New Statistical Perspectives on Bath's Law and Aftershock Productivity

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New Statistical Perspectives on Bath’s Law and Aftershock Productivity

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Abstract. The well-established Bath’s law states that the average magnitude difference between a mainshock and its strongest aftershock is roughly 1.2, independently of the size of the mainshock. The main challenge in calculating this value is the bias introduced by missing data points when the strongest aftershock is below the observed cut-off magnitude. Ignoring missing values leads to a systematic error, because the data points removed are those with particularly large magnitude differences $\Delta M$. The error is minimized, if we restrict the statistics to mainshocks at least two magnitude units above the cut-off, but then the sample size is strongly reduced. This work provides an innovative approach for modelling $\Delta M$ by adapting methods for time-to-event data, which often suffers from incomplete observation (censoring). In doing so, we adequately account for unobserved values and estimate a fully parametric distribution of the magnitude differences $\Delta M$ for $M > 6$ mainshocks. Results show that magnitude differences are best modeled by the Gompertz distribution, and that larger $\Delta M$ are expected at increasing depths and higher heat flows. A simulation experiment suggests that $\Delta M$ is mainly driven by the number and the magnitude distribution of aftershocks. Therefore, in a second study, we modelled the variation of aftershock productivity in a stochastically declustered local catalog for New Zealand, using a generalized additive model approach. Results confirm that aftershock counts can be better modelled by a Negative Binomial than a Poisson distribution. Interestingly, there is indication that triggered earthquakes trigger themselves two to three times more aftershocks than comparable
background events. This effect can either be an indicator for incorrect trigger pair assignments as a result of the declustering approach, or it may represent an actual effect due to a higher prevalent energy level in the tectonic system during on-going earthquake sequences. If confirmed, this effect must be carefully considered in forward simulations of earthquake sequences, as otherwise there is a risk of substantially underestimating cluster sizes and consequently overestimating $\Delta M$.

**Keywords:** Bath’s law, survival models, aftershock productivity, generalized additive models.

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1 Introduction

As energy is released in the event of a strong earthquake, tectonic stress redistributes in the surroundings of the initial rupture and typically results in further earthquakes, so-called *aftershocks* (Utsu et al., 1995). The cascade of aftershocks is commonly referred to as an *earthquake sequence*, and the strongest event of the sequence is called the *mainshock*. Typically, events that occurred shortly before the mainshock, so-called *foreshocks*, are included in the sequence since they are believed to be physically related to the upcoming major earthquake (e.g. Helmstetter and Sornette, 2003).

Extensive research has been carried out to analyze and model the spatio-temporal properties of earthquake sequences, e.g. through the Epidemic Type Aftershock Sequence (ETAS) model (Ogata, 1988, 1998; Zhuang et al., 2002). Studies found a well-established power-law decay of aftershock rates as a function of the time after the mainshock (Omori, 1895), while the spatial cluster is typically elongated rather than isotropic around the mainshock’s rupture plane (e.g. Grimm et al., 2022, 2021; Hainzl et al., 2008; Ogata, 2011; Ogata and Zhuang, 2006; Zhang et al., 2018).

Aftershocks are a relevant risk driver since even moderate events can substantially increase damage in buildings and infrastructure destabilized by a prior mainshock. Similarly, foreshocks can
set the stage for more severe mainshock damage (Abdelnaby, 2012; Kagermanov and Gee, 2019; Papadopoulos et al., 2020). Therefore, one of the central questions for emergency and risk management purposes is: What is the second strongest magnitude to be expected in a sequence?

To date, the literature only provides a starting point for answers to this question. The well-established Bath’s law states that the average magnitude difference $\Delta M$ between a mainshock and its strongest aftershock is roughly 1.2, independently of the size of the mainshock (Bath, 1965). The main challenge in calculating this value is the bias introduced by missing data, if no aftershock was observed above the cut-off magnitude $M_c$ of the catalog and therefore $\Delta M$ cannot be computed. We cannot simply ignore missing values, as these are the ones with particularly large magnitude differences $\Delta M$. Therefore, leaving them out would lead to a systematic bias. Several authors found that the statistics is robust, if we restrict the sample to mainshocks at least two magnitude units above $M_c$, but then the sample size is strongly reduced (e.g. Tahir et al., 2012; Zakharova et al., 2013). Another work around was suggested by Zakharova et al. (2013), who modeled the seismic moment ratio between aftershocks and the mainshock, rather than $\Delta M$, approximating the ratio by zero if no aftershocks were recorded.

In any case, Bath’s law only makes a statement about the average value of the $\Delta M$, but not about their distribution (and its parameters) or any important quantiles in the lower tail of the distribution. Another term that appears occasionally in the literature is that of an earthquake doublet. Doublets are generally defined as a pair of two similarly strong earthquakes, occurring temporarily and spatially close to each other (e.g. Felzer et al., 2004; Grimm et al., 2021; Kagan and Jackson, 1999). Kagan and Jackson (1999) found that approximately 22% of the $M > 7.5$ earthquakes worldwide occurred accompanied by another $M > 7.5$ event within a distance of one rupture length and with an inter-event time of considerably less than their recurrence time estimated from plate motion.
Grimm et al. (2021) showed that roughly 17% of the global $M \geq 6$ mainshocks and more than 20% of the mainshocks in Japan were part of an earthquake doublet, defining them as a pair of earthquakes with no more than 0.4 magnitude units difference, occurring within 365 days and a radius of 2.5 rupture lengths.

1.1 Survival Model Regression of $\Delta M$

In the first part of this work, we propose an innovative approach that models the full, parametric distribution of $\Delta M$ by adapting so-called survival models, originally developed for medical applications. Survival models are a class of regression models that account for data with a censored (or truncated) response variable (see e.g. Klein and Moeschberger (2003) for comprehensive overviews). As the term ”survival” suggests, these models were originally developed in applications where the response represents the non-negative lifetime of a patient in medical studies or the lifetime of a device in engineering contexts (so-called reliability or failure time analysis). The above applications have in common that the exact value of the response is unknown, if the event has not occurred until the end of the study period.

