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Dynamic characteristics reconfiguration of fixture-workpiece system for vibration suppression in milling of thin-walled workpieces based on MR damping fixture

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Abstract:
Milling of the low rigidity and large deformation thin-walled workpiece is a critical challenging task due to the obvious machining vibration, which greatly affects the machining accuracy, surface quality of the final part. Compared with the conventional methods, this paper focuses on suppressing the machining vibration of the flexible workpiece by the MR damping fixture. A novel dynamic characteristics reconstruction strategy of MR damping fixture-workpiece system is proposed to improve the system dynamic characteristics of milling system considering the effect of material removal. Then, the fixture-workpiece system dynamic characteristics reconstruction model is established, in process, the time-varying modal parameters in different conditions are iteratively identified and the stable depth of cut is obtained in any moment. Based on this, the control currents of MR damping fixture are calculated to offset the change of the damping and stiffness properties of the milling system caused by material removal. Subsequently, the feasibility and effectiveness of the proposed method are validated by several experiments. Experimental results show that compared with the initial machining state, the dynamic properties of the fixture-workpiece system are reconstructed by the MR damping fixture, and the vibration response is reduced by 30%~70% and the machined surface is improved effectively.

Keywords: Machining vibration suppression; Dynamic characteristics reconstruction; Dynamic stability; Fixture design; Milling

1. Introduction

Milling plays an important role in aerospace industry, which is widely used in milling of the low rigidity and large deformation workpiece such as aeroengine blades, casings, impellers, blisks et al. However, in milling, vibration is a great obstacle in obtaining higher machining accuracy, better surface integrity and less tool wear [1,2]. Therefore, it is necessary to investigate the system dynamic characteristics of the thin-walled workpiece for suppressing vibration in milling.

Over the past few decades, for the efforts of investigating system dynamic characteristic to mitigate the vibration of thin-walled flexible workpiece, there are mainly three kinds of methods: cutting parameters optimization, passive control method and active control method. For the cutting parameters optimization, the analytical prediction
method is used to evaluate the machining state [3], then, the reasonable cutting parameters can be selected. Yang et al. [4] proposed a new dynamic model of tool-workpiece system to investigate the effect of varying workpiece dynamics on the stability in peripheral milling of thin-walled workpieces. Ding et al. [5] proposed a new system stability analysis method which can realize stable machining by variable spindle speed cutting method. Qin et al. [6] developed a novel milling stability prediction method based Chebyshev-wavelet to identify the tool chatter, and the method is compared with the representative existing methods. Lou et al. [7] proposed a Cotes-formula-based stability prediction method and validated by experiments. Dong et al. [8] proposed an updated numerical integration method, which is verified by machining tests. Li et al. [9] presented a stability prediction approach based on the Newton-Cotes rules, which suppressed chatter, and improve machining efficiency and surface quality of the workpiece. Shi et al. [10] derived a new computational model and obtained an updated frequency response function to analyze the machining stability, which was verified by experiments. However, these methods mainly focus on how to control the calculation accuracy and speed, furthermore, it can only be used for cutting parameters selection, which cannot enlarge the stability region and improve the milling efficiency with the material removal. In addition, this method cannot be widely applied due to the limitation of machining quality and NC equipment performance.

For the passive control method, to pursuing machining stability, process stiffness enhancement by clamping method, support rod, mass block, material removal sequence used are investigated by many researchers. Namazi et al. [11] analyzed the effect of contact stiffness and damping performance between tool shank and spindle to the stability. Luo et al. [12] optimized machining allowance distribution and material removal sequence based on stability lobes for improving machining stability. In addition, Mori et al. [13] presented viscoelastic damper support for vibration and chatter suppression, and the machining stability is obviously improved compared with other used methods by experiments. Fei et al. [14] proposed a moving fixture element method to suppress milling deformation, and the effectiveness of the fixture in machining process is verified. Craig et al. [15] presented a novel fixture concept which used the deformable flexure pins to fill the space the workpiece and the locators for improving the stiffness of the workpiece. A support systems using pivot mechanism for suppressing vibrations during thin-wall milling is designed by Matsubara et al. [16]. Then, Wang et al. [17] designed a special fixture using low-shrinkage of low-melting-point alloy to clamp thin-walled parts to guarantee machining accuracy. From above analysis, the existing process stiffness enhancement methods do not have the characteristics of adjustable support force and dynamic characteristics, and it cannot compensate for the spatio-temporal evolution of the system dynamic characteristics, which caused by material removal and different dynamic processing positions.

