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Investigation and modification of a CSSM-based elastic–thermoviscoplastic model for clay

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ABSTRACT

This paper examines the accuracy of a new elastic-thermoviscoplastic (E-TVP) constitutive model developed based on Critical State Soil Mechanics. The model can be used for simulating the temperature dependent, and strain-rate dependent behavior of clay soils. The study compares the E-TVP behavior of a single soil element with previously published thermo-mechanical experimental results performed on saturated clay specimens at different temperatures. Suggestions regarding unloading and reloading at constant temperatures as well as thermal consolidation under constant loads are presented. A modification for unloading-reloading adds a new criterion to the volumetric thermoviscoplastic strain rate formulation. A physics-based term is added to the current specific volume of the soil to include the viscous effect induced by temperature change. These modifications improve the convergence of laboratory data and simulated model responses. Comparisons of
results from an earlier E-TVP model and the newly improved model provide evidence of improved predictive capabilities.

INTRODUCTION

Engineered structures on foundations in soft clay often experience long-term creep settlements after initial pore water pressures have dissipated. Long periods of loading and changes in ground temperature can produce significant and non-recoverable deformations in highly plastic clays. Oil and gas pipelines, buried high voltage electrical cables, geothermal energy storages, and climate warming impacts on infrastructure are all affected by soil consolidation under coupled thermomechanical loadings.

Time-dependent behavior of soft soils was originally studied by Buisman (1936). Later, Taylor (1942) proposed a family of curves with various preconsolidation stresses and different duration of applied loads to demonstrate the non-uniqueness of stress-strain relationship in clay consolidation. He recommended the separation of volumetric strain into two parts of instant and delayed volumetric strains. Based on the observations of Taylor (1942), Bjerrum (1967) suggested a series of lines in $e - \log(p)$ diagrams representing the delayed compression characteristics of clay. Each line expresses the equilibrium void ratio at a specified effective surcharge pressure and specific time with constant loading.

For calculating long-term deformations, some researchers have adopted Bjerrum (1967) identification of ‘instant’ and ‘delayed’ components of settlements, rather than the more conventional, and successive, ‘primary consolidation’ and ‘secondary compression’. The instant component is associated with largely reversible (elastic) deformations of the clay particles themselves. The delayed component includes non-recoverable (plastic), time-dependent, hydro-mechanical consolidation, and simultaneous viscoplastic reorganization of the inter-particle microstructure. The latter can also be considerably affected by changes in the temperature profile in the ground (Hueckel et al. 2009).

Perzyna (1966)’s modeling and experimental tests provide an initial framework for explaining the viscous behaviour of clay. Following this initial work, Adachi and Okano (1974) proposed
a rate-dependent viscoplastic model based on the Cam Clay model. They presented two loading surfaces known as static and dynamic loading surfaces. At a very slow rate (zero strain rate), the stress state lies on the static yield surface \( f_s \), and in a case where the loading is at a higher rate, the stress state lies on the dynamic yield surface \( f_d \) with a stiffer response. They assumed that the soil skeleton and the adsorbed water behave viscously and as an EVP continuum. Adachi and Oka (1982) improved the earlier work using the viscoplastic flow rule of Perzyna (1966) and defining a viscoplastic parameter as a hardening parameter. Later, Adachi et al. (1987) developed a damage creep law to model undrained creep rupture.

Yin and Graham (1989) developed a one-dimensional creep model based on Bjerrum (1967) timeline for stepped loading using a concept of equivalent time. Later, in 1999, Yin generalized his earlier work and formulated a three-dimensional elastic-viscoplastic constitutive model for overconsolidated clays. In following developments, Zhou et al. (2005) developed an anisotropic elastic-viscoplastic model encompassing the \( K_0 \) consolidation of the soil. It is worth mentioning that the models proposed by Zhou et al. (2005) was based on Perzyna (1966) viscoplasticity and the modified Cam Clay model.

Although the elastic-viscoplastic model proposed by Yin and Graham (1999), and later improved by Yin et al. (2002), was innovative, it described the compressive response of soil specimens in terms of volume strain rather than more fundamental state parameters, namely specific volume or void ratio.

In future development of Yin’s work, Kelln (2007, 2008a, 2008b) proposed an elastic-viscoplastic soil model (EVP) based on the framework of Critical State Soil Mechanics and utilized specific volume instead of strains in their formulations. This has the advantage that decreases in the creep coefficient with time can be expressed without defining an additional material constant. They used this model to simulate vertical and horizontal deformations of a highway embankment on soft estuarine clay near Limavady in Northern Ireland. The simulations produced results that compared well with data measured by field instruments.

Temperature-dependent behavior of soft soils has also been extensively examined by Gupta
(1964), Campanella and Mitchell (1968), Green (1969), Hueckel and Baldi (1990), Tanaka (1995), Cui et al. (2000), Graham et al. (2001), Cekerevac and Laloui (2004), Mašín and Khalili (2012), Xiong et al. (2016) and by other researchers. The models proposed by Modaressi and Laloui (1997) and Yashima et al. (1998) were among the first models investigating the thermal-induced viscous effect in soft soils. These models were based on Perzyna (1966) overstress viscoplastic theory. Temperature fluctuations induce changes in diffuse double layers (DDL) in clay and this affects soil properties such as strength, hydraulic conductivity, pore water pressure, and soil compressibility (Tanaka 1995).

The next step in these early studies was an examination of the coupled effects of loading and changes in temperature on clay soils. Yashima et al. (1998) carried out different constant strain-rate oedometer tests. They found that the preconsolidation pressure of clay is related to temperature change and strain rate. Tsutsumi and Tanaka (2012) performed several constant strain-rate tests and showed that the secondary compression of clay is caused by clay viscosity, which varies with temperature and strain rate.

In real-life engineering applications, for example, foundations in areas affected by climate change, high-temperature waste contaminants, and geothermal storage systems, clays are subjected to mechanical, environmental, and thermal loadings. To accurately calculate soil deformations, both time-dependent and temperature-dependent behaviors of soils should be considered in the design process.

Leroueil (1996), described the soil creep model based on a stress-strain-strain rate-temperature formulation to correctly model the compressibility of soils. Marques et al. (2004) performed different oedometer tests and constant strain-rate tests to investigate the viscous behaviour of soft clay soils. They showed that the vertical yield stress depends on the strain rate and the temperature. Furthermore, the plots of logarithm of strain rate versus preconsolidation pressure are parallel lines for various temperatures. Yashima et al. (1998), extended the elasto-viscoplastic model of Adachi and Oka (1982) to take into account the effect of both temperature and strain rate on the viscosity of soft soils. Laloui and Cekerevac (2008) presented a formulation to consider the effects of both
strain rate and temperature on preconsolidation pressure of clays based on Cekerevac and Laloui (2004).

Kurz et al. (2016) extended the elastic-viscoplastic model proposed by Kelln et al. (2008b) to obtain a model that considers the effects of both temperature and time on clays behavior. Their elastic-thermoviscoplastic soil model (E-TVP) model assumed that the creep coefficient $\psi_T$ varied with temperature. Other recent studies have been reported by Qiao and Ding (2017) and Hamidi (2020) on thermoviscoplastic constitutive modeling.

The research described by Kurz et al. (2016) has been developed by the current authors as a possible tool for studying temperature-related projects. It is based on earlier research by two of the senior authors; Graham et al. (2001) on modeling of reconstituted illitic specimens; Maghoul et al. (2010) and Gatmiri et al. (2010) on thermo-hydro-mechanical modeling for waste disposal; and Saaly et al. (2020) and Anonghouth et al. (2020) on geothermal energy harvesting.