Replacing lifetimes by magnitude differences, we can therefore use survival models to account for the missing $\Delta M$ values where we only have the partial information that $\Delta M > M - M_c$, given mainshock magnitude $M$.

To do so, we decluster a global catalog using a window method, and compute the (partially right-censored) $\Delta M$ between the mainshock and the second strongest event of each cluster. Note that the latter may be a foreshock or an aftershock, as both are relevant in a risk management context. Then, we enrich the cluster set by a plate boundary classification, relative plate velocities, sea floor age and heat flow data, to investigate the regression effects of these large-scale geophysical
conditions on the distribution of $\Delta M$.

1.2 Drivers of $\Delta M$

The magnitude difference $\Delta M$ is controlled by the two drivers (Grimm et al., 2021)

1. number of aftershocks (hereafter called *aftershock productivity*) and

2. frequency-magnitude distribution (FMD) of triggered events.

A simple simulation shall demonstrate the effect of both factors. Assume an initial mainshock of magnitude $M = 8$. Let the expected aftershock productivity of an earthquake with magnitude $M$ be

$$k(M) = A \exp(\alpha (M - M_c)), \quad M \geq M_c,$$

(1)

and the FMD be the exponential distribution with probability density function (pdf)

$$f(M) = \beta e^{-\beta(M-M_c)}, \quad \beta > 0, \ M \geq M_c,$$

(2)

where $\beta$ is related to the Gutenberg-Richter $b$-value by $b = \beta/\ln(10)$ (Gutenberg and Richter, 1944). If we assume the realistic parameters $A = 0.13, \alpha = 2.0$ and $b = 1.0$ and, for simplicity, a Poisson distributed number of aftershocks with trigger-magnitude dependent parameter $\lambda = k(M)$, we can simulate a distribution of $\Delta M$ with a mean of 1.2, consistent with Bath’s Law.

A doubled aftershock productivity (i.e. $A = 0.26$) would lead to a pronounced drop of the average magnitude difference down to 0.66. If the increase of $A$ does only apply to secondary triggering, but the number of direct aftershocks of the $M8$ mainshock remains unchanged, the mean of $\Delta M$
still decreases to below 0.9. A similar reduction of the average $\Delta M$ is achieved, if instead we modify the FMD, sampling magnitudes according to $b = 0.85$.

### 1.3 Regression of Aftershock Productivity

The simplified simulation experiment above illustrates the leverage of both aftershock productivity and FMD on the magnitude differences $\Delta M$. This gave motivation for a more in-depth analysis of the variation in aftershock productivity in the second part of this study. To do so, we decluster a local earthquake catalog for New Zealand and fit a generalized additive model (GAM) to the estimated number of direct aftershocks per event, comparing a Poisson with a Negative Binomial distribution. Here, we use the stochastic declustering method introduced by Zhuang et al. (2002), based on the ETAS-Incomplete model proposed by Grimm et al. (2022) that accounts for anisotropic ruptures as well as short-term aftershock incompleteness to reduce estimation biases. We enrich the local catalog by classifying the events to the main tectonic contexts in the surrounding of the Hikurangi fault, and by concluding the slip type from additional focal mechanism data.

Extensive research has been done on the variation of aftershock productivity. Kagan (2017) and Shebalin et al. (2018) showed that aftershock counts are best modeled by the Negative Binomial distribution due to their large variance. Page et al. (2016) found that aftershock productivity may regionally vary by a factor of almost 10, which would explain the variation in $\Delta M$ to large extent. Marsan and Helmstetter (2017) found that 40-80% of the aftershock variability may be related to variation in the mainshock stress drop. Dascher-Cousineau et al. (2020) investigated a large number of source and site effects on aftershock productivity and showed individual correlations of stress drop and rupture dimension with the number of aftershocks. Wetzler et al. (2016) suggest a larger productivity in subduction zones of the western circum-Pazific, compared to the eastern
side. Zhuang et al. (2004) proposed that triggered events produce more aftershocks than comparable background events, which would provide an additional boost in clustering and decrease expected $\Delta M$’s.

Potential correlations of the FMD of triggered events with the magnitude of the direct ancestor or the cluster mainshock were found by Zhuang et al. (2004), Gulia et al. (2018) and Nandan et al. (2019), and may considerably increase the chance of small $\Delta M$. However, they are out of the scope of this work.

1.4 Scope and Outline

This work consists of two regression studies, the analysis of magnitude differences (hereafter called $\Delta M$-regression), and the analysis of the aftershock productivity (referred to as productivity regression). The focus is on the innovate approach to estimate a fully parametric distribution of $\Delta M$, using survival models that take into account right-censored data rather than avoid it. To our knowledge, no similar approach has been pursued in the literature so far. Especially in the $\Delta M$-regression, covariates represent rather large-scale regional effects. Attempts to consider small-scale variations of these covariates or to include further event specific data are out of the scope of this paper.

Sections 2 and 3 introduce the utilized datasets, the declustering approaches and the compilation of the covariate datasets for both regression studies, respectively. Next, Section 4 rigorously explains the survival model and GAM methodological approaches. Then, the results of the regression studies are shown and discussed in Sections 5 ($\Delta M$-regression) and 6 (productivity regression). Finally, conclusions are drawn from a joint interpretation and related future research topics are recommended.
2 Data for $\Delta M$-Regression

This section summarizes the compilation of the regression dataset for the analysis of global magnitude differences between the mainshock and the second strongest event in the cluster. First, we justify the choice of the underlying global earthquake catalog. Then, we outline the window method to decluster the catalog, followed by the definition of the response variable. Finally, the enrichment of further geophysical variables as regression covariates is explained.

2.1 Global Earthquake Catalog

The choice of an appropriate global earthquake catalog for the regression of magnitude differences raises two requirements which, however, are not fully met by any currently available catalog and therefore requires a trade-off. On the one hand, the catalog should ideally have homogeneous magnitude scales, and be reliably complete in any part of the world, including far off-shore regions and aftershocks occurring shortly after the mainshock. On the other hand, it should be complete to the smallest possible magnitude level to assure a sufficient observable magnitude range of at least one unit below the smallest mainshock magnitude of interest, $M > 6.0$.

