Breather and its interaction with rogue wave of the coupled modified nonlinear Schrödinger equation

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Breather and its interaction with rogue wave of the coupled modified nonlinear Schrödinger equation

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Abstract
In this paper, we investigate the coupled modified nonlinear Schrödinger equation. Through the traditional Darboux transformation, we construct the first-order breather solution which can exhibit Akhmediev breather and general breather. To obtain the higher-order localized wave solution, \( N \)-fold generalized Darboux transformation is given. Under the condition that the characteristic equation admits a double-root, we present the expression of first-order interactional solution. Then we graphically analyze the dynamics of breather and rogue wave. Due to the simultaneous existence of nonlinear and derivative terms in the equation, there presents different profile in two components for the breathers.

Keywords: Coupled modified nonlinear Schrödinger equation; Darboux transformation; Breather; Rogue wave;

1. Introduction

The nonlinear Schrödinger (NLS) equation, as one of nonlinear partial differential equations, has been studied in such fields as nonlinear optics, plasma physics and condensed matter physics due to its potential mathematical properties and physical applications [1–4]. It has been found that the NLS equation can describe the dynamic of nonlinear localized wave usually associated with soliton, breather and rogue wave. The analysis of dynamic mechanism about localized wave can be done by considering certain kind of exact solution of the NLS equation, and has been done to the scalar, coupled and multi-dimensional NLS equation [5–7]. There are different methods to derive the solution of the NLS equation, such as Darboux transformation, Hirota method [8, 9]. One generalization of NLS equation is the modified NLS equation, and the dynamics of modified NLS equation have been investigated [10–12]. Another extension is from the single-component NLS model to the multi-component one, which opens up new fields for the study of related area [13].

In this paper, we will consider a coupled modified NLS equation [14],

\[
\begin{align*}
    iu_t + u_{xx} + \mu(|u|^2 + |v|^2)u + i\gamma(|u|^2 + |v|^2)u_x &= 0, \\
    iv_t + v_{xx} + \mu(|u|^2 + |v|^2)v + i\gamma(|u|^2 + |v|^2)v_x &= 0,
\end{align*}
\]

where \( u \) and \( v \) are complex functions of \( t \) and \( x \), and designate the slowly varying envelopes for the two polarizations, \( t \) and \( x \) respectively denote the normalized time and distance, \( \mu \) and \( \gamma \) are real constants referred to the nonlinearity and derivative nonlinearity coefficients. Eqs. (1) is complete integrability in view of admitting \( N \)-soliton solution via Darboux transformation, dark and anti-dark vector solitons through Hirota method reported in former literatures [14–16]. Double-pole soliton and first-order rogue wave for Eqs. (1) have been discussed [17, 18].

When \( \mu = 0 \), Eqs. (1) reduce to the coupled derivative NLS equations associated with the polarized Alfvén waves in the plasma physics [19]. Darboux transformation and multi-soliton solution formulae, conservation laws, modulation instability and semirational solutions have been given for the coupled derivative NLS equations in [20, 21]. Eqs. (1) is simplified to the coupled NLS equation (or Manakov model) under the condition \( \gamma = 0 \). A mass of work has been done about this case, such as vector rogue waves, breathers and bright-dark-rouge solution [22–25].

As is well known, since the breather solution plays an important part in the excitation of rogue wave, we focus on generating the breather solution and analyzing the dynamic of nonlinear localized wave for Eqs. (1). To our knowledge, there are few reports on the dynamics of multi-breather for the coupled modified NLS system (1), because of the mixed case between the coupled NLS equation and the derivative NLS equation, that is the aim of

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this paper. With the above consideration, in Sec. 2, we give the multi-breather solutions by the traditional Darboux transformation and discuss the corresponding dynamics, such as the Akhmediev and general breathers. In Sec. 3, we present the expression of \(N\)-fold generalized Darboux transformation for Eqs. (1) and construct the interaction solutions between multi-breather and higher-order rogue waves. The influence of parameters will be considered. Sec. 4 will be the conclusions.

2. Breather for the coupled modified NLS equation

In order to understand the excitation mechanism of the interactional solutions, the multi-breather solutions are firstly suggested to discuss in Eqs. (1) by utilizing the traditional Darboux transformation.