Of particular interest in the modeling presented here is its emphasis on plastic potentials (PP) that define the slope of plastic strain increments when clay yields. The article assesses the accuracy and credibility of the E-TVP model by comparing simulated results with experimental data from isotropically consolidated triaxial tests carried out by Tanaka (1995), Crilly (1996), and Ghahremannejad (2003) on saturated specimens of clay. The specimens were subjected to various confining pressures and temperatures that varied from 27°C to 100°C.

In this paper, after briefly presenting the model, we aim to study the ability of the E-TVP model presented by Kurz et al. (2016) to capture a wide range of rate-dependent and temperature-dependent characteristics of clays. Test results that have been examined come from isotropic unloading and reloading at constant temperatures, triaxial undrained shear at three constant temperatures, triaxial drained shear under constant temperatures, and thermal consolidation under constant mechanical loads. Results from the model have been compared with laboratory measurements. Some modifications have been made to improve the stability and robustness of the proposed model.

**PARTITIONING OF STRAINS**

Total strains can be partitioned into elastic and viscoplastic components. That is, $e^{total} =$
\( \varepsilon^{\text{elastic}} + \varepsilon^{\text{viscoplastic}} \) \cite{Yin2002,Kelln2008a}. Partitioning in this way applies to both compression and shear components of the strain tensor.

The assumptions underlying this formulation are that elastic strains are time-independent and isotropic, and this remains valid when the soil is yielding. Plastic strains vary with time, (that is, the duration of loading), and temperature. The effects of changing temperatures have received less attention than those of isothermal viscous effects, but must be considered in projects such as design of foundations for furnaces, transmission towers, high temperature waste containment, deep excavations, thermal piles for accessing geothermal energy, and climate warming that produces thermal gradients in thawing permafrost under roadways.

One outcome of this approach is that preconsolidation pressures are not unique but vary with the strain rate in constant-rate-of-strain tests or with the duration of mechanical loading in constant-load tests \cite{Saellfors1975}. In the normally consolidated range of pressures, the slope \( \lambda \) of the Normal Consolidation Line (NCL) is constant, but NCLs for different mechanical loading rates move to lower values of specific volume \( V \) with decreasing rates of mechanical loading, increasing mechanical load durations, and increasing temperature (Figure 1). Here, \( V \) is specific volume, the volume occupied by unit volume of solids. In the overconsolidated range, preconsolidation pressures and the slope \( \kappa \) of the Unload-Reload Line (URL) vary with the strain rate \cite{Graham1983}. It is now known that this apparent variation of \( \kappa \) is because of deformations that include both elastic recoverable strains and viscoplastic non-recoverable strains which vary with the duration of testing \cite{Kelln2008a}; and temperature \cite{Kurz2016}.

**OUTLINE OF THE E-TVP MODEL**

The following section provides a brief overview. More details can be found in Kurz \cite{Kurz2014} and Kurz et al. \cite{Kurz2016}.

Viscous behaviour in clays means that deformations depend on the duration of loading, the rate at which loading is applied, the soil temperature, or some combination of these processes. It also influences measured properties like preconsolidation pressure, yielding, and undrained shear strength \cite{Graham1983,Graham2001,Yin2002,Kelln2008b}. An
effective constitutive model must therefore include relationships between stress, stress rate, strain, strain rate, time, and temperature. This implies that creep will be present during what are commonly known as primary consolidation and secondary compression, and also in both compression and shear.

Creep of soil particles is non-recoverable, that is, 'plastic', and involves localized movement of water, even if the overall volume is constant. Rates of creep movements decrease exponentially with increasing time and increase with increasing temperature.

Figure 1 shows the general structure of the E-TVP model in compression space defined by mean effective stress $p'$ (logarithmic scale) and specific volume $V$ for a stress status of non-zero deviator stress. Loading can either be one-dimensional in oedometer tests, or isotropic in triaxial compression tests. Here $p' = \left(\sigma'_1 + 2\sigma'_3\right)/3$ and the deviator stress $q = (\sigma'_1 - \sigma'_3)$.

The NCL, with slope $\lambda$, represents isotropic first-time compression. The URL, with slope $\kappa$ represents unloading from the isotropic normal compression line. The isotropic compression state $p'_m, q = 0, V_m$ of any effective stress status is considered to be at the end of consolidation with acceptably small values of pore water pressure. More significantly for our interests here, “Delayed Compression” representing the viscous compression from a starting condition $a_0$ on the NCL corresponding to strain status of A at time $t_0$, will reach $b_1$ with the strain status of B at time $t_1$, and define the then-current position of the URL under constant mechanical and thermal loading, with the same stress state at A and B. Here, $t_0$ is an assumed initial time usually taken as 24 hours after the application of each load increment to 1-D or triaxial compression specimens, $t$ is the duration of the present loading. Viscous straining is also present during hydro-mechanical plastic compression, commonly called ‘consolidation’, during the time period before $t_0$. The actual specific volume of each strain status is represented by $V_x$; however, the specific volume corresponding to $p'_m$ on the relevant unloading-reloading line is illustrated by $V_m$.

Figure 2 shows a CSSM-based schematic of the creep model developed by Kelln et al. (2008b) for a constant isotropic stress status. After long periods of mechanical loading, soil particles will become positioned together as closely as possible and void spaces will have been minimized.
Typically, hydro-mechanical (consolidation) straining will already have been completed. Further creeping is terminated at the Viscoelastic Limit Line (VPL) corresponding to the locus $y_lC$ which is the maximum size of possible yield locus for the mechanical loading of $p'_A$. With passing time, the size of yield locus increases from the $y_lA$ to reach the plastic potential surface (PPS) $PPS_{A,B,C} = y_lC$. A plastic potential represents the slopes $\delta\varepsilon_q^p/\delta\varepsilon_p^p$ for constant ratios of yield stresses $p'_y, q_y$ when maximum purely elastic straining is reached and combined elastic and plastic straining begins (Wood 1990). Here $\delta\varepsilon_q^p$ and $\delta\varepsilon_p^p$ represent plastic strain increments, and $p'_y, q_y$ represent the values of $p'$ and $q$ at yielding. The size of the plastic potential passing through the effective stress state $p', q$ is reflected in an isotropic effective mean stress $p'_m$. The net outcome of this approach is that plastic potentials and yield loci are related in shape but not identical.

The position of the VPL can also be chosen so that the time needed to reach it corresponds with anticipated lifetimes of engineering structures, (Kelln et al. 2008a). The VPL and NCL are parallel and are separated by a constant specific volume of $(\lambda - \kappa)\ln(p'_n/p')$, where $p'_n$ is mean effective stress on the normal compression line at its intersection with the URL that corresponds to the current viscoplastic deformation.

To the left of the VPL, corresponding to deformations inside each locus, in Figure 1, and Figure 2b, deformations are purely elastic and no viscoplastic strains will be experienced. To the right of the VPL, elastic deformations are instantaneous and viscoplastic behaviour begins immediately. If an isotropic condition $(p'_m, q = 0, V_m)$ is to the left of, or on, the VPL in Figure 2b, the stress state is inside, or on, the current yield locus (Figure 2a). Yield loci, as well as the plastic potential in the $(p', q)$ diagram, are assumed to be elliptical, as in the Critical State Model.