For the active control method, system dynamic characteristic reconstruction by complex special fixture system [18], piezoelectric intelligent unit [19], electromagnetic actuator [20] are investigated in lots of works. With the development of sensing Technology, signal recognition and intelligent materials, active self-reconfiguration intelligent fixture system has become a research hotspot by foreign scholars at present. Papastathis et al. [21] proposed a model which characterized the dynamic response between active fixture components and thin-walled workpieces under dynamic moving loads. Subsequently, Parus et al. [22] proposed an active control system considering the damp to suppress regenerative chatter in thin-walled workpiece milling. Considering the dynamic characteristics of the fixture itself to suppress machining vibration, Ransom et al. [23] designed a fixture system based on eddy current damper to suppress machining vibration effectively. Greiner-Petter et al. [24] proposed semi-active fluid mechanism based on the variable stiffness damping using magnetorheological valves and springs.
Moradi et al. [25, 26] designed an adjustable vibration absorber for chatter suppression in milling and proposed nonlinear delay differential equation to describe system coupled dynamic model, then, the position, stiffness and damping parameters of the adjustable shock absorber were optimized. Similarly, Ma et al. [27] designed a MR flexible fixture for suppressing the machining vibration in milling of thin-walled plate. Guo et al. [28] proposed a mechanical/magnetorheological composite clamping method for milling process to avoid chatting, which was verified in milling of thin-walled parts. From these, the methods are only applied to simple parts, and the stability of milling system can be improved by regulating the stiffness and damping of the fixture-workpiece system. However, for the thin-walled workpiece, these methods need to design special fixture system, paste mass block and other auxiliary components, and these methods are not adjustable under different machining conditions.

In milling of thin-walled workpiece, due to material removal, cutting stage and other factors, the time-varying dynamic characteristics of fixture-workpiece system change significantly, which causes machining instability, and the effect of the whole fixture properties to suppress machining vibration are investigated comprehensively in limited works. Consequently, to address this problem, a novel MR damping fixture is designed for milling of thin-walled parts, and the fixture have the controllable damping and stiffness characteristics without changing the original fixture layout, which can reconstruct the dynamic characteristics of the thin-walled workpiece-fixture system according to the actual machining state. Then, based on this, an active control method based on MR damping fixture is proposed. The remainder of this paper is organized as follows. Section 2 describes the dynamic characteristic reconstruction scheme of fixture-workpiece system. Fixture-workpiece system dynamic characteristics reconstruction model is proposed, the time-varying modal parameters in different conditions are iteratively identified, then the control currents of MR damping fixture are calculated to offset the change of the damping and stiffness properties of the milling system caused by material removal, which is shown in Section 3. Subsequently, in Section 4, several experiments are conducted to validate the feasibility and efficiency of the proposed method. Finally, some conclusions are summarized.

2. Problem formulation

For the problem of vibration caused by dynamic characteristics evolution of the fixture-workpiece system in milling of thin-walled workpiece, compared with conventional process parameters optimization, uncontrollable process stiffness enhancement for suppressing vibration, the proposed method using controllable MR dampers can reconstruct the dynamic characteristics of the fixture-workpiece system to improve the machining stability, and the schematic diagram is shown in Fig. 1.

![Fig. 1 Schematic diagram of dynamic characteristics reconstructed MR damping fixture-workpiece system.](image)

Based on the theory of machining dynamics, when MR dampers have no effect on the thin-walled workpiece,
the dynamic equation of fixture-workpiece system is described by a \( n \)-dimensional equation:

\[
M\ddot{q}(t) + C\dot{q}(t) + Kq(t) = F_c(t)
\]  

(1)

where \( M, C \) and \( K \) denote the mass, damping and stiffness matrices of the system. \( \ddot{q}(t), \dot{q}(t) \) and \( q(t) \) are the acceleration, velocity and displacement vectors. \( F_c(t) \) represents the normal cutting force vector acting on the different cutting positions.

Considering the geometry of thin-walled workpiece and its clamping constraints, the weak rigidity regions of the thin-walled workpiece are enforced using MR dampers, which can improved the dynamic stiffness and damping. Then, the thin-walled workpiece-fixture system is evolved into a statically indeterminate system. Therefore, the dynamic equation of the workpiece-fixture system after local modification can be rewritten as

\[
M\ddot{q}(t) + C\dot{q}(t) + Kq(t) + P_z\dot{z} = F_c(t)
\]  

(2)

where \( C_f \) and \( K_f \) denote the damping and stiffness matrices of the MR dampers in fixture. \( P_z\dot{z} \) is the hysteresis force produced by the MR dampers in fixture.