The Lax pair for Eqs. (1) can be given as [15]

\[
\Psi_x = U \Psi = \sum_{k=0}^{2} \zeta^{2-k} U_k \Psi, \tag{2a}
\]

\[
\Psi_t = V \Psi = \sum_{j=0}^{3} \zeta^{j-2} V_j \Psi, \tag{2b}
\]

where \(\Psi = (\psi_1, \psi_2, \psi_3)^T\) is the vector eigenfunction with \(\psi_j\)'s \((j = 1, 2, 3)\) as the corresponding elements, \(\zeta\) is the spectral parameter, \(U, V, U_k, (k = 0, 1, 2)\) and \(V_j, (j = 0, 1, \cdots, 4)\) are \(3 \times 3\) matrices as follows:

\[
U_0 = \frac{i}{6\gamma} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_1 = \frac{1}{2} \begin{pmatrix} 0 & u & v \\ -u^* & 0 & 0 \\ -v^* & 0 & 0 \end{pmatrix},
\]

\[
U_2 = -2\mu U_0, \quad V_0 = \frac{1}{2\gamma} U_0, \quad V_1 = \frac{1}{2\gamma} U_1, \quad V_4 = 4\mu^2 V_0,
\]

\[
V_2 = -4\mu V_0 + \frac{i}{4} \begin{pmatrix} |u|^2 + |v|^2 & 0 & 0 \\ 0 & -|u|^2 & -u^*v \\ 0 & -uv^* & -|v|^2 \end{pmatrix},
\]

\[
V_3 = 2\mu V_1 + \frac{i}{2} \begin{pmatrix} -u^*(|u|^2 + |v|^2) & -u(|u|^2 + |v|^2) & 0 \\ -v^*(|u|^2 + |v|^2) & -|u|^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{\gamma}{2} \begin{pmatrix} 0 & u & v \\ u^* & 0 & 0 \\ v^* & 0 & 0 \end{pmatrix},
\]

the asterisk * represents the complex conjugation. From the compatibility condition of Eqs. (2), i.e., the zero-curvature equation,

\[
U_t - V_x + [U, V] = 0, \tag{3}
\]

one can obtain Eqs. (1), where the square brackets denote the matrix commutator.

Setting the nontrivial initial solution \(u[0] = k_1 e^{2(\xi^2 + \eta_0^2)}\) and \(v[0] = k_2 e^{2(\xi^2 + \eta_0^2)}\) in Eqs. (1), we consider the special solutions of the Lax pair (2) as follows:

\[
\Psi = (\psi_1, \psi_2, \psi_3)^T = MQ\Omega L,
\]

\[
M = diag(e^{\frac{\beta}{2} (\xi_0^2 + \eta_0^2)}, e^{\frac{\beta}{2} (\xi_0^2 + \eta_0^2)}, e^{-\frac{\beta}{2} (\xi_0^2 + \eta_0^2)}), \quad \Omega = diag(\Omega_1, \Omega_2, \Omega_3),
\]

\[
Q = \begin{pmatrix} 0 & \frac{i}{2} (\xi^2 - 4 - \beta) & \frac{i}{2} (\xi^2 - 4 + \beta) \\ -k_2 & k_1 \xi & k_2 \xi \\ k_1 & k_2 \xi & k_2 \xi \end{pmatrix}, \quad L = (l_1, l_2, l_3)^T,
\]

\[
\Omega_1 = e^{\eta_1 + |\eta_0|^2 + \frac{1}{2} (\xi_0^2 - 4 - \gamma_0 |\eta_0|^2)}, \quad \Omega_2 = e^{\eta_2 + |\eta_0|^2 + \frac{1}{2} (\xi_0^2 - 4 - \gamma_0 |\eta_0|^2)},
\]

\[
\Omega_3 = e^{\eta_3 + |\eta_0|^2 + \frac{1}{2} (\xi_0^2 - 4 - \gamma_0 |\eta_0|^2)},
\]

\[
\eta_1 = \frac{i}{6} (\xi^2 - 4), \quad \eta_2 = -\frac{i}{12} \xi^2 + \frac{i}{3} + \frac{i}{4} \beta, \quad \eta_3 = -\frac{i}{12} \xi^2 + \frac{i}{3} - \frac{i}{4} \beta,
\]

\[
\beta = \sqrt{(\xi^2 - 4)^2 + 4\xi^2 (k_1^2 + k_2^2)},
\]

where \(l_1, l_2, l_3, k_1\) and \(k_2\) are arbitrary constants. Next, we will discuss the dynamics of breather to Eqs. (1). Based on the above special solutions of the Lax pair, the one-breather can be given in the following expression by setting
\( \zeta = \zeta_1 \) via the traditional Darboux transformation:

\[
\begin{align*}
    u &= u[0] + 2(\zeta_1^2 - \zeta_1^1) \left( \frac{i \mu \psi_1(\zeta_1)}{\gamma \Theta} |\psi_1(\zeta_1)|^2 \right), \\
    v &= v[0] + 2(\zeta_1^2 - \zeta_1^1) \left( \frac{i \mu \psi_1(\zeta_1)}{\gamma \Theta} |\psi_1(\zeta_1)|^2 \right). \\
\end{align*}
\]

with

\[
\begin{align*}
    \Theta &= |\zeta_1^1| |\psi_1(\zeta_1)|^2 + |\psi_1(\zeta_1)|^2, \\
    \psi_1[\zeta_1] &= l_1^1 i(\zeta_1^2 + \beta_1 - 4) e^{-\frac{i}{2} \left( (\zeta_1^2 + 3) \left( \zeta_1^1 \zeta_1^2 - 6 \zeta_1^1 \right) - 8 \right) + 2x \zeta_1^2 - 24 \zeta_1^2 |t - 12|(|t - 6| + x - 16 |t| + 8x)}, \\
    \psi_2[\zeta_1] &= l_1^2 i(\zeta_1^2 + \beta_1 - 4) e^{-\frac{i}{2} \left( (\zeta_1^2 + 3) \left( \zeta_1^1 \zeta_1^2 - 6 \zeta_1^1 \right) - 8 \right) + 2x \zeta_1^2 - 24 \zeta_1^2 |t - 12|(|t - 6| + x - 16 |t| + 8x)}, \\
    \psi_3[\zeta_1] &= l_1^1 l_2^2 e^{-\frac{i}{2} \left( (\zeta_1^2 + 3) \left( \zeta_1^1 \zeta_1^2 - 6 \zeta_1^1 \right) - 8 \right) + 2x \zeta_1^2 - 24 \zeta_1^2 |t - 12|(|t - 6| + x - 16 |t| + 8x)}, \\
    \beta_1 &= \sqrt{(\zeta_1^2 - 4)^2 + 4 \zeta_1^2 (k_1^2 + k_2^2)}. \\
\end{align*}
\]

When \( l_1 = 0, l_2 = l_3 = 1 \), one can observe the Akhmediev breather, as shown in Figs.1. The solution is periodic in \( x \) direction and localized in \( t \) in both components. While changing the value of the parameters, Figs.2 illustrates the dynamic of general breather, which is periodic in both space and time. Meanwhile, two-breather solution can be derived by setting \( N = 2 \) in the traditional Darboux transformation and choosing different spectral parameter. Interaction between the Akhmediev breather and general breather is depicted in Figs.3, where two breathers keep their profile propagating periodically except the increase of amplitude in the crossing area.

Figs. 1. Akhmediev breather for two components with parameters \( \mu = 2, \gamma = 1, \zeta = 1.3 - 1.3i, l_1 = 0, l_2 = 1, l_3 = 1, k_1 = 1 \) and \( k_2 = -1 \).
Figs. 2. General breather for two components with parameters $\zeta = 1.5 - i$, $l_1 = 1$, $l_2 = 0$, $l_3 = 1$, $k_1 = 1$ and $k_2 = -1$.

Figs. 3. Two breathers for two components with parameters $\zeta_1 = 1 - i$, $\zeta_2 = 2 - i$, $l_1 = 0$, $l_2 = 1$, $l_3 = 1$ and $k_2 = -1$.

3. Interaction of multi-breather and higher-order rogue waves

In the previous section, one and two breathers are discussed based on the traditional Darboux transformation. In the following contents, we will consider the interaction of the breather and rogue wave for Eqs. (1) by the generalized Darboux transformation.