Definitions of some of the model’s parameters differ from those in conventional elastic-plastic soil models. For example, the size of the yield locus, defined by $p'_{0A}, p'_{0B}, p'_{0C}$ in Figure 2a, is controlled by the VPL instead of the isotropic NCL. A normally consolidated specimen under constant stress at A in Figure 2b creeps with increasing time to B and then to C. In each of these cases, the current size of the yield locus is now inside the plastic potential surface (PPS) and on the left of the normal consolidation line. Generation of viscoplastic volumetric compression
increments $\delta V_{AB}^{Tvp}$ and $\delta V_{BC}^{Tvp}$ is now consistent with the basic rule of plastic deformations. That is, plastic deformations occur when a $p', q, V$ stress state exceeds the current yield locus. Remember that normal consolidation lines, unload-reload lines, yield loci, creep trajectories and volumetric limit lines are all three-dimensional in nature, and are only represented as two-dimensional in Figures 2a and 2b.

The model requires that a plastic potential passing through an effective stress $(p', q)$ and controlled in size by $p'_m$, has to be compared with the size of the related yield locus $p'_0$. When isotropic stress states $(p'_m, 0, V_m)$ lie on or below the viscoplastic limit line, for example due to earlier unloading, the stress state is inside the yield locus $f(p', q, p'_0) < 0$ and $p'_0 > p'_m$. The soil response is then purely elastic.

When $p'_0 < p'_m$, the stress state is in theory outside the current yield locus ($f(p', q, p'_0) > 0$). This means that, from a physics viewpoint, additional loading can produce both hydromechanical and thermoviscoplastic strains (Kurz et al. 2016). The strains vary with changes in loading, the duration of loading, and temperature.

In this model, the second-order strain-rate tensor is written as:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^{Tvp}_{ij}$$

(1)

where $\dot{\varepsilon}^e_{ij}$ and $\dot{\varepsilon}^{Tvp}_{ij}$ denote, respectively, the elastic and thermoviscoplastic increments of the strain-rate tensor.

From isotropic elasticity, the elastic part of the strain-rate tensor $\dot{\varepsilon}^e_{ij}$ can be written as,

$$\dot{\varepsilon}^e_{ij} = \frac{\kappa}{3V} \frac{p'}{p'} \delta_{ij} + \frac{1}{2G'} \dot{s}_{ij}$$

(2)

where $p'$ and $s_{ij}$ are the mean effective principal stress and deviator stress components of the stress-rate tensor, and $\kappa$, $V$, and $G'$ represent the URL coefficient, specific volume and shear modulus, respectively. Kurz et al. (2016) assumed that temperature change only causes the plastic part of the straining.
In Figure 1 and Figure 2, the subscript \( m \) indicates the isotropic stress state of the yield locus. The specific volume \( V_m \) of the specimen, which is related to the loading time \( t \) and the isotropic mean effective stress \( p'_m \) (the reference size of the plastic potential), can then be derived as follows from the hyperbolic creep function of Yin et al. (2002).

\[
V_m = N - \lambda \ln(p'_m) - \frac{\psi_T \ln \left( \frac{t_0 + t}{t_0} \right)}{1 + \frac{\psi_T}{N-Z} \ln \left( \frac{t_0 + t}{t_0} \right)}
\]  

(3)

In Figure 1, \( N \) and \( Z \) are respectively time-independent values of specific volume \( V \) on the normal consolidation line (NCL) and the viscoplastic limit line (VPL) at \( p' = 1 \) kPa, \( \psi_T \) is the temperature-dependent creep rate expressed as the slope of a delayed compression line in the compression plane \( V \) vs. \( \ln(t) \), \( \lambda \) is the slope of the NCL and the VPL, \( t \) is the duration of loading, and \( t_0 \) is an assumed initial time for calculating creep deformations, usually taken as 24 hours after the application of load. The NCL therefore represents only hydromechanical plastic strains (consolidation) in the first 24 hours after a new load increment has been added.

Yin and Graham (1989, 1994, 1999) introduced a logarithmic function to model the creep strains in clays. In long-term consolidation calculations, the logarithmic creep function may lead to significant errors. The logarithmic creep function in infinite time produces infinite strain, which is incorrect. The coefficient of secondary compression, which is the slope of creep strain against \( \log \) (time), diminishes with time. Accordingly, the logarithmic creep function overestimates the creep deformations. Yin (1999) suggested a hyperbolic creep function to overcome the above-mentioned deficiency in the logarithmic creep term. Yin et al. (2002) adopted this hyperbolic nonlinear creep function for time-dependent creep volume strain under isotropic stress. This approach is followed by Yin and Tong (2011) with a new one-dimensional elastic viscoplastic model of creep and swelling. Also, Feng and Yin (2017) utilized the previous effort on the creep function for calculating the creep consolidation of a double soil layer.

Kurz et al. (2016) adopted the creep rate coefficient \( \psi_T \) from the following exponential relationship for the creep coefficient \( C_\alpha \) proposed by Fox and Edil (1996) as a function of temperature.
\[ \psi_{T2} = \psi_{T1} \exp \left[ \Omega(T_2 - T_1) \right] \]  

where \( \Omega \) is a material constant found experimentally, and the parameters \( \psi_{T1} \) and \( \psi_{T2} \) are the coefficients of delayed (thermoviscous) compression corresponding respectively to \( T_1 \) and \( T_2 \) Kelvin. 

Kurz et al. (2016) recommended that \( \psi \) should be measured at different temperatures in the laboratory and then used as “anchor points” to find the \( \Omega \) parameter, which is assumed to be constant, but with different values for different clays and different pore water chemistries. To the author’s knowledge, literature pertaining to \( \Omega \) is not available. This project used a constant value of \( \Omega = 0.015/K \) (Kurz et al. 2016), which provides a near-zero value for \( \psi \) at 0 Kelvin. In Equation 4, \( \psi \) is substituted instead of \( C_\alpha \) in the original Fox and Edil (1996) relationship. The value of \( C_\alpha / C_c \) is assumed to be in the range of 0.02 - 0.10 as reported by Mesri and Choi (1985) for natural soils in which \( C_c \) is compression index. The variation of \( C_c \) is large and expressed by plasticity index \( (I_P) \). The average value of 0.06 is chosen for \( C_\alpha \) at the temperature of 28°C. By knowing the parameter \( \Omega \) and one anchor point \( (\psi_1,T_1) \), the amount of \( \psi_2 \) is updated in the spreadsheet for each \( \Delta T \) increment.

The first two terms in Equation 3 provide the specific volume \( (V_{NCL}) \) on the NCL at time \( t_0 \) at constant \( p'_m \). The third term calculates the change in specific volume during a loading time \( t \) under constant temperature \( T \) and constant mechanical loading \( p'_m \). Figure 1 also present the creep-induced compression line \( (a_0 - b_1) \), which is a vertical line starting from the current stress state in the compression plane. It describes the development of thermoviscoplastic (temperature and time) strain at a constant load with increasing time.