Despite not providing homogenized magnitude scales, we chose the U.S. Geological Survey National Earthquake Information Center (USGS-NEIC) catalog. We extracted all events from 1973 until 2021 with depths smaller than 70 km that occurred at a maximum of 300 km distance to a tectonic plate boundary according to the digital model by Bird (2003). The completeness magnitude of this dataset is $M_c = 5.0$ according to Kagan and Jackson (2010) and Tahir et al. (2012), which allows us to apply the regression model to clusters with mainshock magnitude larger than 6.0.

To test the influence of inhomogeneous magnitude scales, we performed sensitivity analyzes using the International Seismological Centre – Global Earthquake Model (ISC-GEM) instrumental cat-
alogue, which is a relocated global event set with homogenized magnitude scales (Bondár et al., 2015; Di Giacomo et al., 2015a,b, 2018; Storchak et al., 2015). Due to its higher level of magnitude completeness, $M_c = 5.6$ according to Di Giacomo et al. (2015b) and $M_c = 6.0$ according to Michael (2014) since 1964, we have to limit our statistical analysis to mainshocks with $M \geq 6.6$.

2.2 Declustering of Global Catalog

In order to obtain a set of independent clusters, including the information about the magnitude difference $\Delta M$ between the mainshock and the largest aftershock (or foreshock), we declustered the global earthquake catalog using a rather simple window method (see e.g. Gardner and Knopoff, 1974; Uhrhammer, 1986; van Stiphout et al., 2012). To do so, we first sorted the catalog in descending magnitude order. Then, we consecutively searched aftershocks occurring within a time window of $T = 100$ days and a spatial radius of $R(m) = 2.5L(m)$, where $L(m) = 10^{-2.44+0.59m}$ is the expected rupture length of the mainshock, depending on its magnitude $m$, according to Wells and Coppersmith (1994). Similar to Reasenberg (1985), we linked clusters if an event $B$ is found to trigger the potential aftershock $A$, but $A$ is the mainshock of an already identified cluster and therefore, due to prior re-ordering of the catalog, has the larger magnitude, $m_A \geq m_B$. In this case, event $B$ is called a foreshock of $A$.

We conducted sensitivity studies that showed that the regression results are insensitive to varying definitions such as $T = 365$ days and $R(m)$ varying between $1.0L(m)$ and $2.5L(m)$.

2.3 Response Variable

For each cluster, the magnitude difference $\Delta M$ is computed between the mainshock (i.e., the strongest event of the cluster) and the second-strongest event, be it a foreshock or aftershock. In
total, we obtain 2,933 clusters with mainshock magnitudes $M > 6.0$.

Note that 1,180 of these are *single-event* clusters, i.e., no associated foreshock or aftershock was found in the corresponding time-space window. Based on seismological reasoning we can assume that these mainshocks actually triggered aftershocks, but that these were simply too weak to be recorded in the dataset, given its cut-off magnitude $M_c$. Therefore, if for a mainshock $i$ with magnitude $M_i \geq M_c$ no second event is listed, we have the partial information that the magnitude difference is $\Delta M_i > M_i - M_c$. The single clusters are the reason why we need advanced regression models that can deal with censored data.

### 2.4 Covariates

We enriched the declustered catalog by additional geophysical site information interpolated to the mainshock locations by a nearest neighbor approach.

Using the digital plate boundary model of Bird (2003), we categorized each event into one of seven *plate boundary classes*: continental convergence boundary (CCB), continental transform fault (CTF), continental rift boundary (CRB), oceanic spreading ridge (OSR), oceanic transform fault (OTF), oceanic convergent boundary (OCB) and subduction zone (SUB). Fig. 1 shows the mainshock locations of the declustered catalog, color-coded by the corresponding plate boundary class assigned to them. Table 1 lists the number of clusters with censored and observed $\Delta M$ value per boundary class, respectively. Note that almost half of the clusters are assigned to a subduction zone, and that oceanic spreading ridges and transform faults host more censored than observed data points.

From the same digital model, we assigned estimates of the *relative plate velocity* and *sea floor age* from the next boundary segment point to the mainshock locations. Likewise, using a near-
est neighbor approach, we interpolated values from the scattered heat flow dataset of Bird et al. (2008), provided to us by the author. Fig. 2 illustrates the marginal distributions of the interpolated covariate data at the mainshock locations, grouped by the assigned plate boundary class. Subduction zones show the largest relative plate velocities (values range between 0.4 and 262 mm/a, see Fig. 2(a)), while oceanic spreading ridges and transform faults provide the youngest sea floor ages (between 0 and 262 Ma, see Fig. 2(b)) and the largest heat flows (between 0.025 and 0.3 $W m^{-2}$, see Fig. 2(b)).

3 Data for Productivity-Regression

This section summarizes the dataset compilation for the regression of aftershock productivity. First, we briefly introduce the chosen local event set for New Zealand. Then, we rigorously describe the stochastic declustering method which is applied in order to estimate the number of aftershocks as the response variable. Finally, we describe the enrichment of the local event set by further geophysical properties.

3.1 Local New Zealand Catalog

We limited this study to the Hikurangi subduction zone in New Zealand. A local event set was provided by GNS Science as an input to the ongoing 2022 revision of the New Zealand National Seismic Hazard Model. Using the algorithm of Stepp (1972), we computed that the catalog is complete down to $M_c = 3.5$ from 1982; however, to be conservative, we assumed $M_c = 3.5$ from 1987, concurrent with an improvement to the seismic network. Fig. 3 shows the chosen extract of 11,091 events surrounding the Hikurangi fault, between 1987 and end of 2020, at depths down to 80 km.
3.2 Declustering of Local Catalog

For the regression of aftershock counts, we cannot use the window declustering method, as it does not distinguish direct from secondary aftershocks. Instead, we used the stochastic declustering approach based on the Epidemic Type Aftershock Sequence (ETAS) model, as introduced by Zhuang et al. (2002).

