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Application of grey BP neural network model based on wavelet denoising to predict the residual settlement of goafs

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ABSTRACT

The residual settlement of goafs is a nonlinear process with time series. To study its settlement law and prediction model, we chose the Mentougou mining area in Beijing as an example. The wavelet threshold denoising method was used to optimize the measured data, and the Grey GM (1,1) and BP neural network models were combined in series. A grey BP neural network model based on wavelet denoising was proposed, the prediction accuracy of different models was calculated, and the prediction results were compared with the original data. The results showed that the prediction accuracy of the grey BP neural network combined model was higher than that of the single GM (1,1) model. Furthermore, the mean absolute percentage and root mean square errors of the grey BP neural network model after wavelet denoising were 0.35% and 16.05, respectively, both less than the errors of the combined prediction model using the original data. Thus, the combination model optimized by wavelet analysis had a high prediction accuracy, strong stability, and accorded with the law of change of measured data. It accurately reflected the goaf surface subsidence process, and therefore has a strong popularization and application value.

Introduction

To perfect new urbanization strategies for accelerated development, utilizing goaf sites as a building foundation has become an important measure to solve the problem of land shortage¹⁻². Therefore, it is necessary to establish a prediction model to predict the residual settlement of the old goaf surface and ensure the safety and stability of new buildings on its surface³⁻⁵.

Among the prediction models proposed by scholars, the grey model and BP neural network model prediction methods are most widely used to predict surface subsidence in underground mining areas⁶⁻⁸. In 1982, Professor Deng Julong, a Chinese scholar, proposed the grey system theory integrating automatic control and operations research, aiming at the in-depth exploration of grey problems. The theory can effectively predict the unknown information, which is beneficial for development, and has thus, been widely used in many fields of engineering technology⁹. Xu Liangji et al.¹⁰ considered remote sensing interpretation data and measured data as data sources. The GM (1,1) model was used to dynamically predict the ground subsidence of goaf and results showed that the maximum residual between the predicted and actual value was only 2.4 mm. The correlation coefficient (R²) was greater than 0.95, suggesting a good degree of curve fitting. Due to the uncertainty of mining subsidence, Huafeng Xu et al.¹¹ established the row vector average sequence GM (1,1), column vector average sequence GM (1,1), and cell volume sequence GM (1,1) models respectively, to model and analyse the monitoring data, for practical verification, and for model accuracy. The test showed that the predicted results were very close to the actual settlement value and met engineering requirements. Furthermore, Wang Zhengshuai et al.¹²⁻¹³ conducted a comparison test of various grey models for the nonlinear change of residual settlement in goaf. They found that the predicted values of the discrete DGM (1,1) and the GM-Markov models were closer to the actual values than the traditional GM model, with a higher stability. This validated the accuracy and feasibility of grey prediction models supporting their use in mining area surface subsidence research. In 1986, American scientists, Rumelhart and McCelland, put forward the concept of BP neural networks based on artificial neural network algorithms using nonlinear neuron processing functions. Its advantages lay in its simple structure, flexibility, and convenience, making it suitable for the study of nonlinear problems, such as surface subsidence caused by coal mining¹⁴. Lee S et al.¹⁵ combined this artificial neural network model with the geographic information system to evaluate and predict land subsidence changes in an abandoned coal mine in South Korea, based on the existing land subsidence information. Thus, they verified that the subsidence development trend predicted by ANN was consistent with the actual conditions, and predicted that the land subsidence in this region would continue to occur in the future. Pei Yanyu et al.¹⁶ used a genetic algorithm to optimise the parameters of the BP neural network model, using 36 groups of training samples and 4 groups of test samples for evaluation.
and analysis. Their results showed that the prediction was consistent with actual engineering conditions, suggesting that this model has a significant role in the study of nonlinear change problems with randomness. The surface subsidence value predicted by Li Yongfa using the GA-BP neural network model was compared with the monitoring value obtained by PS-INSAR technology, and the deviation between both results was found to be within a reasonable range, thereby verifying the feasibility and accuracy of the GA-BP neural network model for predicting ground subsidence. Wavelet analysis, based on Fourier analysis, can effectively distinguish unstable signal, refine the signal, and analyze the local situation of the signal. Wavelet transform weakens the noise in the signal and restores the reconstructed signal; thus, it is widely used in engineering practice.