With workpiece material removal, the dynamic characteristics of fixture-workpiece system evolve continuously, which causes the stable boundary of the machining system decreases and gradually closes to the unstable machining region. To ensure stable machining, the MR dampers are regulated by the external current to improve the dynamic damping and stiffness characteristics of the fixture-workpiece system. Then, for the reconstructed dynamic characteristics of the fixture-workpiece system with MR dampers, the dynamic equation of motion can be expressed as

\[
(M + \Delta M)(C + \Delta C + C_f)\ddot{q} + (K + \Delta K + K_f)q + P_z\dot{z} = F_c(t)
\]  

(3)

where \( \Delta M \), \( \Delta C \) and \( \Delta K \) are the change of the modal mass, damping, stiffness matrices of workpiece surface caused by material removal.

According to the above analysis, under determined fixture layout and initial cutting parameters, with the materials removal, the dynamic damping and stiffness of the fixture-workpiece system are reconstructed using MR dampers in milling of the thin-walled workpiece, which can improve the dynamic characteristics of machining system and keep the machining system stable for a long time.

3. Fixture-workpiece system dynamic characteristics reconstruction model

3.1 Dynamic characteristics reconstruction strategy

With loss of generality, the dynamic milling force is viewed as external load exerted on the fixture-workpiece system, then, the stable milling boundary is calculated and the stable milling parameters are selected. For obtaining the dynamic characteristics evolution process of MR damping fixture-workpiece system considering material removal, the dynamic milling forces in time domain given by Altintas [29] are used to calculate the stability limit depth of cut and the spindle speed, which can express the damping and stiffness parameters explicitly for reconstructing the dynamic characteristics of the machining system. Therefore, the dynamic milling forces can be expressed as
\[ F_x(t) = N a_p K_p A_0 \Delta x(t) \]
\[ F_y(t) = N a_p K_p A_0 \Delta y(t) \]
\[ F_z(t) = N a_p K_p A_0 \Delta z(t) \]  
(4)

where \( A_0 \) denotes time-varying directional dynamic milling force coefficients, which is determined by Fourier series expansion method. \( a_p \) represents the depth of cut. \( \Delta x(t), \Delta y(t), \Delta z(t) \) are the dynamic displacements of cutter in X, Y and Z directions.

In milling, significant interaction occurs between the cutter and the workpiece. Hence, considering the complexity of frequency response function, the dynamic milling force Eq.(4) in frequency domain gives

\[ \begin{bmatrix} F_x(i\omega_c) \\ F_y(i\omega_c) \\ F_z(i\omega_c) \end{bmatrix} = \frac{N}{4\pi} a_p K_p \left(1 - e^{-i\omega_c T}\right) A_0 G \begin{bmatrix} F_x(i\omega_c) \\ F_y(i\omega_c) \\ F_z(i\omega_c) \end{bmatrix} \]  
(5)

where \( \omega_c \) denotes chatter frequency in milling. \( T \) is the cutter tooth passing period. \( G \) represents the transfer function between tool and workpiece and can be defined as follows:

\[ G = G_e + G_c = \begin{bmatrix} G_{xx}(i\omega_c) & G_{yx}(i\omega_c) & G_{xz}(i\omega_c) \\ G_{yx}(i\omega_c) & G_{yy}(i\omega_c) & G_{yz}(i\omega_c) \\ G_{xz}(i\omega_c) & G_{yz}(i\omega_c) & G_{zz}(i\omega_c) \end{bmatrix} \]  
(6)

For thin-walled workpiece, the cutter is regarded as rigidity compared to workpiece. Considering the workpiece geometry and tool path in milling of thin-walled parts, several experiments shows that the vibration in Z direction is the dominant mode compared with that in X, Y directions, which can be verified by Zhou et al. [30]. Hence, for simplification, the dynamic milling forces in X and Y directions can be ignored. Subsequently, the 3D chatter stability model described here can be simplified as a 1D model in the Z direction. Then, the stability problem of Eq.(5) is reduced to the following:

\[ F_z(i\omega_c) = \frac{N}{2\pi} a_p K_p \alpha_{zz} Re\left[G_z(i\omega_c)\right] F_z(i\omega_c) \]  
(7)