The ETAS model describes the spatio-temporal clustering behavior of the entire catalog, and models a dynamic event rate at time $t$ and location $(x, y)$, given the prior event history $H_t$, through two overlapping processes,

$$R(t, x, y|H_t) = u(x, y) + \sum_{i:t_i \leq t} R_{trig}^{i}(t, x, y, i),$$  \hspace{1cm} (3)

where $u(x, y)$ denotes the time-invariant, aftershock-independent seismic background rate, and $\sum_{i:t_i \leq t} R_{trig}^{i}(t, x, y, i)$ is the sum of the trigger rate contributions by all events $i$ that occurred prior to time $t$. For more details about the ETAS model, see e.g. Jalilian (2019); Ogata (1988, 1998); Zhuang et al. (2002).

From Equation (3), Zhuang et al. (2002) concluded that the probability, that the event $j$ at time $t_j$ and location $(x_j, y_j)$ was an aftershock of the prior event $i$, is

$$P_{j,i} = \frac{R_{trig}^{i}(t_j, x_j, y_j, i)}{R(t_j, x_j, y_j|H_t)}.$$

Similarly, the probability that event $j$ is a seismic background event and therefore independent of any prior trigger, is

$$P_{j,\text{backgr}} = \frac{u(x, y)}{R(t_j, x_j, y_j|H_t)}.$$
Thus, unlike the window method, the ETAS model provides probabilistic trigger associations between event pairs, and does not require the arbitrary choice of a fixed space-time window to search aftershocks. Instead, it optimizes built-in parametric functions that model the expected number of aftershocks (for trigger magnitude $m_i$),

$$K(m_i) = A e^{\alpha (m_i - M_c)}, \quad A > 0, \quad \alpha > 0,$$

the temporal decay of aftershock rates (e.g. Omori-Utsu power law, see Omori, 1895), and a typically isotropic spatial distribution of aftershocks.

In this work, we used the ETAS-Incomplete model version of Grimm et al. (2022), who introduced a novel, anisotropic and locally restricted spatial kernel and accounted for incomplete records of aftershock sequences. The estimation source code is available in a public github repository (see Data and Resources). The new features solve the estimation biases due to the misfit of mostly elongated aftershock clouds by an isotropic kernel, and an underestimation of the trigger potential of strong mainshocks as a consequence of missing aftershock data. Both biases were shown to heavily affect our response variable, the aftershock productivity (Grimm et al., 2022, 2021; Hainzl, 2021; Hainzl et al., 2013; Page et al., 2016; Seif et al., 2017).

### 3.3 Response Variable

As the response variable of the regression model, we defined the estimated number of direct aftershocks for each event $i$ in the catalog. We did this by counting the number of subsequent events $j$, for which $i$ is the most probable trigger event, i.e. $P_{j,i} > P_{j,k} \quad \forall \ k \neq i$, and that are more likely triggered by $i$ than being a background event, i.e. $P_{j,i} > P_{j,\text{backgr}}$. 

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Note that the response is inevitably affected by the short-term incomplete records of aftershock sequences, as we can only count aftershocks that are recorded in the dataset. Nevertheless, the ETAS-Incomplete model approach avoids a biased parameter estimation, that would lead to manipulated declustering probabilities.

3.4 Covariates

The local catalog provides us the magnitude and depth for each event as immediate covariates for the regression model. Additionally, if the triggering event $i$ was itself already triggered by a previous event, we traced back the trigger sequence and identified the largest magnitude in the cluster, that occurred before event $i$. This covariate tests whether an event, that is a member of a cluster, is more or less productive than an independent background event, and whether its aftershock productivity is influenced by the previous mainshock.

A focal mechanism data set comprising 1,581 events of the chosen catalog extract was provided by GNS Science. We used the `nodal-plane.py` function from the public `GEMScienceTools/oq-mbtk` repository, based on the algorithm in Álvarez-Gómez (2019), to translate the given focal mechanisms into the slip type categories normal (N), strike-slip (SS) and reverse (R) and mixed categories.

We classified each earthquake in the catalogue to the main tectonic regions. These regions are the shallow crust, subduction interface, subduction intraslab deep (within the subducting plate, but deeper than the zone of contact between the subducting and overriding plate), and subduction intraslab shallow (within the subducting plate, but in the shallow part of the plate beneath the interface, e.g. Reyners et al., 2010). To do so, we used the methodology described by Pagani et al. (2020), which classifies each earthquake based on its hypocentral position relative to surfaces.
with buffers) that demarcate the boundaries of these regions. The surface used to represent the Hikurangi subduction interface and the top of the subducting plate was derived from a 3D model provided by GNS Science. Earthquakes shallower than 40 km and within 5 km of this surface were classified as interface; those shallower than 40 km and more than 5 km below the surface were classified as shallow slab; and those deeper than 40 km and within 5 km above or 60 km below this surface were classified as deep slab (the large below-slab buffer helps to capture earthquakes with large depth errors). Earthquakes shallower than the Moho (depths defined by LITHO1.0, Pasyanos et al., 2014) with a 10 km buffer were classified as crustal; crustal earthquakes were then sub-classified as occurring within or outside of the surface projection of the subduction zone. If an earthquake was classified into more than one tectonic region, then the following hierarchy was applied: interface is more likely than shallow slab, which is more likely than deep slab, which is more likely than shallow crustal. All other earthquakes were labelled as “unclassified” and not used in further analyses.

4 Regression Methods

In this section, we summarize the statistical models used in the two regression studies. Subsection 4.1 introduces survival regression models that can account for the censored $\Delta M$ response data due to unobserved aftershocks. Subsection 4.2 explains the use of a generalized additive model (GAM) for modeling aftershock counts in the local New Zealand catalog. All statistical analyses were performed with the open source software $\mathbb{R}$ (R Core Team, 2021).
4.1 Survival Models

4.1.1 Why Using a Survival Model for Earthquakes?

The magnitude difference $\Delta M$ between the mainshock and the second-largest earthquake of a sequence is only known, if at least one foreshock or aftershock was observed and assigned to the mainshock. Indeed, roughly 40% of the global clusters consist of a stand-alone mainshock. For these clusters, we can conclude that the second strongest event must be smaller than the cut-off magnitude $M_c$, i.e., that $\Delta M_i > M_i - M_c$, where $M_i$ is the magnitude of mainshock $i$. In statistics, data points which are capped by such an upper observable threshold are called right-censored (Klein and Moeschberger, 2003, section 3.2). Classical statistical models would substantially underestimate $\Delta M$ due to the relevant proportion of censored observations.