From Kurz et al. (2016), the thermoviscoplastic volumetric strain-rate \( \dot{\varepsilon}_p^{T\nu p} \) can be obtained from Equation 5.

\[
\dot{\varepsilon}_p^{T\nu p} = \left( \frac{\psi_T}{V_m t_0} \right) \left( 1 - \frac{N - \lambda \ln(p'_m) - V_m}{N - Z} \right)^2 \exp \left[ \frac{N - \lambda \ln(p'_m) - V_m}{(N - \lambda \ln(p'_m) - V_m - 1)} \psi_T \right] \tag{5}
\]

Identifying and quantifying the components of plastic deformations requires definition of a flow
rule. The plastic potential $g$, as shown by $PPS_{A,B,C}$ in Figures 2a,2b, has the form

$$g(p', q, \xi) = 0$$  \hspace{1cm} (6)

and takes the elliptical shape of the MCC yield locus. Here $\xi = p'_m = p'_{mA,B}$ in Figure 1 defines the size of the plastic potential related to a constant effective stress $p', q$. The plastic strain-rate tensor is assumed normal to the plastic potential, Equation 6. However, the shape of plastic potential and yield surface is elliptical, their locations during the time of testing are not the same. That is, the flow rule is ‘non-associated’.

$$\dot{\varepsilon}^{Tvp}_{ij} = S \frac{\partial g}{\partial \sigma'_{ij}} \delta_{ij}$$  \hspace{1cm} (7)

The term $\partial g/\partial \sigma'_{ij}$ determines the sign of the volumetric thermoviscoplastic strain. The scalar function $S$ is defined as

$$S = \left( \frac{\psi_T}{V_{mt0}} \right) \left( 1 - \frac{N - \lambda \ln(p'_m) - V_m}{N - Z} \right)^2 \exp \left[ \frac{N - \lambda \ln(p'_m) - V_m}{\left( \frac{N - \lambda \ln(p'_m) - V_m}{N - Z} - 1 \right) \psi_T} \right] \frac{1}{|\partial g/\partial p'|}.$$  \hspace{1cm} (8)

The scalar function $S$ is assumed to be positive. The expression $\partial g/\partial p'$ is always positive for normally consolidated soils; using the absolute value is for consideration of overconsolidated soil. It is worth mentioning that this framework may cause instability when the term $\partial g/\partial p'$ approaches zero (reaching the critical state condition). The thermoviscoplastic volumetric strain-rate tensor generated by changes in temperature is therefore:

$$\dot{\varepsilon}^{Tvp}_{ij} = \left( \frac{\psi_T}{V_{mt0}} \right) \left( 1 - \frac{N - \lambda \ln(p'_m) - V_m}{N - Z} \right)^2 \exp \left[ \frac{N - \lambda \ln(p'_m) - V_m}{\left( \frac{N - \lambda \ln(p'_m) - V_m}{N - Z} - 1 \right) \psi_T} \right] \frac{1}{|\partial g/\partial p'|}.$$  \hspace{1cm} (9)

Additional discussion of thermal effects will be found in the following section.
The E-TVP model needs the determination of eight parameters. The conventional critical state soil mechanics parameters in the model can be obtained from load-unload-reload consolidation tests as well as strain-controlled consolidated drained triaxial tests (CD) or consolidated undrained triaxial tests (CU). The creep parameter $\psi_{T1}$, which is $dV/d(ln(t))$, can be obtained from a creep test. The material parameter $\Omega$ is estimated using Equation 4, knowing $\psi_{T1}$ and having an anchor point of $\psi_{T2} = 0$ at $T_2 = 0$ Kelvin. Moreover, the constant $t_0$ can be obtained using curve fitting.

COMPARISON OF MEASURED AND SIMULATED RESULTS

Results from the E-TVP model presented in the previous sections will now be compared with experimental data obtained by Tanaka (1995) and Crilly (1996), who tested 50 mm diameter x 100 mm high specimens of reconstituted illite; and Ghahremannejad (2003), who tested reconstituted Kentucky M44 clay, which consists predominantly of approximately equal portions of illite and kaolin. The tests included mechanical unloading-reloading cycles at different constant temperatures, drained and undrained triaxial compression tests under several fixed temperatures, and changes in volume due to temperature changes under drained constant loading. For the comparisons, the E-TVP model was implemented in a spreadsheet as a single-element model representing a specimen in a triaxial cell. Section “The effects of heating: thermal consolidation under constant loads” shows that the ‘original’ model should be modified for the heated, constant loading tests. The necessary modification is explained in Section “Model modification”.

Isotropic unloading and reloading at constant temperatures

The main objective of Crilly’s tests was to examine the relationship between temperature and the slope $\kappa$ of the URL in the compression plane (Crilly 1996). Here, we compare modeling results with laboratory data from Crilly’s triaxial test specimens T1500, (Figure 3), and T1508, (Figure 4 and Figure 5). The tests consisted of an isotropic consolidation phase that included one or more mechanical unloading-reloading cycles, and a subsequent shearing phase. The following section deals only with the unload-reload cycles, indicated here by square symbols and dashed lines.

Both specimens were initially consolidated at 27°C and about 0.6 MPa mean effective compression pressure. The mean effective stress is the difference between the externally-applied cell
pressure and 1 MPa of internal pore water pressure applied to ensure saturation. Consolidation was assumed to have ended when the axial strain rate dropped below 0.1%/day. The unload-reload phase was begun after the initial consolidation phase had been completed.

After being consolidated to 0.588 MPa, specimen T1500, was unloaded in three steps to 0.264 MPa and later reloaded in three steps to 0.560 MPa, (Figure 3). The temperature was held constant at 27°C. The total duration of the mechanical unloading-reloading phase for specimen T1500 was about 16 days. In test T1508, the specimen was first loaded isotropically to $p' = 1.479$ MPa, (Figure 4), and T = 27°C. After reaching equilibrium, the temperature was raised to 65°C. The specimen was then unloaded to 0.737 MPa ($OCR$ = 2) and reloaded to 1.541 MPa during a test period of 6.79 days. The temperature was then increased to 100°C and a second unload-reload cycle was performed (Figure 5), with the specimen being unloaded to 0.771 MPa and reloaded to 1.524 MPa during 5.09 days.

As is common in laboratory testing, the data in Figures 3, 4, and 5 nearly closed loops with slightly different paths for unloading and reloading, and slightly lower specific volume when the final reloading pressure was reached. The graph implies hysteresis and different volume change behaviour during shrinking and swelling.

Figures 3, 4, and 5 also indicate results simulated using the original Kurz et al. (2016) E-TVP model, (shown as solid lines); and results from a modified model developed by the authors (shown as dash-dotted lines). The solid lines show significant differences between the measured and modelled results. The dash-dotted lines obtained from the modified model described later show much better agreement with Crilly’s data.

Modifications to the original model serve two purposes. One relates to unload-reload behaviour at constant temperature and will be described in following paragraphs. The second relates to the effects of temperature changes at constant pressures. It will be described in Section “The effects of heating: thermal consolidation under constant loads”.

The original E-TVP model assumed that deformations due to both mechanical unloading and reloading were elastic-viscoplastic. For the modified model, deformations in mechanical unloading-
reloading cycles have been assumed to be purely elastic during unloading. Reloading remains elastic-viscoplastic.

Both the original and the modified model assume that the unloading-reloading parameter (κ) is independent of the stress level and temperature. In Crilly’s tests, the reloading phase was largely above the VPL, (Figures 2a, 2b), and it was reasonable to assume that viscoplastic straining started immediately after reloading began. Since the E-TVP model assumes that the material properties κ, λ, N, and M are independent of temperature, Crilly’s measured soil parameters were considered a single population and averaged for use in the E-TVP modeling (Table 1).