with

\[ \alpha_{zz} = \phi \left[-K_t (\cos 2\kappa + 1) + K_s \sin 2\kappa\right]^{\frac{\alpha_{zz}}{\phi}} \]  
(8)

\[ Re\left[G_z(i\omega_c)\right] = \frac{1-r^2}{\kappa \left(1-r^2\right)^2 + (2\zeta r)^2} \]  
(9)

where \( K_t, K_r, K_s \) represent the cutting force coefficients in tangential, radial, and axial directions, respectively. \( k \) is the normal stiffness of the workpiece. \( \zeta \) denotes the damping ratio. \( r = \omega_c / \omega_n \), \( \omega_n \) is the natural frequency.

Therefore, the stability limit depth of cut and the spindle speed are calculated as follows:

\[ a_{hm} = \frac{2\pi}{NK_t \alpha_{zz} Re\left[G_z(i\omega_c)\right]} \]  
(10)
From Eq.(10) to Eq.(11), the limit depth of cut increase by means of regulating the damping and stiffness parameters of the machining system. Hence, the proposed method is used to reconstruct the dynamic characteristics using the MR dampers. For fixture-workpiece system, under the determined process parameters and fixture layout, with the materials removal, the dynamic characteristics of machining system reduce and the vibration enhances, then, the output dynamic characteristics of MR dampers are regulated by current, which can offset the energy loss with the damping and stiffness properties of MR dampers.

Subsequently, for the fixture-workpiece system, the dynamic characteristic reconstruction method of the system must satisfy the constraint of spindle speed \( n \):

\[
    \begin{cases}
        0 < n \leq n_{\text{max}} \\
        n = \frac{60f}{N \cdot k} \quad (k = 1, 2, 3L)
    \end{cases}
\]

where \( f \) is the tool passing frequency. \( n_{\text{max}} \) denotes the maximum allowable spindle speed of machine tool.

For the axial depth of cut \( a_p \), it needs to meet the following constraints:

\[
    \begin{cases}
        0 < a_p \leq a_{p_{\text{max}}} \\
        0 < a_p \leq a_{p_{\text{lim}}} (n)
    \end{cases}
\]

where \( a_p \) is the initial selected axial cutting depth in the process. \( a_{p_{\text{max}}} \) denotes the maximum depth of cut. \( a_{p_{\text{lim}}}(n) \) represents the stable limit depth of cut, which needs to match with the spindle speed \( n \).

According to the Eq.(12) and (13), with the material removal, to ensure that the optimal cutting parameters keep in a stable cutting area for a long time, the dynamic parameters of the fixture-workpiece system in each cutting state are calculated based on structural dynamic modification method, then, the stability lobes are plotted as shown in Fig. 2. Next, it can be determined whether the optimal cutting parameters are still in the absolute stable region. If it is in the absolute stable region, machining continues. Otherwise, the stability limit depth of cut \( a'_{p_{\text{lim}}} \) in this state is calculated.

Considering the initial stable limit depth of cut \( a_{p_{\text{lim}}} \) before machining, the change of stable limit depth of cut \( \Delta a_{p_{\text{lim}}} \) before and after machining can be calculated. Therefore, the change of stable limit depth of cut \( \Delta a_{p_{\text{lim}}} \) can be determined by

\[
    \begin{cases}
        \Delta a_p = a_{p_{\text{lim}}} - a_p \\
        \Delta a_{p_{\text{lim}}} = a_{p_{\text{lim}}} - a_{p_{\text{lim}}}^i \quad (i = 1, 2, 3L)
    \end{cases}
\]

where \( \Delta a_p \) is the allowance value of initial selected depth of cut \( a_p \) corresponding to initial limit depth of cut \( a_{p_{\text{lim}}} \), \( a_{p_{\text{lim}}}^i \) is the stable limit depth of cut under the \( i \)-th cutting state.

When \( a_{p_{\text{lim}}}^i \) is greater than \( a_p \), the machining process is stable and continue. While \( a_{p_{\text{lim}}}^i \) is less than \( a_p \), the machining process is unstable, and the dynamic characteristic of machining system need to adjust for further machining. Therefore, it can be seen form the Fig. 2 that for obtaining better dynamic characteristic reconstruction
effect of the fixture-workpiece system, the limit depth of cut \(a_{p_{\text{lim}}}^h\) of the reconstructed fixture-workpiece system must meet:

\[
a_{p_{\text{lim}}}^h \geq a_p + \Delta a_p
\]  

(15)

Fig. 2 Dynamic characteristic reconstruction strategy of MR damping fixture-workpiece system.