For simplicity, the $N$-fold generalized Darboux transformation of Eqs. (1) will be directly presented in the following form

\[
\Psi[N] = T[N]T[N-1] \cdots T[1]\Psi_1, \quad T[k] = \zeta^2 P_2[k] + \zeta^2 P_1[k] - I, \quad (k = 1, 2, \cdots, N), \quad (4a)
\]

\[
u[n] = u[n-1] + 2(\zeta_1^2 - \zeta_2^2) \left[ \frac{\mu \nu_1[n-1] \nu_2[n-1]}{\gamma \Gamma_1[N]} \frac{\psi_1[n-1] \psi_2[n-1]^*}{\Gamma_1[N]} \right], \quad (4b)
\]

\[
u[n] = v[n-1] + 2(\zeta_1^2 - \zeta_2^2) \left[ \frac{\mu \nu_1[n-1] \nu_3[n-1]}{\gamma \Gamma_1[N]} \frac{\psi_1[n-1] \psi_3[n-1]^*}{\Gamma_1[N]} \right], \quad (4c)
\]
where

\[ \Psi_1[0] = \Psi_1^0 = (\psi_1[0], \psi_2[0], \psi_3[0])^T, \]

\[ \Psi_1[j-1] = \lim_{\epsilon \to 0} \frac{\Psi_1[j-1]|_{\epsilon = \epsilon + \epsilon}}{e^{\epsilon}} = \lim_{\epsilon \to 0} \frac{T[j-1]|_{j-2} \cdots T[1]|_{j-1}}{e^{\epsilon}}, \]

\[ \Psi_1[j-1] = (\psi_1[j-1], \psi_2[j-1], \psi_3[j-1])^T \quad (j = 2, 3, \cdots, N), \]

and the matrices \( P_1[k] \), \( k = 1, 2, \cdots, N \) in the Darboux transformation (4a) are given

\[
P_1[k] = \begin{pmatrix}
0 & \frac{(\zeta_1^2 - \zeta_1^2)\psi_1[k-1]\psi_2[k-1]}{\Gamma_1[k]} & \frac{(\zeta_1^2 - \zeta_1^2)\psi_1[k-1]\psi_3[k-1]}{\Gamma_1[k]} \\
\frac{(\zeta_1^2 - \zeta_1^2)\psi_2[k-1]\psi_1[k-1]}{\Gamma_1[k]} & 0 & \frac{(\zeta_1^2 - \zeta_1^2)\psi_2[k-1]\psi_3[k-1]}{\Gamma_1[k]} \\
\frac{(\zeta_1^2 - \zeta_1^2)\psi_3[k-1]\psi_1[k-1]}{\Gamma_1[k]} & \frac{(\zeta_1^2 - \zeta_1^2)\psi_3[k-1]\psi_2[k-1]}{\Gamma_1[k]} & 0
\end{pmatrix},
\]

\[
P_2[k] = \begin{pmatrix}
\Lambda_1[k] & 0 & 0 \\
0 & \frac{(\zeta_1^2 - \zeta_1^2)\psi_2[k-1]\psi_3[k-1]}{\Gamma_2[k]} & \frac{(\zeta_1^2 - \zeta_1^2)\psi_2[k-1]\psi_3[k-1]}{\Gamma_2[k]} \\
0 & \frac{(\zeta_1^2 - \zeta_1^2)\psi_3[k-1]\psi_2[k-1]}{\Gamma_2[k]} & \Lambda_3[k]
\end{pmatrix},
\]

with

\[
\Lambda_1[k] = \frac{\zeta_1^2|\psi_1[k-1]|^2}{\Gamma_1[k]} + \zeta_1(|\psi_2[k-1]|^2 + |\psi_3[k-1]|^2),
\]

\[
\Lambda_2[k] = \frac{|\zeta_1|^2|\psi_1[k-1]|^2 + |\psi_2[k-1]|^2 + |\psi_3[k-1]|^2}{\Gamma_2[k]},
\]

\[
\Lambda_3[k] = \frac{|\zeta_1|^2|\psi_1[k-1]|^2 + |\psi_2[k-1]|^2 + |\psi_3[k-1]|^2}{\Gamma_2[k]},
\]

\[
\Gamma_1[k] = |\zeta_1|^2 \left[ |\psi_1[k-1]|^2 + \zeta_1^2(5|\psi_2[k-1]|^2 + |\psi_3[k-1]|^2) \right],
\]

\[
\Gamma_2[k] = |\zeta_1|^2 \left[ |\zeta_1|^2|\psi_1[k-1]|^2 + |\psi_2[k-1]|^2 + |\psi_3[k-1]|^2 \right],
\]

Based on the \( N \)-fold generalized Darboux transformation (4) and choosing proper eigenfunction \( \Psi_1 \), one can derive the \( N \)-order localized wave solution through \( N \) times iteration.