In Figures 3, 4, and 5, results from the modified version of the E-TVP model illustrate reasonable agreement with laboratory data for both reloading and unloading phases at different constant temperatures. The vertical axes have been drawn to a fine scale that suggests some variations of the level of agreement between measured and modelled specific volumes with temperature, particularly during unloading.

**Triaxial undrained shear results at three constant temperatures**

Crilly also tested triaxial compression specimens in undrained (CIU) and drained (CID) shear tests at constant temperatures. Figures 6, 7, and 8 compare data from three of the undrained specimens, T1503, T1505, and T1508 with results from E-TVP modeling. The specimens were isotropically consolidated to 1.56 MPa from a starting mean effective pressure of 0.63 MPa. A back-pressure of 1 MPa ensured saturation and avoided boiling. The consolidation pressure of T1503 was then reduced to 1.25 MPa, producing a lightly overconsolidated ratio (LOC) of 1.24 at a constant test temperature of 27°C. Specimens T1505 and T1508 were normally consolidated (NC) at 1.5 MPa and sheared at test temperatures of 65°C and 100°C respectively. Undrained shearing was performed at an axial strain rate of 0.9%/hr.

Figures 6, 7, and 8 show simulated and measured data of (a) stress paths plotted as deviator stress q versus mean effective stress p′, (b) deviator stress versus axial strain ε₁ and (c) pore water pressure versus ε₁.

The patterns of behaviour from the E-TVP model are broadly similar to those of the measured
data but they suggest rather more axial straining, higher pore water pressures, and in the LOC specimen in Figure 6, some remaining anisotropy associated with early stages of elastic straining. The initial forming of the specimens was one-dimensional in cylindrical tubes. Differences are particularly evident in the simulated graphs of deviator stress \( q \) versus effective mean stress \( p' \) that show marked strain-softening. Modeling by Hueckel et al. (2009) also shows significant strain-softening in modeling undrained tests. Crilly (1996) showed that while the slope \( M \) of the Critical State Line remains constant, the actual shape of the yield surface changes with temperature.

Maximum simulated pore water pressures \( \delta u_f \) in the three tests are approximately 0.8 MPa, 1.1 MPa, and 1.1 MPa, respectively. Remember, however, that in Figure 6, the specimen was lightly overconsolidated, while in Figures 7 and 8, they were normally consolidated. Normalized estimates of \( \delta u_f / p'_c \) where \( p'_c \) is the overconsolidation pressure of the sample, are approximately 0.64 to 0.73, and 0.73, respectively. They suggesting that generation of pore water pressures depends more on consolidation pressure and consolidation history than on temperature.

When approaching the critical state condition in Figures 6, and 7, some instability in behavior is observed. This is due to the definition of the volumetric viscoplastic strain in the E-TVp formulation (Kurz et al. 2016), as explained in Section “Outline of the E-TVp model”. The function \( S \) is defined to be positive. However, by reaching the critical state condition, the term \( \partial g / \partial p' \) approaches zero, and numerical instability occurs.

**Triaxial drained shear under constant temperatures**

Ghahremannejad’s study on reconstituted M44 clay included drained triaxial compression tests on three normally consolidated specimens with consolidation pressures of 0.45, 0.4, and 0.3 MPa; and different, but constant, cell temperature of 22°C, 75°C and 100°C. The axial strain rate during shearing was 0.12%/hour, slow enough to prevent build-up of pore water pressures. Table 2 shows consolidation pressures \( p'_c \), specific volumes \( V_c \) at the beginning of shearing, and temperatures during shearing (°C) for the three specimens.

Here RTND denotes a drained, normally consolidated test at room temperature and HTDN denotes drained, normally consolidated tests at high temperature.
Parameters used in the E-TVP modeling are shown in Table 3.

Figures 9, 10, and 11 compare measured and simulated values of deviator stress $q$ and volumetric strain $\varepsilon_v$ with increasing axial strain $\varepsilon_1$. Thermally-induced volume change is related to the clay interparticle forces and pore water viscous behavior, which affects the fabric change by variation in particle resistance (Abuel-Naga 2006). In general, the graphs show reasonably good agreement for measured and simulated relationships between $q$ and $\varepsilon_1$. Although the shapes of the relationships are generally similar, modelled values of $\varepsilon_v$ were consistently higher than measured values.

The effects of heating: thermal consolidation under constant loads

The previous section examined results from specimens that had been loaded to become normally consolidated and were then sheared in both drained and undrained tests at constant pressures. One specimen, T150, (Figure 6), was unloaded to become lightly overconsolidated before shearing. The testing and modeling were related to tests performed at three different constant temperatures. With some exceptions, especially modeling that exaggerated strain-softening in undrained tests, results from the modeling agreed fairly well with experimental data.

This section examines test results in which the loading was kept constant but the temperature was increased. This process is known as ‘thermal consolidation’, (Hueckel et al. 2009).

Data from tests T1442 and T1460 by Tanaka (1995) were used to examine the ability of the E-TVP model to simulate volumetric strains developed in isotropically consolidated specimens kept at constant stress levels, but experiencing increases in temperature from 28°C to 65°C, (Figure 12 and Figure 13). In these tests, the soil temperatures reached 65°C after about 0.6 days and then remained nominally constant. The testing laboratory was not temperature-controlled. As seen elsewhere, opening and closing of access doors caused small irregular temperature changes in the specimens.

Specimens were first consolidated isotropically to 1.0 MPa at 28°C. They were then unloaded to 0.5 MPa to obtain an overconsolidation ratio of 2.0. The temperature of T1442 was then raised to 65°C with the drainage valve open. Specimen T1460 was first loaded at the original 28°C to an isotropic consolidation pressure of 1.5 MPa, once again becoming normally consolidated. Then,
with drainage still open and the loading \( p'_c \) kept constant, its temperature was raised from 28°C to 65°C.

The test data were first simulated using the material properties in Table 4 and the E-TVP model outlined in Figure 2 and Section “Outline of the E-TVP model”.

In Figure 12, the test results showed that volume strains \( \varepsilon_v \) in the lightly overconsolidated T1442 peaked at about the time the target temperature had been reached (0.6 days). Strains then decreased before reaching a new equilibrium about 1.5 days after heating began. Broadly similar laboratory results are seen in Figure 13 for T1460 in terms of timing, but the reduction in \( \varepsilon_v \) from its initial peak value at 0.6 days was much smaller.

The original E-TVP modeling produces patterns of volume strains that are significantly smaller than those seen in the laboratory tests. Figure 12 and Figure 13 show modelled peak volumetric strains by the original model that are respectively about 50% and 32% lower than measured values.

**MODEL MODIFICATION**

Figure 14 and Figure 15 show the effects of heating and cooling in the original E-TVP model, and the need to modify the model to improve its results. As outlined in Sections “Isotropic un-loading and reloading at constant temperatures” and “The effects of heating: thermal consolidation under constant loads”, mechanical unloading-reloading cycles, thermal expansion, and thermal contraction need to be modelled rather differently to take account of the non-reversible nature of thermal consolidation. This is dealt with in following paragraphs and represents the second set of changes in the original E-TVP model.