Replacing lifetimes by magnitude differences, our data meets the necessary requirements of a survival model,

- **non-negative responses** ($\Delta M \geq 0$)
- **independent responses** (mainshocks result from declustered catalog)
- **non-informative censoring** (i.e., conditional on covariates, censored clusters are not suspected to deviate structurally in their $\Delta M$-distribution from non-censored clusters).

4.1.2 Model Formulation and Software

In order to estimate both covariate effects and the entire distribution of magnitude differences $\Delta M$, we need a fully parametric survival model approach. As will be shown in the results section, the best model fits were achieved assuming a Gompertz distribution for the magnitude differences, rather than other candidates such as Weibull or Generalized Gamma. The Gompertz distribution is
defined on $(0, \infty)$. Therefore data points with $\Delta M = 0$ were substituted by the value 0.01. In the R package flexsurv (Jackson, 2016), the Gompertz distribution is parameterized by its probability density function

$$f(x|a, b) = be^{ax} \exp \left( -\frac{b}{a} (e^{ax} - 1) \right)$$

with shape parameter $a \in \mathbb{R}$ and scale parameter $b > 0$. Besides the categorical plate boundary class, we modeled the effects of the mainshock magnitude $x_{mag}$ and depth $x_{depth}$, as well as the locally interpolated relative plate velocity $x_{veloc}$, heat flow $x_{heat}$ and sea floor age $x_{age}$. In the resulting full Gompertz survival model, we regressed the scale parameter $b$ through all covariates for observation $i$ by

$$\log(b(x_i)) = \beta_0 + \beta_1 x_{class=CCB,i} + \ldots + \beta_6 x_{class=OTF,i} + f_{mag}(x_{mag,i}) + f_{depth}(x_{depth,i}) + f_{veloc}(x_{veloc,i}) + f_{heat}(x_{heat,i}) + f_{age}(x_{age,i}),$$

where $\beta_0, \beta_1, \ldots, \beta_6$ are the coefficients related to categorical variables, where boundary class "SUB" is the reference category, represented by the intercept $\beta_0$, and the $f$ terms denote coefficients related to categorical variables. Similarly, we modeled the shape parameter $a$ depending on the linear effects of the plate boundary class, i.e.

$$\log(a(x)) = \alpha_0 + \alpha_1 x_{class=CCB} + \ldots + \alpha_6 x_{class=OTF}.$$

In this work, we fitted models using the function flexsurvreg from the flexsurv package, which estimates parameters by optimizing a parametric likelihood adapted for censored data (Jackson, 2016). To allow for flexible non-linear effects, all metric variables are modeled by the penalized
spline function \textit{pspline} from the \texttt{R} package \textit{survival} (Therneau, 2016), consistently using $df = 2$ degrees of freedom and $n = 2.5 \times df$ basis functions (Eilers and Marx, 1996; Hurvich et al., 1998).

4.2 Generalized Additive Count Models

In the second part of this study, we model the number of aftershocks $N_i$ of each event $i$. The starting point for modeling count data response variables are so-called generalized linear models. Given covariate values $x_{i1}, \ldots, x_{ik}$, the expected aftershock productivity of event $i$ is modeled by the log-linear relationship

$$E[N_i] = \exp(\eta_i)$$

where $\eta_i = \beta_0 + \beta_1 x_{i1} + \ldots + \beta_k x_{ik}$ is the linear predictor and $\beta_0, \beta_1, \ldots, \beta_k$ are the estimated coefficients. Note that the covariates have an \textit{exponentially multiplicative effect} on the expected number of aftershocks (see e.g. Fahrmeir et al., 2013, section 5.2).

In this work, we used a GAM approach and replaced the linear effects of all metric covariates by potentially smooth functions that can more flexibly represent varying effects in different value ranges of the covariates (e.g., see Fahrmeir et al., 2013, section 9.1). The full model is then specified by the predictor

$$\eta_i = \beta_0 + \sum_{k=1}^5 \beta_k I(x_{TR,i} = k) + \sum_{h=6}^{12} \beta_h I(x_{SL,i} = h) + \ldots$$

$$f_{mag}(x_{mag,i}) + f_{depth}(x_{depth,i}) + \ldots$$

$$f_{mainshMag}(x_{mainshMag,i}) I(x_{isBackground,i} = false),$$

where the $\beta$’s are the estimated coefficients for the linear categorical effects of the tectonic region (TR, $k = 1, \ldots, 5$) and slip type (SL, $h = 6, \ldots, 12$), and $\beta_0$ is the intercept representing the refer-
ence categories \( TR=\text{crustal outside} \) and \( SL=\text{unknown} \). The functions \( f_{\text{mag}}, f_{\text{depth}} \) and \( f_{\text{mainshMag}} \) represent the smooth effects of the magnitude and depth of the triggering event as well as of the prior mainshock magnitude, and \( I(\ldots) \) is the indicator function that is 1 if the inside condition is fulfilled, and 0 otherwise.

To fit the model, we used the function \( \text{gam} \) from the \( \mathbb{R} \) package \textit{mgcv} (Wood, 2017), using a logarithmic link function and the restricted maximum likelihood estimator (REML) as the smoothing parameter estimation method. Penalized splines based on a basic spline basis (P-Splines) were used to model the unspecified smooth functions (e.g., see Fahrmeir et al., 2013, section 8.1). We used the standard smooth term function of \textit{mgcv}, choosing \( k = 5 \) and \( k = 8 \) (for depth) as the dimensions of the basis.