Through the nontrivial transformation \( \Psi = M\tilde{\Psi} \), the original Lax pair (2) can be transformed into the following form

\[
\tilde{\Psi}_x = \tilde{U}\tilde{\Psi} \equiv (M^{-1}UM - M^{-1}M_x)\Psi, \quad (5a)
\]

\[
\tilde{\Psi}_t = \tilde{V}\Psi \equiv (M^{-1}VM - M^{-1}M_t)\Psi, \quad (5b)
\]

with

\[
\tilde{U} = \begin{pmatrix}
-\frac{\imath(\zeta_1^2 - 2\mu)}{3y} & \frac{\imath \zeta_1 k_1}{6y} & \frac{\imath \zeta_2 k_2}{6y} \\
-\frac{\imath \zeta_1 k_1}{6y} & 0 & \frac{\imath \zeta_2 k_2}{6y} \\
-\frac{\imath \zeta_2 k_2}{6y} & 0 & \frac{\imath(\zeta_1^2 - 2\mu)}{6y}
\end{pmatrix},
\]

\[
\tilde{V} = \begin{pmatrix}
\Lambda_{11} & \frac{\imath k_1[2\zeta_1^2(k_1^2 + k_2^2) - \zeta_1^2 + 2\mu]}{4y} & \frac{\imath k_2[2\zeta_1^2(k_1^2 + k_2^2) - \zeta_1^2 + 2\mu]}{4y} \\
\frac{\imath k_1[2\zeta_1^2(k_1^2 + k_2^2) - \zeta_1^2 + 2\mu]}{4y} & \Lambda_{22} & -\frac{\imath k_2^2 k_1 k_2}{4} \\
\frac{\imath k_2[2\zeta_1^2(k_1^2 + k_2^2) - \zeta_1^2 + 2\mu]}{4y} & -\frac{\imath k_2^2 k_1 k_2}{4} & \Lambda_{33}
\end{pmatrix},
\]

\[
\Lambda_{11} = \frac{i y^2(3 \zeta_1^2 - 8\mu)(k_1^2 + k_2^2) - 2\zeta_1^2 + 8\zeta_2^2\mu - 8\mu^2}{12y^2},
\]

\[
\Lambda_{22} = \frac{i y^2(3 \zeta_1^2 k_2^2 - 4\mu k_2^2 - 4\mu k_2^2 - \zeta_1^2 + 4\zeta_2^2\mu - 4\mu^2)}{12y^2},
\]

\[
\Lambda_{33} = \frac{i y^2(3 \zeta_1^2 k_2^2 - 4\mu k_2^2 - 4\mu k_2^2 - \zeta_1^2 + 4\zeta_2^2\mu - 4\mu^2)}{12y^2}.
\]
To solve the Lax pair (5), the spectral characteristic equation of \( \tilde{U} \) in the new Lax pair (5) should be

\[
\xi^3 + \frac{1}{12\gamma^2}[3\gamma^2\xi^2(k_1^2 + k_2^2) + (\xi^2 - 2\mu^2)]\xi - i\frac{1}{2160}(\xi^2 - 2\mu)[9\gamma^2\xi^2(k_1^2 + k_2^2) + 2(\xi^2 - 2\mu)^2] = 0, \tag{6}
\]

here, \( \zeta \) is the spectral parameter and \( \xi \) is the unknown number.

As is well known, the rogue waves may be excited when the above mentioned characteristic equation (6) possesses multiple root. In this paper, we consider the interactional solution between rogue waves and breathers under the condition that Eq. (6) owns a double-root. By choosing \( k_1 = -\frac{1}{2}, k_2 = 1, \gamma = 1, \mu = 2 \) and the spectral parameter \( \zeta = \xi_1 = \frac{1}{i\sqrt{1 + i\sqrt{11}}} \), and substituting them into Eq. (6), we can get a double-root \( \xi_1 = \xi_2 = \frac{1}{i\sqrt{1 + i\sqrt{11}}} \) and a single root \( \xi_3 = -\frac{1}{2} \sqrt{55} - \frac{i}{2} \) for Eq. (6).