Delage and Lefebvre (1984) indicated that during compression of clays, macropores collapse under stresses higher than the preconsolidation pressure. This produces irreversible deformations. Smaller intra-aggregate pores then compress, and chemistry and microstructure determine the physiochemical interactions between the clay particles. The combination of mechanical and physiochemical effects, and the competition between them, control volume changes of the soil (Olson and Mesri 1970; Cui et al. 2013). During loading, which involves interactions of mechanical and physiochemical effects, the macropores compress. Elastic and viscoplastic straining can both take
place. During unloading, the effect of mechanical loading decreases. In the competition between
the mechanical and physiochemical effects, the mechanical repulsive forces become dominant,
and swelling can be observed. The physiochemical attractive forces remain, and the associated
physiochemical strains do not recover during unloading.

The modification of the original E-TVP model proposed by the authors is therefore to treat
unloading, at least in a relatively short timeframe, as an elastic process. Reloading the external
mechanical force necessitates both elastic and viscoplastic volume changes.

Therefore, the loading criterion can be added to Equation 9 as:

\[
\dot{\varepsilon}_{ij}^{TVP} = \left( \frac{\phi_T}{V_{m(l)}} \right) \left( 1 - \frac{N - \lambda \ln(p_m') - V_m}{N - Z} \right)^2 \exp \left[ \frac{N - \lambda \ln(p_m') - V_m}{N - Z - 1} \phi_T \right]
\]

\[
\dot{p}' > 0 \quad \text{for} \quad \frac{1}{|\partial g/\partial p'|} \frac{\partial g}{\partial \varepsilon_{ij}^{TVP}} \quad \text{and} \quad f > 0
\]

Hueckel et al. (2009) described how clay soils react to changes in temperature. Volume
changes are associated with thermal expansion of clay particles and water in the soil, whether
as free water in macropores or as adsorbed water between the sheets that form clay minerals.
Increasing temperatures disrupt adsorbed water and strongly affect the viscosity of interparticle
contacts. Because of differences in their chemistry and microstructure, kaolinitic, illitic, and
montmorillonitic specimens react differently to changes in temperature. The complex behaviour of
different clay minerals means that the data base of laboratory results often appears confusing.

In undrained testing, heating increases pore water pressures, which then become time-dependent
(Graham et al. 2001). Behavioral changes associated with heating are usually not fully recoverable
on cooling and therefore represent a thermoplastic component of straining. Hueckel et al. (2009)
reported that when temperatures increase in drained tests, normally consolidated and lightly over-
consolidated clays contract, $\kappa$ increases, the preconsolidation pressure $p'_c$ decreases, and both $\lambda$
and the slope $M$ of the Critical State Line (CSL) remain constant. Heavily overconsolidated clays
often expand. Yield loci, the state boundary surface and the overconsolidation ratio ($OCR$) vary
with thermomechanical loading, stress history, and drainage conditions.

Under field conditions, localized heating is associated with initially higher pore water pressures
and therefore lower mean effective stresses $p'$. This produces decreases in strength, and changes in
the geometry of yield loci. In structural applications such as thermopiles, these changes can mean
that locations where stress levels are currently safe may transition into failure during future heating.

Non-reversibility of thermoplastic loading requires addition of a ‘scale factor’ to the preconsol-
idation pressure, and therefore creep strains, in the original E-TVP model.

The effect of temperature on free and adsorbed water in soil needs to be considered when
determining the size of the plastic potential, as expressed by the preconsolidation pressure. The
effect of the expansion of free water and changes in adsorption forces in adsorbed water is not
considered in the original E-TVP model. In addressing this question, Laloui and Cekerevac (2003,
2008), and Hueckel et al. (2009) modified the apparent preconsolidation pressure $p'_c$ in the Cam-
clay model to reflect temperature variations, Equation 11. Rather than working in terms of absolute
temperature, they developed a scale factor to explain the thermal softening and hardening in terms of
temperature changes $\Delta T = T - T_0$ that can be positive or negative relative to a reference temperature
value $T_0$. After a change $\Delta T$ in the temperature of a specimen, the apparent preconsolidation
pressure $p'_c$ can then be related to the initial apparent preconsolidation pressure $p'_{c0}$ at temperature
$T_0$ as shown in Equation 11, and to the plastic volumetric strain $\varepsilon^p_V$.

$$ p'_c = p'_{c0} \cdot \exp \left( \frac{1 + e_0}{\lambda - \kappa} \varepsilon^p_V \right) \left[ 1 - \gamma \log \left( 1 + \frac{\Delta T}{T_0} \right) \right] $$

(11)

Here, $e_0$ is the initial voids ratio, $\varepsilon^p_V$ is the plastic component of the volume strain, and
$\gamma$, a material parameter for the evolution of preconsolidation pressure with temperature, can
be determined by curve-fitting to experimental data. Equation 12 stems from the last part of
Equation 11, which is considered to be the thermal modification of Equation 3. A positive \( \Delta T \) represents heating. Because of the negative sign in the bracketed term in Equation 11, it reduces the amount of thermal consolidation from the value obtained using the original Equation 3. Cooling, represented by a negative \( \Delta T \), increases the thermal consolidation. With no increment of temperature, the modifying function will be 1.0 and the new function of \( V_m \) in Equation 12 will be the same as the previous definition of \( V_m \) in Equation 3.

The new elastic-thermoviscoplastic specific volume \( V_m \) is defined as:

\[
V_m = N - \lambda \ln\left(p'_m\right) - \frac{\psi_T \ln\left(\frac{t_{0+\psi}}{t_0}\right)}{1 + \frac{\psi_T}{N-Z} \ln\left(\frac{t_{0+\psi}}{t_0}\right)} \left[ 1 - \gamma \log\left(1 + \frac{\Delta T}{T_0}\right) \right]
\]  

(12)

in which the unit of temperature is Kelvin. Curve-fitting of data from the tests by Tanaka (1995) and Crilly (1996) suggest \( \gamma = 0.2 \) for the clay they were working with.

Figures 14 and 15 show a graphical representation of the process described by Equation 12 for the heating and cooling mode, respectively. In these figures, the isotropic mean effective pressure \( p'_m \) for a normally consolidated specimen denotes the size of the elliptical plastic potential.

In the original model (Figure 2), increasing time was associated with viscous compression \( \delta V^{Tvp}_{AB} \) from A to B in Figure 14b for heating and 15b for cooling. With the same time duration, the apparent plastic potential surface moved to the ‘apparent’ isotropic consolidation stress \( p'_nB \) corresponding to point \( S \) on the NCL.

Now, using the modified model in Figure 14, we have to take account of the new higher temperature, the ‘scale factor’ \( \gamma \) in Equation 12, and the coefficient of viscosity \( \psi_T \). With heating, the change in specific volume will be reduced to \( \delta V^{Tvp}_{AB} \) at \( B' \); and the apparent plastic potential will move by a small amount to \( S' \) and \( p'_nB' \). The changes in apparent preconsolidation pressure are associated with ‘aging’ and ‘heating’ of the clay.

For cooling, (Figure 15), there will be expansion in the size of apparent plastic potential locus and apparent preconsolidation from \( p'_nB \) and \( S \) to \( p'_nB' \) and \( S' \).

With these adjustments, the modified E-TVP model in Equation 12 was then used to simulate...
the results measured in T1442 and T1460, (Tanaka 1995). The results are shown as the ‘modified’ values in Figures 12, and 13. Compared with the original E-TVP simulation, the modified E-TVP model shows much-improved agreement between simulated and measured volumetric strains.

It should, perhaps, be remembered that the modeling is being applied to ‘single element’ laboratory specimens in relatively short-term laboratory tests. Different modeling behaviour may be required in large-scale field applications.