Hence, the grey model and BP neural network models are more effective at predicting series changes; however, there are limitations. The disadvantage of the grey model is that the mathematical formula by which it deals with error and nonlinear fitting is not ideal. Similarly, the BP neural network model has a slow convergence speed and high demand for data samples. With the continuous expansion of their application, the shortcomings and deficiencies of single prediction models are gradually being highlighted. In this study, using the Mentougou mining area in Beijing as the study subject, the BP neural network and wavelet theory prediction mechanism based on grey theory were used to predict and analyze the residual settlement of the old mined-out area. This combination model provides a new basis for the theoretical exploration and practical application of surface deformation monitoring and prediction engineering in mining areas.

### Theoretical basis of residual subsidence prediction in goaf

#### GM (1,1) prediction model. Data monitoring in practical engineering is usually limited due to short monitoring cycles, therefore, it is necessary to adopt a targeted prediction method to study the settlement and deformation trend of goaf. The grey prediction models do not require much known historical data to predict the unknown information, and among these, the GM (1,1) model is most widely used as a single-sequence first-order gray linear model. Its modelling process is as follows:

1. **Establish the original data sequence**:
   \[
   x^{(0)} = \left[ x^{(0)}(1) \oplus x^{(0)}(2) \oplus \cdots \oplus x^{(0)}(n) \right]
   \]

2. **Stepwise accumulation (1-AGO)** generates a new prediction sequence:
   \[
   x^{(1)} = \left[ x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n) \right]
   \]

3. **Establishing grey differential equations for cumulative sequences**:
   \[
   x^{(0)}(t) + az^{(1)}(t) = b, \quad t = 1, 2, 3, \ldots, n
   \]
   Where, \( z^{(1)} \) adjacent mean sequence generated for \( x^{(1)} \):
   \[
   z^{(1)} = \left[ z^{(1)}(2), z^{(1)}(3), \ldots, z^{(1)}(n) \right]
   \]

4. **The corresponding whitening differential equation**:
   \[
   \frac{dx^{(1)}}{dt} + ax^{(1)} = b
   \]
   In the formula \( a \) is the development coefficient and \( b \) is the grey action.

3. **Construct coefficient matrix and constant term, calculate \( a \) and \( b \) by least square method**:
   \[
   B = \begin{bmatrix}
   -z^{(1)}(2) & 1 \\
   -z^{(1)}(3) & 1 \\
   \vdots & \vdots \\
   -z^{(1)}(n) & 1 \\
   \end{bmatrix}, \quad Y = \begin{bmatrix}
   x^{(0)}(2) \\
   x^{(0)}(3) \\
   \vdots \\
   x^{(0)}(n) \\
   \end{bmatrix}
   \]
   \[
   u = (a, b)^T = (B^T B)^{-1} B^T Y
   \]

4. **Substitute the values of \( a \) and \( b \) back into the original differential equation and derive the time response formula of grey differential**:
   \[
   x^{(1)}(t + 1) = \left( x^{(0)}(1) - \frac{b}{a} \right) e^{-at} + \frac{b}{a}
   \]