In order to solve the Lax pair (2), we choose the seed solution of Eqs. (1) as presented in the above section. Setting \( N = 1 \) in the generalized Darboux transformation (4) and considering the above choice of parameters, one can construct the interaction solution between one breather and the first-order rogue wave as follows:

\[
u = \frac{e^{i\gamma t} + 4\sqrt{55}G_1 - 2H_1 + 2i\sqrt{55}(G_1 - 2H_1)x}{F},
\]

\[
u = \frac{e^{i\gamma t} + 4\sqrt{55}H_1 - 2G_1 + 2i\sqrt{55}(H_1 - 2G_1)x}{F},
\]

with

\[
G_1 = -\frac{\sqrt{5}}{102400} \left[ ((15\sqrt{55}it - 4\sqrt{55}ix - 16i + 5t + 20x)\sqrt{4\sqrt{5} + 5} + (5it + 20ix + 16) - 15\sqrt{55}it + 4\sqrt{55}x)\sqrt{4\sqrt{5} - 5} ((15\sqrt{55}it - 4\sqrt{55}ix + 16i - 5t - 20x)\sqrt{4\sqrt{5} + 5} + (5it + 20ix + 16) - 15\sqrt{55}it + 4\sqrt{55}x)\sqrt{4\sqrt{5} - 5}) e^{\frac{it}{2} + 5it + 1it}, \right.
\]

\[
H_1 = \frac{\sqrt{5}}{640} \left[ ((15\sqrt{55}it - 4\sqrt{55}ix - 16i + 5t + 20x)\sqrt{4\sqrt{5} + 5} + (5it + 20ix + 16) - 15\sqrt{55}it + 4\sqrt{55}x)\sqrt{4\sqrt{5} - 5}) e^{\frac{it}{2} + 5it + 1it}, \right.
\]

\[
F = (40it + 150ix + 16 - 400ix + 775i^2 + 80x^2) \sqrt{55} \frac{e^{-\frac{it}{2} + 5it + 1it}}{40},
\]

where \( \alpha \) is the constant. The second-order interaction solution can be derived according to the generalized Darboux transformation (4) by setting \( N = 2 \), but we do not demonstrate them here due to the complicated and lengthy expression, which can be verified with the aid of Mathematica or Maple software.

When choosing the proper value of parameters, one can observe the interaction between one breather and first-order rogue wave in Figs. 4(a), 4(b) and 4(c), 4(d), where the parameter \( \alpha \) is smaller than that in Figs. 4(a) and 4(b). Through comparison among the figures, one can verify that the distance between the breather and rogue wave becomes. Here, we find that the parameter \( \alpha \) will influence the distance between the breather and rogue wave. When increasing the value of \( \alpha \), the breather merges with the rogue wave, while \( \alpha \) decreases, the breather and the rogue wave will separate. To better understand the interaction between the breather and first-order rogue wave, we display the profile of temporal evolution corresponding to Figs. 4(a) and 4(b), as those presented in Figs. 5. At \( t = 0 \), one can observe that the first-order rogue wave coexists with one general breather, as the dashed line shown in Figs. 5. The left envelopes of dashed line represent the general breather in two components, and the right ones of dashed line indicate the first-order rogue wave. At \( t = 0 \), the breather has the higher amplitude than the first-order rogue wave in component \( u \), while the breather has the smaller amplitude than the first-order rogue wave in component \( v \). At \( t = 12 \) in component \( u \) and \( t = 16 \) in component \( v \), one can see the profile of the single breather, as the solid line shown in Figs. 5. Here, due to the different period of breathers in two components, the
diverse choice of time at $t = 12$ and $t = 16$ is for the sake of obtaining complete amplitudes of breathers in two components. Meanwhile, one can notice that the breather has the higher amplitude in component $u$ than that in component $v$, as the left contour of dashed lines displayed in Figs.5, while the rogue wave in component $v$ has the higher amplitude than that in component $u$, which can be seen in the right contour of dashed line in Figs. 5.

Figs. 4. The evolution of the first-order interactional solutions between one general breather and the first-order rogue wave with parameters $\mu = 2$, $\gamma = 1$, $\zeta = \frac{1}{2} \sqrt{11} + \frac{1}{2} i \sqrt{5}$, $k_1 = 1$, $k_2 = -\frac{1}{2}$, $\alpha = 10^{-5}$ in (a) and (b), $\alpha = 10^{-7}$ in (c) and (d).
Figs. 5. The evolution of the first-order interactional solutions between one general breather and the first-order rogue wave at different time with parameters \( \mu = 2, \gamma = 1, \zeta = \frac{1}{2} \sqrt{11} + \frac{1}{2} i \sqrt{5}, \alpha = 10^{-3}, k_1 = 1 \) and \( k_2 = -\frac{1}{2} \).