5 Results of the \( \Delta M \)-Regression

In this section, we show and discuss the results of a parametric survival model fitted to the global declustered earthquake catalog in order to describe the magnitude difference \( \Delta M \) between the mainshock and the second strongest event in the cluster. First, we justify and validate the distribution assumption for the response variable. Then we show and interpret the effects of the modeled covariates. Finally, we assess the explanatory power of the model using a response residual plot.

5.1 Choice of Distribution Family

Following the simple simulation model outlined in the section \textit{Introduction}, with parameters \( b = 1 \), \( A = 0.13 \) and \( \alpha = 2 \), we fitted a Weibull, a Gompertz and a Generalized Gamma distribution to the simulated magnitude differences \( \Delta M \). Fig. 4(a) shows the fits of the three distributions to the kernel density estimator of the sampled data. The Gompertz distribution clearly provides the best
fit to the moderately negatively-skewed data.

In order to confirm this assumption based on the actual dataset, we fitted a Gompertz survival model with only the scale parameter depending on the categorical plate boundary class, and compared the predicted survival curves to those provided by the non-parametric Kaplan-Meier estimator, which does not require a specific distribution assumption (Klein and Moeschberger, 2003, ch. 4). In Fig. 4(b), the step functions colored according to the seven boundary classes, refer to the Kaplan-Meier estimates. The Gompertz survival model survival curves are plotted on top by black lines, showing generally good agreement.

5.2 Covariate Effects

Fig. 5 shows the covariate effects for the full parametric Gompertz survival model. The categorical effects in Fig. 5(a) represent predictions of the response $\Delta M$ given the various boundary classes, if the other covariates are held fixed at their median values (magnitude=6.4, depth=23 km, velocity=66.5 mm/a, sea floor age=220 Ma, heat flow $\approx 0.07 \, Wm^{-2}$). The effects of the metric covariates in Fig. 5(b-f) are similarly predicted for a fine grid of values of the considered variable, holding the other covariates fixed and assuming a subducting environment (i.e., boundary class "SUB"). Gray shades represent the 95% confidence interval.

5.2.1 Effect of Boundary Class

Fig. 5(a) reveals no structural effects of specific boundary classes. If we were fitting the same model, but leaving out sea floor age and heat flow, the boundary classes OSR and OTF would show a substantial and OCB a moderate increase in magnitude differences. In other words, mainshocks at oceanic, especially transform and divergent type boundaries, produce weaker second strongest
events than those in continental zones, which fits with the generally limited magnitude sizes in these two boundary classes (Bird et al., 2002; Boettcher and Jordan, 2004). However, this effect seems to be sufficiently represented by the added metric covariates.

5.2.2 Effect of Mainshock Magnitude

For values smaller than $M = 7.8$, the mainshock magnitude effect depicted in Fig. 5(b) confirms the well-established Bath’s law hypothesis that the average magnitude difference $\Delta M$ is roughly 1.2, independently of the mainshock magnitude. For larger magnitudes, there seems to be a tendency toward smaller $\Delta M$.

However, this effect is very uncertain for two reasons. First, the sample size of $M > 7.8$ events (41 data points) is very small compared to the lower magnitude ranges, leading to large standard errors. Second, the mainshock magnitude controls the radius of the spatial window in the declustering approach. Thus, larger mainshocks span an exponentially increasing area, in which potential aftershocks are searched. To test, whether the observed effect of strong mainshocks may be an artifact of a too generous choice of the spatial window radius, we repeated the study for an event set declustered with radius $R(m) = K(m)L(m)$, where the factor $K(m)$ gradually decreases from 2.5 to 1.0 for magnitudes between 6.0 and 9.0. This sensitivity study confirmed the shape of the effect curve, indicating that the second strongest event usually occurred relatively close to the mainshock.

5.2.3 Effect of Mainshock Depth

Fig. 5(c) shows that the effect of the mainshock depth is almost constant for depths smaller than 40 km. Between 40km and 50km, $\Delta M$ increases from roughly 1.2 to a new level of approximately
1.5. This effect is consistent with the observation of Hainzl et al. (2019), who showed that aftershock productivity decreases at higher depths due to reduced seismic coupling, i.e. the energy discharges increasingly through seismic creep rather than through aftershocks. Given a constant magnitude size distribution, this would immediately translate into higher magnitude differences $\Delta M$. A connection with missing data at greater depths is unlikely, as we are only interested in the largest aftershock rather than the entire sequence.

5.2.4 Effect of Relative Plate Velocity

Plate velocities play an important role for the duration of stress re-accumulation at a fault after the occurrence of a large earthquake. However, recurrence intervals of so-called characteristic earthquakes are typically in the range of multiple decades or even centuries. For the short-term recurrence of strong aftershocks, Fig. 5(d) reveals no clear effect of the relative plate velocity. As an alternative covariate representing the velocity of deformation in the tectonic system, we tested global strain rate data (Kreemer et al., 2014), which similarly showed no structural effect.

5.3 Effect of Heat Flow

According to Fig. 5(e), regions with heat flow larger than $0.23 W/m^2$ show a substantial increase of magnitude differences. Warmer rock is known to be more viscous, which discharges stress through seismic creep rather than abrupt fractures, leading to the same aftershock productivity argument as for higher depths. As Fig. 2 shows, high heat flow values are typically prevalent in oceanic ridges and transform faults, which explains why the model predicts larger $\Delta M$ for the plate boundary classes OSR and OTF if heat flow is left out as a covariate.
5.3.1 Effect of Sea Floor Age

Fig. 5(f) shows that magnitude differences are substantially larger in young compared to old oceanic crusts. A potential causal reason for the effect of the plate age cannot be ruled out, but is unknown to the authors. Note that young sea floor typically comes with large heat flows. Therefore, the effects of the two variables are consistent. As new oceanic crust is formed at oceanic ridges, the effect also coincides with the increased magnitude differences in the nested model without sea floor age.

If we fit the full model to the subset of subduction zone mainshocks only, both heat flow and sea floor age show no clear signal. Thus, it is likely that their effect is mainly driven by their tails at oceanic ridges.