According to the following formula, the prediction model of the original sequence can be obtained:

\[
\text{BP neural network model. The Artificial Neural Network is a widely used information processing technology, and among these, the BP neural network shows good performance in the field of data prediction and is more suitable for dealing with changes in surface nonlinear subsidence caused by coal mining. The BP neural network has a strong learning ability and large storage space for mapping the relationship between input and output patterns. It’s structure is not restricted, but is usually divided into the input, hidden, and output layers. The neurons in each layer are independent of each other and connected between the layers, as shown in Fig. 1. and Fig. 2.}
**Grey BP neural network model.** To avoid the shortcomings of using a single model, we combined the GM (1,1) and BP Neural Network Models by serial combination, chosen for their excellent performances in isolation, to build a Grey BP Neural Network Model. The combined model was used to make a preliminary prediction of the data sequence through the grey network, which were used as the learning samples for further prediction by the BP neural network model and acquire the error sequence of the combination model, the specific prediction process is shown in Fig. 3. The combined model effectively reduced the error and improved the prediction accuracy, the specific modelling process is as follows:

1. Based on the raw data sequence, \( x^0 = [x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)] \), the GM (1,1) model was used to prediction, and the prediction sequence was \( \hat{x}^0 = [\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \ldots, \hat{x}^{(0)}(n)] \).
2. The error sequence, \( \epsilon^{(0)} = x^0 - \hat{x}^0 \), was obtained by subtracting the original data sequence \( x^0 \) from the prediction sequence \( \hat{x}^0 \).
3. Taking the prediction sequence \( \hat{x}^0 \) and error sequence \( \epsilon^{(0)} \) as the input and output samples, respectively, the BP neural network model was trained to obtain the corresponding weight \( V \) and threshold \( W \).
4. The error sequence \( \epsilon^{(0)} \) was imputed into the trained BP neural network model for further prediction and to obtain the new error sequence \( \epsilon^{(0)} \).
5. The prediction sequence \( \hat{x}^0 \) and the new error sequence \( \epsilon^{(0)} \) were added to obtain the prediction value \( Z^{(0)} = \hat{x}^{(0)} + \epsilon^{(0)} \) of the grey BP neural network model.

Comparing the original data with the predicted value, the prediction accuracy of the combined model was calculated and evaluated.

**Basic principle of wavelet denoising.** The signal of the original monitoring data had fluctuating noise signal, affecting the real monitoring information and accuracy of derived ground subsidence data. When dealing with such nonlinear signals, wavelet transform can reduce or eliminate random signals, extract system signals, and provide more accurate data support for deformation predictions.

The wavelet transform of any continuous function signal \( f(t) \) is defined as:

\[
WT_f(a, b) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t) \psi \left( \frac{t-b}{a} \right) dt
\]

Where, \( \psi_{a,b}(t) = |a|^{-1/2} \psi \left( \frac{t-b}{a} \right) \), to make the contravariant transformation exist, \( \psi(t) \) needs to meet the admissible row condition:

\[
C_{\psi} = \int_{-\infty}^{\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < \infty
\]

Where, \( \hat{\psi}(\omega) \) is the Fourier transform of \( \psi(t) \), then the inverse transform is:

\[
f(t) = C_{\psi}^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_{a,b}(t) WT_f(a, b) \frac{da}{|a|^2}
\]

The field of wavelet theory has facilitated gradual exploration of a more mature and perfect theoretical system in its wide application, from which the wavelet threshold denoising method has gradually been developed. The method is flexible and accurate, and the principle is simple. It can effectively remove noise and retain real signal characteristics and has a wide range of applications in many fields. Normally, the frequency of the real signal is low while that of noise is high. Thus, the principle of wavelet threshold denoising is to reduce or remove the noise distributed in the high frequency wavelet coefficients. The one-dimensional signal model containing noise is:

\[
s(n) = f(n) + \sigma e(i), \quad i = 1, 2, \ldots, n-1
\]

Where, \( s(n) \) is the monitoring signal, \( f(n) \) is the real signal, \( \sigma \) is the noise level, and \( e(i) \) is the noise signal.