**DISCUSSION AND CONCLUSIONS**

This paper has used earlier laboratory data to examine the viability of an elastic-thermoviscoplastic (E-TVP) model based on Critical State principles for soft saturated clays, (Kurz et al. 2016). The model simulates the coupled time-dependent thermo-mechanical behaviour of saturated clays.

Results from E-TVP analysis were compared with data from triaxial tests that included mechanical unloading-reloading cycles; drained and undrained shearing at constant temperatures; and changes in temperature (thermal loading) under constant effective stress. Modifications have been suggested that improve simulations under thermal consolidation and unloading-reloading cycles. Unloading-reloading cycles are often used for evaluating the unload-reload coefficient $\kappa$. Assumptions that formed the basis of the original E-TVP model required that behavior is only elastic, with no plastic component, when the isotropic consolidation pressure $p'_c$ is on or below the viscoplastic limit $p'_m$. The authors have assumed that in principle, mechanical unloading behaviour is purely elastic, while reloading is thermoviscoplastic.

Figures 3, 4, and 5 show that these assumptions and the modified model shown in Equation 12 produce moderate simulations of volume changes at different temperatures in compression space. Figures 6, 7, and 8 compare simulated and measured results from undrained shear tests. The model underestimates peak deviator stresses prediction by about 15% and overestimates peak pore water pressure by nearly 20%. The model shows higher strain-softening and higher pore water pressures when tests are done at higher temperatures. Higher temperatures decrease the viscosity of water in the soil voids and diffuse double layers. Decreasing the viscosity increases creep rates in the soil. Differences in comparing modelled results with measured values may be due to uncertainties
about temperature-dependent factors such as thermal expansion rate of the soil structure, or values of the function $\psi_T$.

In drained triaxial tests, the E-TVP formulation assumes that the rate of straining is slow enough to prevent build-up of pore water pressures. Changes of pore water pressure and deviator strain with temperature have been tracked in Figures 9, 10, and 11. Although the model implies lower volumetric expansion at higher temperatures, it consistently over-estimates the measured volumetric strains. The figures also show decreases in deviator strain at higher temperatures and this agrees with the experimental data.

Data from triaxial tests on illitic clay have shown that modeling results for undrained and drained shear at constant temperatures are close to experimental measurements.

Figures 12 and 13 represent the volumetric straining associated with warming from 28 to 65°C at constant mechanical loading for overconsolidated (OC) and normally consolidated (NC) specimens. The irreversible thermally-induced volume change could be related to the reduction in clay particle resistance to fabric change. Moreover, the variation of volume change depends on stress history, so the increase of OCR value causes less volume change. This can be seen in Figure 12.

DATA AVAILABILITY

All the data that support the findings of this study are available from the corresponding author upon reasonable request.

ACKNOWLEDGMENTS

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LIST OF ABBREVIATIONS

The following abbreviations are used in this paper:

\( NCL = \) Normally Consolidation Line;
\( PPS = \) Plastic Potential Surface;
\( URL = \) Unload-Reload Line; and
\( VPL = \) Viscoplastic Line.
The following symbols are used in this paper:

\( g \) = plastic potential;

\( G' \) = effective shear modulus;

\( M \) = slope of critical state line;

\( N \) = location of isotropic normal compression line in \( V - ln(p') \) plane at \( p' = 1 \text{ kPa} \);

\( p' \) = mean effective stress;

\( p'_m = \xi \) = isotropic mean effective stress (reference size of plastic potential);

\( p'_n \) = mean effective stress on the normal compression line at its intersection with the URL;

\( p'_0 \) = reference size of yield locus;

\( q \) = deviator stress;

\( s_{ij} \) = deviator stress tensor;

\( S \) = scalar multiplier;

\( t \) = time elapsed from the end of primary consolidation;

\( t_0 \) = time corresponding to the end of primary consolidation;

\( T \) = temperature;

\( u \) = pore water pressure;

\( V \) = specific volume;

\( V_m \) = isotropic specific volume;

\( \delta_{ij} \) = Kronecker delta;

\( \gamma \) = viscosity parameter;

\( \varepsilon_p \) = volumetric strain;

\( \varepsilon_q \) = deviator strain;

\( \varepsilon_{ij} \) = strain tensor;

\( \dot{\varepsilon}_{vp} \) = viscoplastic volumetric strain rate;

\( Z \) = location of viscoplastic limit line in \( V - ln(p') \) plane at \( p' = 1 \text{ kPa} \);

\( \kappa \) = slope of unloading-reloading line in \( V - ln(p') \) plane;

\( \lambda \) = slope of normal compression line in \( V - ln(p') \) plane; and

\( \psi_T \) = temperature-dependent slope of secondary compression line in \( V - ln(p') \).


List of Tables

1 Averaged model parameters of reconstituted illite for mechanical unloading-reloading and triaxial undrained shear tests (Crilly 1996) .......................................................... 32
2 Sample characteristics at the beginning of drained shear (Ghahremannejad 2003) .... 33
3 Model parameters of M44 clay for drained shear tests .............................................. 34
4 Model parameters of reconstituted illite for thermal consolidation tests, (Tanaka 1995) ................................................................................................................................... 35
**TABLE 1.** Averaged model parameters of reconstituted illite for mechanical unloading-reloading and triaxial undrained shear tests (Crilly 1996).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$\kappa$</th>
<th>$\lambda$</th>
<th>$N$</th>
<th>$M$</th>
</tr>
</thead>
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<tr>
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<td>0.090</td>
<td>2.097</td>
<td>0.986</td>
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### TABLE 2. Sample characteristics at the beginning of drained shear (Ghahremannejad 2003)

<table>
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<th>Specimen</th>
<th>$p'_c$ (MPa)</th>
<th>$V_c$</th>
<th>Temperature (°C)</th>
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<tr>
<td>RTDN7</td>
<td>0.45</td>
<td>1.8458</td>
<td>22</td>
</tr>
<tr>
<td>HTDN20</td>
<td>0.4</td>
<td>1.8817</td>
<td>75</td>
</tr>
<tr>
<td>HTDN11</td>
<td>0.3</td>
<td>1.8615</td>
<td>100</td>
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**TABLE 3.** Model parameters of M44 clay for drained shear tests

<table>
<thead>
<tr>
<th>Parameters</th>
<th>κ</th>
<th>λ</th>
<th>N</th>
<th>M</th>
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</table>

<table>
<thead>
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<th>Parameters (\kappa)</th>
<th>(\lambda)</th>
<th>(N)</th>
<th>(M)</th>
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<tr>
<td>Value</td>
<td>0.028</td>
<td>0.087</td>
<td>2.113</td>
</tr>
</tbody>
</table>
List of Figures

1 Representation of creep in the E-TVP model for a stress state \((p, q \neq 0)\) at constant mechanical loading and constant temperature in: (a) stress plane, and (b) compression plane ................................................................. 39

2 Representation of the yield surface and plastic potential surfaces with development of viscoplastic volumetric strains at constant isotropic stress and constant temperature in: (a) stress plane, and (b) compression plane .................................................. 40

3 Specific volume versus mean effective stress \(p'\); unload-reload test T1500 at 27°C, ((Crilly 1996)). Line patterns are as follows: laboratory test data - dashed line with square symbols; original E-TVP model - solid line; modified E-TVP modeling - dash-dotted line ............................................................ 41

4 Specific volume versus mean effective stress \(p'\); unload-reload test T1508 at 65°C, ((Crilly 1996)). Line patterns are as follows: laboratory test data - dashed line with square symbols; original E-TVP model - solid line; modified E-TVP modeling - dash-dotted line ............................................................ 42