5.4 Response Residuals

Fig. 6 shows the response residuals (i.e., observed minus predicted values) plotted against predicted magnitude differences. Note that, as observations are censored, residuals are censored as well. Therefore, we show only residuals for non-censored observations here. This explains the superiority of negative residuals. The blue line represents the linear trend of the residuals.

The large variation of the residuals suggests a weak predictive power of the model. Residuals of more than one magnitude unit are not rare, and can even reach up to almost two units. Small observations are typically substantially overestimated, and vice versa. The root mean square error for predictions by the full model, 0.62, is only minimally better than by a Gompertz intercept model, 0.63. However, these values only account for predictions of non-censored observations. The majority of substantial covariate effects identified above explain increases of the expected magnitude difference, which means that related observations (e.g. events with larger depth or heat flow, or
at younger sea floors) are considerably more likely to be censored and therefore left out of the residuals statistics.

A similar argument holds for the negative linear trend of the residuals. For instance, the lowest mainshock magnitude, $M = 6.1$, can only have observed magnitude differences up to $\Delta M_{\text{maxObs}} = M - M_c = 1.1$. Therefore, if the model predicts $\Delta M > 1.1$, only negative residuals will occur in the statistic. As we move to larger predicted values on the x-axis, the selection bias affects even larger mainshock magnitudes.

The censoring of observations and residuals hinder a rigorous diagnosis of the model. Despite the covariates showing some relevant signals, it is evident that the model misses additional high-resolution geophysical variables for local site effects or event-specific properties that can help explain a larger proportion of the variance in the data.

5.5 Sensitivity Studies

As partly mentioned above, we tested the influence of varying time windows (e.g. $T = 365$ days) and spatial windows (e.g. $R(m) = L(m)$ or $R(m) = K(m) L(m)$ with gradually decreasing $K(m)$ as described above) in the declustering approach on the regression results. The covariate effects are very insensitive, indicating that the second strongest event typically occurs close to and shortly after (or before) the mainshock. In other words, the contamination of the response variable through background seismicity is negligible.

6 Results of Productivity-Regression

In this section, we present the results of the GAM regression of aftershock count data in New Zealand. First, we justify and validate our choice of the Negative Binomial distribution instead
of the commonly used Poisson distribution for aftershock counts. Then we show and interpret the
effects of the modeled covariates. Finally, we illustrate the impact of the results in a simulation
experiment.

6.1 Choice of Distribution Family

Fig. 7(a) and (b) show the quantile-quantile (Q-Q) plots of the deviance residuals for the Poisson
distribution (a) and the Negative Binomial distribution (b). The latter fits the data better, as it
adjusts the variance independently of the mean and therefore allows for larger variation than the
Poisson distribution, resulting in a more adequate representation of the upper and lower tails of the
distribution. The dispersion parameter of the Negative Binomial fit is 2.3.

In Fig. 7(c) and (d), the corresponding model residuals are plotted against the linear predictors $\eta_i$
(see Equation 6) for the Poisson (c) and Negative Binomial distribution (d). The Negative Binomial
fit shows substantially less spread in the residuals than the Poisson fit, confirming that aftershock
count data is rather Negative Binomial distributed.

Note that, according to our simplified simulation model, a larger variance of aftershock counts
directly translates into a larger variance of $\Delta M$. In other words, the Negative Binomial distribution
increases the likelihood of particularly small $\Delta M$.

Alternative approaches such as a Quasi Poisson or a zero-inflated model were tested, but did not
stand out substantially from the respective basic models. Additionally, from a substantive point of
view, there seems to be no causal reason for "excess zeros" that would suggest the use of zero-
inflated approaches.
6.2 Covariate Effects

Fig. 8 and 9 present the exponential, multiplicative effects of the categorical and metrical covariates on the expected number of aftershocks according to the relationship in Equation (5). That is, if the exponential effect of a category is larger than 1, it has a positive impact on aftershock counts, and vice-versa. If the exponential effect is equal to 1, the model shows no effect of the respective category.

6.2.1 Effects of Categorical Variables

Fig. 8 shows the effects of the categorical covariates Tectonic Region and Slip Type. The effects of the tectonic region are presented relative to their reference category crustal outside. Crustal events in the subduction zone show a substantially increased aftershock productivity. The expected number of aftershocks is approximately 1.8 times larger than for crustal events outside of the subduction zone. For interface and slab events, both shallow and deep, no clear signal is found, as their confidence intervals overlap with the reference line at \( \exp(0) = 1 \). Unclassified events appear to have a slightly positive effect, but the uncertainty is large. Sensitivity tests with different buffer sizes of the slab do not consistently confirm the effect for unclassified events. The positive effect of crustal events on the aftershock productivity, compared to interface, might be explained by reduced seismic coupling in subduction zones (Hainzl et al., 2019). On the interface, a substantial part of the deformation is often aseismic (Lay et al., 2012).

A second possibility is that, in the proximity of the study region, the crust contains a dense network of faults with a wide range of orientations, and therefore more structures that could be brought closer to failure by a change in stress conditions due to mainshock earthquakes. Slip type effects
are depicted relatively to their reference category unknown. None of the focal mechanisms appears
to be substantially more or less productive. Additionally, there seems to be no selection effect in
the sense that events where the focal mechanism is known have a common effect on the aftershock
productivity.

6.2.2 Effects of Magnitude and Depth

Fig. 9(a) shows an exponential effect of the triggering magnitude on aftershock productivity. How-
ever, this effect is enforced by the declustering approach, as the ETAS model fits the exponential
aftershock productivity function (4) to optimize the aftershock trigger rates \( R_{t}^{\text{trig}}(t, x, y, i) \). There-
fore, the effect only has control character.

The effect of the depth, shown in Fig. 9(b), confirms the argumentation in the discussion of the
\( \Delta M \) regression results, that increasing magnitude differences may coincide with reduced after-
shock productivity at higher depths. The physical reason may again be reduced seismic coupling
along the subduction interface relative to the shallow crust (Hainzl et al., 2019; Lay et al., 2012).

6.2.3 Effect of Mainshock Magnitude

Fig. 9(c) shows no clear trend in the effect of varying mainshock magnitudes. However, indepen-
dently of the size of the mainshock, triggered events in general appear to be two to three times
more productive than a comparable background event.