**Engineering examples and applications**

**Geological conditions and monitoring.** The planned site is located in the Longquan Town, Mentougou District. The geological structure of the site is moderately complex and its digital surface model is shown in Fig. 4. The proposed site was located above the 9# coal seam of the Mentougou mine field, on-site data collection and visits to the surrounding residents revealed that the shallow surrounding the field is a historical small coal mining site. Most of these coal mines were mined privately or collectively using the basic coal mining method of room-and-pillar mining with a low recovery rate. The mining depth was generally no more than 60 m and the dip angle was 6°–8°, which is a gently inclined coal seam. To identify the engineering geological conditions in the proposed site, a Dpp-100 automobile drilling rig was used for a geological survey of the site (Fig. 5).

Based on drilling samples, in-situ testing, and geotechnical test results, the strata in this area was divided into three layers, according to the characteristics of rock and soil. The first layer was artificial filling soil layer, the second layer was a general
Quaternary sedimentary layer, mainly composed of silty clay and gravel, and the third layer was sandstone and cal sams with different wathering degrees in jrasic strata. Considering the shallow mining depth of small coal kiln without support measures, the area was presumed to be at risk of ground collapse.

To study the basic law of surface residual settlement deformation after coal mining was restricted by the terrain, a dip observation line was arranged on the south side of the planning land from east to west using Surface Subsidence Data by Field Measurement. The number of monitoring points was N1-N39, the interval between the two monitoring sessions was about 1y, with a total of 12 monitoring sessions recording work carried out using observation stations in accordance with the relevant provisions of "Engineering Survey Specification". Based on the data of rock movement monitoring the curves of monitoring points on the tendency observation, lines were drawn (Fig. 6).

As shown in Fig. 6, the overall change process of surface movement and deformation was continuous and gradual, presenting an asymmetrical distribution. Over time, the surface observation values from the monitoring points on both sides of the east and west to the central goaf showed a decreasing trend. The subsidence basin was mainly concentrated on the surface above the goaf and the curve shape conformed to the general law of surface subsidence. Maximum settlement at point N22 near the goaf boundary, by computing the cumulative subsidence of stage 12, was 1166.9 mm, the average annual subsidence was 106.1 mm, and the settlement value of the easternmost monitoring point N1 was the lowest at 82.2 mm, indicating that the surface was still in the process of settlement. The measured data of N22 monitoring points were shown (Table 1).

As mining has been discontinued in the Mentougou coal mine and other small coal mines, the surface above has now gone through a period of rapid deformation, and is entering the residual deformation stage. According to the observation data, the cumulative settlement curve and settlement velocity curve of the maximum subsidence point N22 monitoring site during the monitoring period were calculated and drawn (Fig. 7). The accumulated settlement of N22 point increased gradually with monitoring time, while the decline curve appeared as a gradually gentle phenomenon, presenting a "semi-parabolic" slow downward trend. The settlement velocity curve of the N22 point generally showed a trend of gradual decrease, but the curve appeared to have an "inflection point", not in conformity with general laws. This may be because the shanty towns established near the monitoring points, along with new buildings and human activities has increased the load on the surface of the goaf, resulting in a sudden increase in the sinking speed and a subsequent gradual decrease. The maximum subsidence velocity of monitoring point N22 reached 0.61 mm/d, less than the subsidence speed during the active period of surface movement (1.7 mm/d) stipulated in The Code for Coal Pillar Establishment and Coal Pressing Mining of Buildings, Water Bodies, and Railways and Main Shafts and lanes. During this time, the subsidence process was gentle and in the recession period of surface movement, which had little influence on buildings.

**GM (1,1) Model Prediction.** According to the principle of maximum subsidence, the actual settlement data of the N22 monitoring point on the surface observation line of the goaf were selected as the original sequence, generating Calculation Sequence by One - time Accumulation Method, by establishing the first-order linear differential equation to solve the development coefficient $a$ and the gray action $b$. The fitting GM (1,1) prediction model of the accumulated settlement of the N22 point was obtained by substituting the original differential equation, and the prediction accuracy was tested (Table 2). Subsequently, the original and predicted values were compared and analysed to obtain residual and relative errors predicted by the model (Table 3).

