varpi = 2,\ \theta_0 = 0$ and $y = 1.5$.

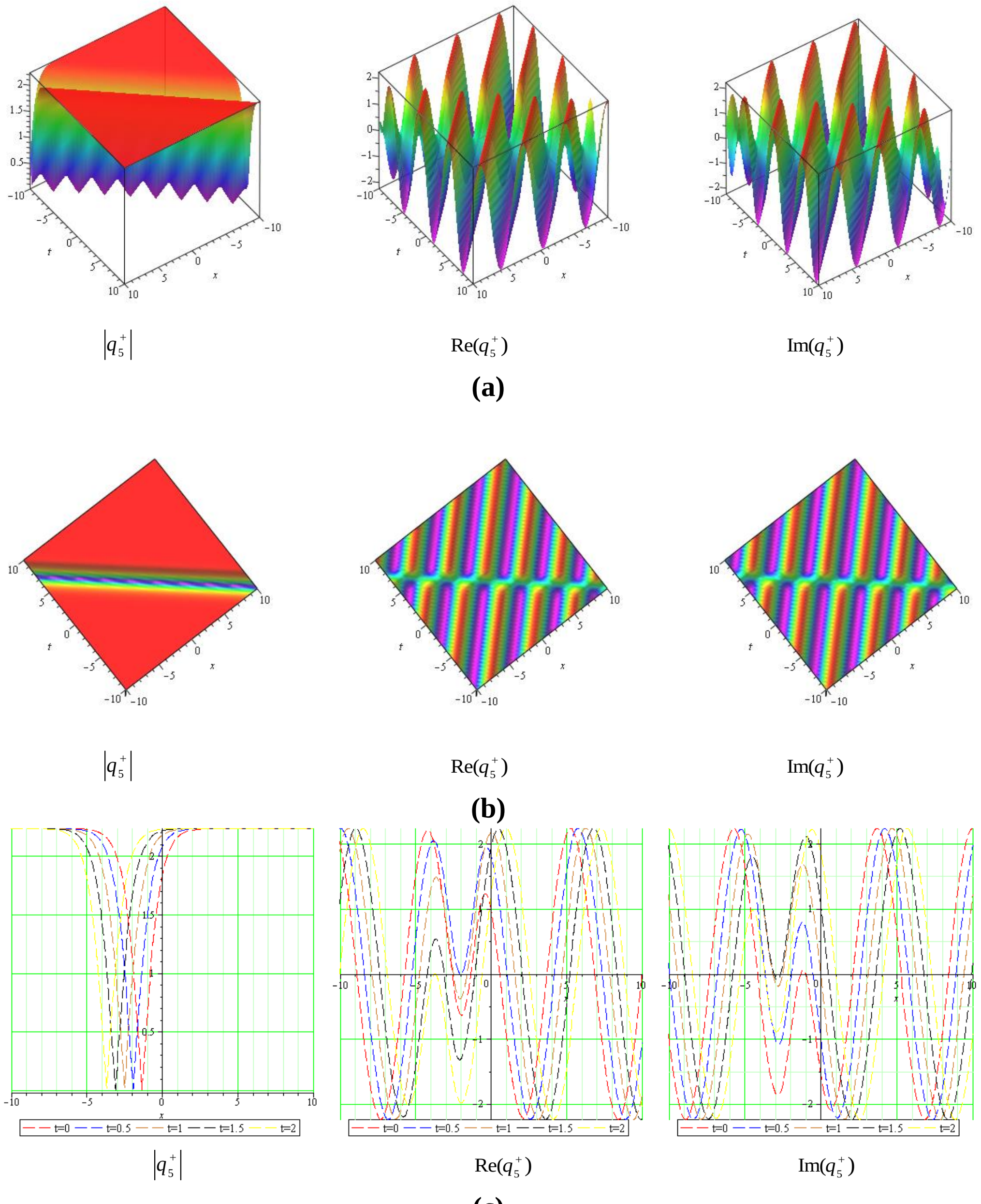


Fig 3. (a) 3D-plot for q_1^+ **(b)** the contour plot for q_1^+ **(c)** 2D-plot for q_1^+ at t = 0, t = 0.5, t = 1, t=1.5, t=2 .respectively, when $\omega = -2,\ \alpha = 1,\ \vartheta_1 = 1,\ \vartheta_2 = 1,\ \beta = 0.1,\ \chi_1 = 1.5,\ \chi_2 = 1,\ \eta_1 = 0.75,\ \eta_2 = 1,\ \varpi = 1,\ \theta_0 = 1$ and $y = 2$.

5. Conclusion

In this article functional variable method is applied to KMN model successfully to get the wave solutions of considered model. This model was first suggested not only to express the oceanic rogue waves but also to model the optical fiber communication. The solutions show that considered method fit well for nonlinear KMN model with complex structure.

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