This finding has two possible explanations. On the one hand, it may be an indicator that the ETAS
model based declustering approach does not adequately disentangle spatio-temporal clusters in
the catalog, and incorrectly assigns too much aftershock productivity to the smaller events, at the
cost of the mainshock productivity. In other words, in seismically active periods, small events
may be simply more likely to be assigned offsprings than in seismically quiet times, but in reality, many of these aftershocks should perhaps be direct rather than secondary ones of the mainshock. Such a rearrangement of trigger relationships would have a strongly distorting effect on our model, in which we consider the number of direct aftershocks. Note, however, that we used here an ETAS model approach that accounts for short-term aftershock incompleteness as well as locally restricted, anisotropic spatial kernels and therefore already improves some of the major biases of common ETAS models (de Arcangelis et al., 2018; Grimm et al., 2022, 2021; Hainzl et al., 2008; Hainzl, 2021; Seif et al., 2017).

On the other hand, Zhuang et al. (2004) proposed that triggered events are more productive than background events, based on a similar study. It seems reasonable that during an on-going sequence the aftershock productivity could temporarily increase due to a higher level of energy prevalent in the tectonic system, compared to seismically quiet periods with occasional background activity. A doubling of the productivity parameter $A$ in the simulation model (see Introduction), applied only to secondary triggering, led to a reduction of the expected magnitude difference $\Delta M$ from 1.2 to below 0.9 due to the increasing cluster sizes. This additional “boost” in triggering illustrates the relevance of the observed effect. The finding may also contribute to an explanation as to why the ETAS model tends to underestimate cluster sizes and doublet probabilities in forward simulations, as observed in my first contribution (Grimm et al., 2021). Further research is recommended to evaluate this finding.

7 Conclusions

We adapted a survival regression model approach from medical studies to estimate the parametric distribution of the magnitude difference $\Delta M$ between the mainshock and its strongest foreshock or
The highlight of this regression class is that it accounts for right-censored observations. In our case, these are mainshocks for which no aftershock or foreshock is recorded above cut-off magnitude $M_c$, and for which we therefore have only the partial information that $\Delta M > M - M_c$, where $M$ is the mainshock magnitude.

We declustered a global earthquake catalog using a window method and computed $\Delta M$ for each of the independent clusters. Then, we enriched the cluster dataset with a plate boundary classification, relative plate velocities and sea floor ages obtained from the digital plate boundary model of Bird (2003) and with heat flow from Bird et al. (2008). From a simplified simulation model, assuming an exponential aftershock productivity law and the Gutenberg-Richter type magnitude size distribution, we concluded that the Gompertz distribution may be the better choice than Weibull or Generalized Gamma.

The regression results show that larger $\Delta M$ are expected at higher depths and in younger ocean crust. This may be an indication, that aftershock productivity is a relevant driver of $\Delta M$, as in these conditions lower aftershock productivity is expected due to reduced seismic coupling (Hainzl et al., 2019).

In the second part of this study, we used the stochastic declustering method of Zhuang et al. (2002) to estimate the aftershock productivity per event in a local catalog for New Zealand. To do so, we used the anisotropic ETAS-Incomplete model (Grimm et al., 2022) to disentangle the trigger relations between events. We further enriched the event set by a categorization of events in tectonic regions and slip types and used a generalized additive regression approach to model the aftershock productivity.

The results clearly confirm that aftershock counts follow a Negative Binomial rather than a Poisson distribution (Kagan, 2017; Shebalin et al., 2018). Also, aftershock productivity decreases with
increasing depth, supporting the reasoning regarding the depth effect on $\Delta M$ above. Furthermore, results indicate that triggered earthquakes trigger themselves two to three times more aftershocks than non-triggered ones. In other words, secondary aftershock triggering is substantially stronger than direct triggering by background events. This effect may either be an indicator for wrongly disentangled sequences in the sense that secondary triggering is overestimated at the cost of the productivity of the mainshock, or it may represent an actual effect due to the temporarily higher energy level after the occurrence of a strong mainshock. A causal effect, if confirmed, would have an enormous impact on the expected cluster sizes (compare with Grimm et al., 2021) and could explain some of the rather small $\Delta M$ observations in the first study.

Future research should identify whether small magnitude differences $\Delta M$ are typically characterized rather by above-average aftershock productivity or by magnitude size distributions favoring large aftershocks. To do so, one could compile a sufficiently large set of earthquake sequences and analyze the correlation of their $\Delta M$ with estimates of the aftershock productivity (Equation 4) and frequency-magnitude distribution (Equation 2). Additionally, it should be verified whether triggered events indeed have a larger aftershock productivity, and how this effect impacts $\Delta M$. Similarly, potential correlations of aftershock magnitudes with their ancestors should be evaluated. Finally, an extension of the $\Delta M$-regression model using small-scale covariate data could certainly contribute to a better understanding of magnitude differences in different geophysical settings.

Data and Resources

The U.S. Geological Survey National Earthquake Information Center (USGS-NEIC) catalog has been downloaded from https://earthquake.usgs.gov/earthquakes/search/ (last accessed on March 30, 2022). Global covariate data has been downloaded from http://
The New Zealand event set and focal mechanism data was provided by GNS Science as an input to the ongoing 2022 revision of the New Zealand National Seismic Hazard Model. For the stochastic declustering, we used the ETAS-Incomplete model source code available in the Github repository https://github.com/ChrGrimm/ETASanisotropic (Grimm et al., 2022), implemented using the software Matlab. All statistical analyses were performed with the open source software R.

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Statements and Declarations

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**Contribution of the authors**

CG (first author) designed the study, compiled and declustered the datasets, conducted and evaluated the $\Delta M$ regression model, prepared figures and wrote the paper. TR conducted and evaluated the productivity regression model. SH, KJ and MK contributed significantly to the interpretation of the model results in the geophysical context. Additionally, KJ categorized New Zealand events into tectonic regions. MP und HK contributed to the project idea and advised on data and methodology.

**References**


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