It can be seen from Table 2 that the posterior error ratio of the GM (1, 1) prediction model = 0.0249 < 0.35, and the small error probability value $= 1 > 0.95$, revealing a high precision level of the model which reached the first level prediction precision standard. Comparing the forecast data of the GM (1,1) model with the original data, the overall fitting degree was high, but the residual and relative errors of individual prediction values were large. The relative error ranged from -39.26% to 12.02%, the prediction results were heterogeneous, error polarization occurred, revealing that the prediction results of the single model were not very accurate, and it was difficult to solve nonlinear field problems using the prediction model established by grey theory alone. Therefore, other prediction models should be combined to reduce error and improve the prediction accuracy.

**Grey BP neural network model prediction.** From the predicted results of the grey GM (1,1) model, we found that the prediction error of the single model for the cumulative settlement of the initial monitoring point was large, therefore, MATLAB software programming method was used to achieve multiple training of samples to reduce the error of the predicted value. First, the results of the GM (1,1) model fitting prediction were used as input values for neural network training, the error sequence of the GM (1,1) model was then imputed into the trained BP neural network prediction model. Subsequently, the modified error sequence was added to the predicted sequence of the GM (1,1) model, which was the predicted value of the grey BP neural network model (Table 3).
To further determine the advantages and disadvantages of the two prediction models, the prediction accuracy was evaluated by comparing their mean absolute percentage and root mean square errors (Table 4). The smaller the two reference indicators, the smaller the actual predicted value error, the calculation formulas are:

\[
MAPE = \frac{1}{n} \sum_{t=1}^{n} \frac{|X_t - \hat{X}_t|}{X_t} \quad (16)
\]

\[
RMSE = \sqrt{\frac{1}{n} \sum_{t=1}^{n} (X_t - \hat{X}_t)^2} \quad (17)
\]

where, \(X_t\) is the measured value of cumulative subsidence point, \(\hat{X}_t\) is the predicted value of cumulative subsidence point, and \(n\) is the number of sample data.

It can be seen from Table 4, that the mean absolute percentage and root mean square errors of the grey BP neural network model were less than those of the GM (1,1) model, suggesting that the overall prediction effect of the grey BP neural network model was better than that of the GM (1,1) model. By comparing the prediction results of the two models (Table 3), we found that the predicted value of error compensation by the BP neural network was closer to the original value. Moreover, following preliminary training optimization, the residual error control was smaller, the maximum relative error decreased from -39.26 \% to -22.76 \%, and the non-uniformity of GM (1,1) model prediction error was reduced. To intuitively compare the development trend of the predicted values and original values of the two different models, drawing change curves based on data in Table 4 were used (Fig. 8).

Fig. 8 shows the development trend of the GM (1,1) and grey BP neural network models were similar to that of original value, although there was a small fluctuation suggesting both prediction models could better reflect the cumulative subsidence. However, from a macro point of view, the prediction curve of the grey BP neural network model was closer to the original data curve, the degree of fit was higher, and the relatively moderate and smooth small dispersion degree was better than that of the GM (1,1) Model. Thus, the combined prediction model combined the advantages of the GM (1,1) and grey BP neural network models. It could not only effectively solve the problem of series with volatility and nonlinearity, it minimized the requirement for large sample datasets by the BP neural network. Hence, we showed that the combined prediction model had more significant error optimization effects, a better stability performance, higher prediction accuracy, more accurate and applicable data prediction abilities than the single prediction model.