The second-order rogue wave usually includes two types, for example the fundamental case and the triangular pattern case. For this reason, we will discuss two kinds of the second-order interactional solutions. Firstly, we consider the interaction between two breathers and the fundamental second-order rogue waves, as displayed in Figs. 6. Here, the breathers show different profiles in two components. The amplitudes above the plane wave background of two breathers are bigger than that at the bottom in component \( u \), while the opposite case can be found in component \( v \), where the amplitudes below the plane wave background of two breathers are larger than that at the top. The breather existing in component \( v \) is usually called quasi-breather. To better observing the difference in two components, we also present the profile of two breathers and the fundamental second-order rogue waves at different time, as shown in Figs. 7. At \( t = 0 \), two breathers exist with the second-order rogue wave, and the two left envelopes of solid line correspond to the breathers, and the right one of dashed line denotes the second-order rogue wave. Simultaneously, one can see that the two breathers have the similar amplitude as the rogue wave in component \( u \), as the dashed line shown in Fig. 7(a), while the rogue wave has a very large amplitude in component \( v \), which is the characteristic of rogue wave and associated with the right envelope of dashed line in Fig. 7(b). At \( t = 16 \) in component \( u \) and \( t = 20 \) in component \( v \), only the contours of two breathers can be observed, and the two breathers appear inapparent in component \( v \) as a result of the large amplitude of rogue wave. Here, we choose the different time to display the obvious amplitudes of two breathers in two components due to the different period of breathers.

Figs. 6. The evolution of the second-order interactional solutions between two breathers and the fundamental second-order rogue waves with parameters \( \mu = 2, \gamma = 1, \zeta = \frac{1}{4} \sqrt{117} + \frac{1}{2} i \sqrt{17}, \alpha = 10^{-7}, k_1 = 1 \) and \( k_2 = \frac{1}{2} \).
Changing the parameters, one can obtain the other view of interaction between two breathers and the triangular pattern rogue wave, which is presented in Figs. 8(a) and 8(b). The two breathers in two components still have different profile, namely, breather and quasi-breather, similar with those shown in Figs. 6, but there exists the difference in the rogue wave. Comparing the result with Figs. 6, one can behold that the fundamental second-order rogue wave splits into three first-order ones, which are usually called the triangular pattern rogue wave. Furthermore, one can discover that the amplitude of rogue wave in component $u$ is far less than that in component $v$. To analyze the parameter $\alpha$ how to influence the distance between the breather and rogue wave, we display Figs. 8(c) and 8(d) by changing the value of parameter $\alpha$, which is smaller than that in Figs. 8(a) and 8(b). The similar phenomenon can be found that when $\alpha$ decreases, the two breather and the triangular pattern rogue wave separate, which is consistent with the previous analysis.

Figs. 7. The evolution of the second-order interactional solutions between two breathers and the fundamental second-order rogue waves at different time with parameters $\mu=2, \gamma=1, \xi=\frac{1}{4}\sqrt{47} + \frac{1}{4}i\sqrt{17}, \alpha=10^{-7}, k_1=1$ and $k_2=\frac{1}{4}$. 

Figs. 8. The evolution of the second-order interactional solutions between two breathers and the fundamental second-order rogue waves at different time with parameters $\mu=2, \gamma=1, \xi=\frac{1}{4}\sqrt{47} + \frac{1}{4}i\sqrt{17}, \alpha=10^{-7}, k_1=1$ and $k_2=\frac{1}{4}$. 

(a) (b)
4. Conclusion

In this paper, we investigate the coupled modified NLS system by the Darboux transformation technique. The multi-breather solutions are generated by the traditional Darboux transformation and the dynamics are discussed in detail. In addition, the concrete expression of one breather is given and it is discussed in two types, for example, Akhmediev breather and the general breather. Furthermore, we give the expression of N-fold generalized Darboux transformation and the condition that characteristic equation has a double-root to derive the interactional solution. We exhibit the first-order interactional solution and omit the second-order one due to the length expression here. Under certain parameters, we graphically analyze the dynamics of one general breather and first-order rogue wave, two breathers and second-order rogue wave based on the interactional solutions. For the coupled modified NLS equation, due to the simultaneous existence of nonlinear and derivative terms, there presents different profiles in two components for the breathers and rogue wave, as presented in Figs. 4-8. We hope the result presented can be useful to demonstrate the dynamic of localized wave in the coupled modified NLS equation.

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Data availability

Data sharing not applicable to this article as no data sets were generated or analysed during the current study.

Conflict of interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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