5 Specific volume versus mean effective stress \(p'\); unload-reload test T1508 at 100°C, ((Crilly 1996)). Line patterns are as follows: laboratory test data - dashed line with square symbols; original E-TVP model - solid line; modified E-TVP modeling - dash-dotted line ............................................................ 43

6 Illustration of triaxial undrained response under constant temperature of 27°C for specimen 1503, ((Crilly 1996)). (a) deviator stress vs. mean effective stress, (b) deviator stress vs. axial strain, and (c) pore water pressure vs. axial strain. Line patterns are as follows: dashed lines with square symbols - laboratory test data; solid lines - results from E-TVP modeling (original and modified models) .................................................. 44
Illustration of triaxial undrained response under constant temperature of 65°C for specimen 1505, ((Crilly 1996)). (a) deviator stress vs. mean effective stress, (b) deviator stress vs. axial strain, and (c) pore water pressure vs. axial strain. Line patterns are as follows: dashed lines with square symbols - laboratory test data; solid lines - results from E-TVP modeling (original and modified models).

Illustration of triaxial undrained response under constant temperature of 100°C for specimen 1508, ((Crilly 1996)). (a) deviator stress vs. mean effective stress, (b) deviator stress vs. axial strain, and (c) pore water pressure vs. axial strain. Line patterns are as follows: dashed lines with square symbols - laboratory test data; solid lines - results from E-TVP modeling (original and modified models).

Response of drained triaxial test under constant temperature of 22°C, ((Ghahremannejad 2003); specimen RTDN7): (a) deviator stress vs. axial strain, (b) volumetric strain vs. axial strain. The dashed lines represent experimental data. Solid lines represent data from both the original and modified modeling.

Response of drained triaxial test under constant temperature of 75°C, ((Ghahremannejad 2003); specimen HTDN20): (a) deviator stress vs. axial strain, (b) volumetric strain vs. axial strain. The dashed lines represent experimental data. Solid lines represent data from both the original and modified modeling.

Response of triaxial drained response under constant temperature of 100°C, ((Ghahremannejad 2003); specimen HTDN11): (a) deviator stress vs. axial strain, (b) volumetric strain vs. axial strain. The dashed lines represent experimental data. Solid lines represent data from both the original and modified modeling.

Volumetric straining plotted against time during and following thermal warming from 28°C to 65°C, ((Tanaka 1995)); Specimen T1442, OCR=2). The dashed line with square symbols indicate the original experimental data. The solid line and dash-dotted line represent results from the original and modified E-TVP models respectively.
Volumetric straining plotted against time during and following thermal warming from 28°C to 65°C, ((Tanaka 1995)); Specimen T1460, $OCR=1)$. The dashed line with square symbols indicate the original experimental data. The solid line and dash-dotted line represent results from the original and modified E-TVP models respectively. ............................................................... 51

Representation of the apparent plastic potential surface for the modified elastic-thermoviscoplastic model in heating mode in: (a) stress plane, and (b) compression plane ............................................................... 52

Representation of the apparent plastic potential surface for the modified elastic-thermoviscoplastic model for cooling mode in: (a) stress plane, and (b) compression plane ............................................................... 53
Fig. 1. Representation of creep in the E-TVP model for a stress state \((p, q \neq 0)\) at constant mechanical loading and constant temperature in: (a) stress plane, and (b) compression plane.
Fig. 2. Representation of the yield surface and plastic potential surfaces with development of viscoplastic volumetric strains at constant isotropic stress and constant temperature in: (a) stress plane, and (b) compression plane.
Fig. 3. Specific volume versus mean effective stress $p'_e$; unload-reload test T1500 at 27°C, Crilly 1996). Line patterns are as follows: laboratory test data - dashed line with square symbols; original E-TVP model - solid line; modified E-TVP modeling - dash-dotted line.

![Graph showing specific volume versus mean effective stress](image-url)
Fig. 4. Specific volume versus mean effective stress $p'$; unload-reload test T1508 at 65°C, (Crilly 1996). Line patterns are as follows: laboratory test data - dashed line with square symbols; original E-TVP model - solid line; modified E-TVP modeling - dash-dotted line.
Fig. 5. Specific volume versus mean effective stress $p'$; unload-reload test T1508 at 100°C, (Crilly 1996). Line patterns are as follows: laboratory test data - dashed line with square symbols; original E-TVP model - solid line; modified E-TVP modeling - dash-dotted line.
Fig. 6. Illustration of triaxial undrained response under constant temperature of 27°C for specimen 1503, (Crilly 1996). (a) deviator stress vs. mean effective stress, (b) deviator stress vs. axial strain, and (c) pore water pressure vs. axial strain. Line patterns are as follows: dashed lines with square symbols - laboratory test data; solid lines - results from E-TVP modeling (original and modified models).
Fig. 7. Illustration of triaxial undrained response under constant temperature of 65°C for specimen 1505, (Crilly 1996). (a) deviator stress vs. mean effective stress, (b) deviator stress vs. axial strain, and (c) pore water pressure vs. axial strain. Line patterns are as follows: dashed lines with square symbols - laboratory test data; solid lines - results from E-TVP modeling (original and modified models).
Fig. 8. Illustration of triaxial undrained response under constant temperature of 100°C for specimen 1508, (Crilly 1996). (a) deviator stress vs. mean effective stress, (b) deviator stress vs. axial strain, and (c) pore water pressure vs. axial strain. Line patterns are as follows: dashed lines with square symbols - laboratory test data; solid lines - results from E-TVP modeling (original and modified models).
Fig. 9. Response of drained triaxial test under constant temperature of 22°C, (Ghahremannejad 2003; specimen RTDN7): (a) deviator stress vs. axial strain, (b) volumetric strain vs. axial strain. The dashed lines represent experimental data. Solid lines represent data from both the original and modified modeling.
Fig. 10. Response of drained triaxial test under constant temperature of 75°C. (Ghahremannejad 2003; specimen HTDN20): (a) deviator stress vs. axial strain, (b) volumetric strain vs. axial strain. The dashed lines represent experimental data. Solid lines represent data from both the original and modified modeling.
**Fig. 11.** Response of triaxial drained response under constant temperature of 100°C, (Ghahremannejad 2003; specimen; HTDN11): (a) deviator stress vs. axial strain, (b) volumetric strain vs. axial strain. The dashed lines represent experimental data. Solid lines represent data from both the original and modified modeling.
Fig. 12. Volumetric straining plotted against time during and following thermal warming from 28°C to 65°C, (Tanaka 1995); Specimen T1442, $OCR=2$). The dashed line with square symbols indicate the original experimental data. The solid line and dash-dotted line represent results from the original and modified E-TVP models respectively.
Fig. 13. Volumetric straining plotted against time during and following thermal warming from 28°C to 65°C, (Tanaka 1995); Specimen T1460, $OCR=1$). The dashed line with square symbols indicate the original experimental data. The solid line and dash-dotted line represent results from the original and modified E-TVP models respectively.
Fig. 14. Representation of the apparent plastic potential surface for the modified elastic-thermoviscoplastic model in heating mode in: (a) stress plane, and (b) compression plane.
Fig. 15. Representation of the apparent plastic potential surface for the modified elastic-thermoviscoplastic model for cooling mode in: (a) stress plane, and (b) compression plane