**Grey BP neural network model prediction after wavelet denoising.** Using the field application practice and measured data analysis of monitoring points, the monitoring data of surface subsidence in the goaf is affected by several factors, resulting in forecasting errors when directly using raw data. To avoid this phenomenon and improve research efficiency, the wavelet function was introduced into the wavelet analysis toolbox of MATLAB software to pre-process the original data. In this study, the cumulative settlement of the N22 monitoring point was selected for wavelet threshold denoising analysis. To select the optimal threshold, a state in which all other factors remained constant was controlled by the control variable method, using the Rigrsure, Sqtwolog, Heursure, and Minimaxi methods to denoise the original data with unknown scale white noise, and different denoising effects were obtained. The comparison curves of denoising effects using each of the four different threshold methods are shown in Fig. 9.

The denoising effects differed in the threshold chosen. Subsequently, the reconstructed sinking curve was smoother and more stable, without obvious oscillation and broken line phenomena. Wavelet threshold denoising improved and retained the original signal by removing the noise, thereby achieving the true function of N22 measuring point data denoising and providing a signal that was closer to the real subsidence data. It is difficult to determine the effect of denoising only by curve comparison charts; therefore, the root mean square error and signal to noise ratio were used to further evaluate the wavelet denoising quality. Theoretically, the smaller the root mean square error, the greater the signal to noise ratio, the closer the denoising signal is to the original signal, and thus the better the denoising effect. Since the number of observation periods at the monitoring point was less than 32, the root mean square error of the minimax threshold was zero, and could not effectively denoise and the curve coincided with the original data curve. Therefore, only the denoising effects of the other three threshold methods were compared (Table 5) using the results of the three different threshold evaluation indices. The root-mean-square error of unbiased risk estimation threshold was the minimum and the signal-to-noise ratio maximum. Thus, the denoising effect of this threshold method was relatively better and the prediction more accurate than the others.

Based on these results, the Daubechies3 wavelet, Rigrsure threshold method, and soft threshold principle were selected to denoise the accumulated settlement of monitoring points through one layer decomposition. The sequence of each layer after N22 data denoising and decomposition is shown in Fig. 10. The data denoised by wavelet analyses were applied to the grey
BP neural network model trained above for prediction, providing a new method for analysing the law of surface subsidence in goaf (Table 6).

Regression analysis of the accumulated settlement denoised by wavelet analysis (Table 6) revealed that the denoising value was similar to the measured value, and the maximum relative error was 5.01% with no significant fluctuations. Additionally, the noise fluctuations in the last eight periods decreased gradually, the denoising values were more stable, and the relative errors were ±1%, indicating a high degree of fitting. The real signal extracted by wavelet denoising was highly similar to the real settlement value, in line with the law of surface subsidence, further validating the reliability of the wavelet threshold denoising method. We found, through the prediction results of the grey BP neural network model denoised by wavelet analyses, that the relative errors in the 12 periods were all controlled within ±4%, the predicted value was roughly similar to the original value, and had a higher fitting degree, thus supporting the reliability and stability of this model.

The data sequence was smooth after denoising by wavelet analyses, the errors of the learning sample of the grey BP neural network were adjusted and the results optimised, greatly improving the accuracy of the data and model14. The mean absolute percentage and root mean square errors of the denoised grey BP neural network model were 1.78% and 16.05, respectively, which were significantly smaller than the error derived when the original data was used for prediction. To verify the effects of wavelet denoising on the prediction accuracy of the grey BP neural network model, the residual values of the prediction model before and after wavelet denoising were compared (Fig. 11).

Though some residual values were larger after wavelet denoising, the residual value of most phase wavelets after denoising were less than that before denoising, suggesting that the noise signal in the measured data had an effect on the prediction results of the combined model. Overall, the positive and negative trends of residual values were basically the same before and after wavelet denoising, that is, the predicted values of the two combined models were both higher and lower than the measured values. Furthermore, denoising could only reduce the error of prediction but did not affect the overall prediction trend, suggesting that the error variation of the prediction results of the grey BP neural network model based on the accumulated settlement after wavelet denoising was smaller and more stable than when the original data was used to predict the settlement value directly, and provided results closer to the actual settlement value39. The grey BP neural network model based on wavelet denoising has the advantage of three prediction theories further improving the accuracy and reliability of the predicted results provided by the model. This method has a wide applicability in the analysis of changes with volatility, randomness, and nonlinearity and provides a new method for predicting the settlement of small coal mine goafs or other similar projects. The theoretical basis can also be effectively applied to monitoring subsidence deformation in mining area.

Conclusion

(1) Using a mining area in Mentougou, Beijing as an engineering background, the general trend and settlement velocity of a surface residual settlement in a mined-out area of a small coal mine was analysed, according to general subsidence law, to provide reference and data support for establishing an effective prediction model.

(2) The grey GM (1,1) model was established to predict the surface residual settlement of goaf, and the model equation is given. The accuracy level was high, but the prediction error was non-uniform.

(3) Combining the ability of grey theory to effectively deal with small samples and few data points and that of the BP neural network in solving nonlinear problems, series them into grey BP neural network model for prediction, the mean absolute percentage and root mean square errors of the combined model were evaluated. We found that the optimization error effect of the combined model was significant and the values provided were more similar to that of the raw data that those provided by the single model, suggesting a higher prediction accuracy.

(4) Noise in the original monitoring sequence was processed using the wavelet threshold denoising method and linearizing the nonlinear settlement data. The curve after denoising was smoother with a high degree of fitting with original values. Thus, the denoising value obtained by reconstruction was applied to the combined model of the grey BP neural network, and we calculated mean absolute percentage and root mean square errors of 1.78% and 16.05, respectively, exhibiting a greatly improved prediction accuracy. The prediction effects of the grey BP neural network model based on wavelet denoising meets the needs of engineering practice, has a good theoretical significance, and high application prospects in subsidence prediction.

Data availability

The datasets used or analysed during the current study available from the corresponding author on reasonable request.

References


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Author contributions statement

Xiangdong Zhang and Wenliang Li wrote the main manuscripts and established theoretical models. Xuefeng Zhang provided research ideas and overall control of manuscript text. Guanjun Cai, Kejing Meng and Zhen Shen provided engineering reference materials and analyzed field observation data. All authors reviewed the manuscript.
**Additional information**

**Competing interests:** The authors declare no competing interests.

![BP neural network structure](image1)

**Figure 1.** BP neural network structure

![Flow chart of BP neural network](image2)

**Figure 2.** Flow chart of BP neural network

![Prediction flow chart of grey BP neural network model](image3)

**Figure 3.** Prediction flow chart of grey BP neural network model
Figure 4. Digital surface model diagram

Figure 5. Field drilling situation

Figure 6. The observation curve of monitoring points.
**Figure 7.** Cumulative settlement and settlement velocity curve of N22 monitoring points.

**Figure 8.** Comparison of predicted value and original value curve.

**Figure 9.** Comparison of four threshold denoising effect curves.
Figure 10. Approximate signal and detail signal diagram after decomposition

Figure 11. Comparison of residual value before and after denoising.

Table 1. The measured data of N22 monitoring points.

<table>
<thead>
<tr>
<th>Observation period</th>
<th>Observed value (m)</th>
<th>Cumulative subsidence value(mm)</th>
<th>Observation period</th>
<th>Observed value (m)</th>
<th>Cumulative subsidence value(mm)</th>
<th>Observation period</th>
<th>Observed value (m)</th>
<th>Cumulative subsidence value(mm)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>172.68</td>
<td>0.0</td>
<td>5</td>
<td>171.90</td>
<td>-783.8</td>
<td>9</td>
<td>171.64</td>
<td>-1039.3</td>
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<tr>
<td>2</td>
<td>172.44</td>
<td>-241.7</td>
<td>6</td>
<td>171.83</td>
<td>-850.6</td>
<td>10</td>
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<td>-1112.4</td>
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<tr>
<td>3</td>
<td>172.27</td>
<td>-408.5</td>
<td>7</td>
<td>171.75</td>
<td>-927.6</td>
<td>11</td>
<td>171.52</td>
<td>-1155.3</td>
</tr>
<tr>
<td>4</td>
<td>172.04</td>
<td>-641.7</td>
<td>8</td>
<td>171.70</td>
<td>-981.3</td>
<td>12</td>
<td>171.51</td>
<td>-1166.9</td>
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</table>

Table 2. Model building results

<table>
<thead>
<tr>
<th>Development coefficient</th>
<th>Grey action</th>
<th>Fitting GM (1.1) model</th>
<th>Posterior error ratio</th>
<th>Probability of small error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0772</td>
<td>1931.9822</td>
<td>$S^{(1)}(t+1) = -252.67.4e^{-0.0772t} + 25.025.7$</td>
<td>0.0249</td>
<td>1</td>
</tr>
</tbody>
</table>
### Table 3. Comparison between original and predicted values of the two models

<table>
<thead>
<tr>
<th>Observation period</th>
<th>Original value (mm)</th>
<th>GM(1, 1) model</th>
<th>Grey BP neural network model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Predicted value (mm)</td>
<td>Residual error (mm)</td>
</tr>
<tr>
<td>1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
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<td>-241.7</td>
<td>-336.6</td>
<td>94.9</td>
</tr>
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<td>-408.5</td>
<td>-463.5</td>
<td>55.0</td>
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<td>-580.9</td>
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<td>-689.6</td>
<td>-94.2</td>
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<td>-790.2</td>
<td>-60.4</td>
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<tr>
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<td>-969.6</td>
<td>-11.7</td>
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<tr>
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<td>-1039.3</td>
<td>-1049.5</td>
<td>10.2</td>
</tr>
<tr>
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<td>-1123.4</td>
<td>11.0</td>
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<tr>
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<td>-1155.3</td>
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<td>36.5</td>
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<tr>
<td>11</td>
<td>-1166.9</td>
<td>-1255.1</td>
<td>88.2</td>
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</table>

### Table 4. Prediction accuracy test table

<table>
<thead>
<tr>
<th>Evaluation criterion of prediction performance</th>
<th>GM(1, 1) model</th>
<th>Grey BP neural network model</th>
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</thead>
<tbody>
<tr>
<td>Mean absolute percentage error</td>
<td>9.09%</td>
<td>7.39%</td>
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<tr>
<td>Root mean square error</td>
<td>60.06</td>
<td>49.01</td>
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</table>

### Table 5. Denoising quality evaluation table of different threshold methods.

<table>
<thead>
<tr>
<th>Threshold mode</th>
<th>Root mean square error (mm)</th>
<th>Signal-to-noise ratio</th>
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</thead>
<tbody>
<tr>
<td>Regsure</td>
<td>7.40</td>
<td>33.04</td>
</tr>
<tr>
<td>Sqtwolog</td>
<td>12.49</td>
<td>19.36</td>
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<td>Heursure</td>
<td>21.93</td>
<td>10.17</td>
</tr>
</tbody>
</table>

### Table 6. Cumulative settlement and predicted value of N22 monitoring points after denoising

<table>
<thead>
<tr>
<th>Number of monitoring periods</th>
<th>Original value (mm)</th>
<th>Regression analysis after denoising</th>
<th>Grey BP neural network model after denoising</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Denoising value (mm)</td>
<td>Residual error (mm)</td>
</tr>
<tr>
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<td>0.0</td>
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<td>-12.8</td>
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<td>1.1</td>
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<td>-1155.3</td>
<td>-1153.2</td>
<td>-2.1</td>
</tr>
<tr>
<td>12</td>
<td>-1166.9</td>
<td>-1165.5</td>
<td>-1.4</td>
</tr>
</tbody>
</table>

**Table 6. Cumulative settlement and predicted value of N22 monitoring points after denoising**