In short, according to the stability prediction model Eq.(10), Eq.(11) and the Eq.(15), the appropriated control currents \(i\) of MR dampers in the fixture-workpiece system are calculated, which can be used to reconstruct the dynamic characteristics of the fixture-workpiece system by means of the stiffness and damping factor outputted by MR dampers, and enlarge the stable cutting region and improve the machining stability.

3.2 Solution of the time-varying modal parameters and reconstructed currents in milling

From Eq.(10) to Eq.(11), it can be found that the modal parameters such as natural frequency, damping ratio and stiffness are closely related with milling system stability. Due to material removal and MR dampers, the modal parameters of the milling system are time-varying. Therefore, to reconstruct the dynamic characteristics of the fixture-workpiece system, the time-varying modal parameters should be calculated accurately based on structural dynamic modification method, and the modified process is shown in Fig. 3.
Then, introducing an auxiliary equation is:

$$ W(t) + \omega(t) = 0 $$

(16)

Combining Eq. (3) and Eq. (16), and rearranging the resulting expressions in matrix form yields

$$ (A_{W,0} + A_{W,m} + B_{W,m})x_W(t) + (B_{W,0} + A_{W,m} + B_{W,m})x_W(t) = F_W(t) $$

(17)

where

$$ A_{W,0} = \begin{bmatrix} C_{W,0} & M_{W,0} \\ M_{W,0} & 0 \end{bmatrix}, \quad A_{W,m} = \begin{bmatrix} \Delta C_{W,m} & \Delta M_{W,m} \\ \Delta M_{W,m} & 0 \end{bmatrix}, \quad A_{W,m} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{W,0} = \begin{bmatrix} K_{W,0} & 0 \\ 0 & 0 \end{bmatrix} $$

In addition, the subscript ‘W’ denotes workpiece-fixture system, the subscript ‘m’ expresses the m-th workpiece state, the subscript ‘0’ expresses initial state.

To better determine the modal parameters of milling system under time-varying conditions, the equation of milling system is transformed from physical space coordinate system to modal space coordinate system. Thus, by defining vector $x_W(t)$ as

$$ x_W(t) = U_{W,0}x_{W,0}(t) $$

(18)
where \( \mathbf{f}_{W,0}(t) \) represents initial modal displacement vector of workpiece. \( \mathbf{U}_{W,0} \) denotes initial modal shape matrix of workpiece and can be expressed as

\[
\mathbf{U}_{W,0} = \begin{bmatrix}
\psi_0 & \psi_0^* \\
\psi_0A_0 & \psi_0A_0^*
\end{bmatrix}
\] (19)

where \( A_0 \) and \( A_0^* \) are conjugate complex eigenvalues, \( \Psi_0 \) and \( \Psi_0^* \) are conjugate complex mode shape matrix.

Substituting the Eq.(18) into Eq.(17) and premultiplying this equation by \( \mathbf{U}_{W,0}^T \), we obtain

\[
\mathbf{U}_{W,0}^T(\mathbf{A}_{W,0} + \Delta \mathbf{A}_{W,m} + \mathbf{A}_{i,m}) \mathbf{U}_{W,0}^T \mathbf{f}_{W,0}(t) + \mathbf{U}_{W,0}^T(\mathbf{B}_{W,0} + \Delta \mathbf{B}_{W,m} + \mathbf{B}_{i,m}) \mathbf{U}_{W,0}^T \mathbf{f}_{W,0}(t) = \mathbf{U}_{W,0}^TF_{W,0}(t)
\] (20)

In general, in complex modal coordinate system,

\[
\begin{align*}
\mathbf{U}_{W,0}^T \mathbf{A}_{W,0} \mathbf{U}_{W,0} &= \text{diag}(a \ a^*) \\
\mathbf{U}_{W,0}^T \mathbf{B}_{W,0} \mathbf{U}_{W,0} &= \text{diag}(b \ b^*)
\end{align*}
\] (21)

where \( a \) and \( a^* \), \( b \) and \( b^* \) are conjugate complex numbers, respectively.

Therefore, Eq.(21) is reduced to

\[
\mathbf{U}_{W,0}^T \mathbf{A}_{W,0} \mathbf{U}_{W,0} \mathbf{f}_{W,0}(t) + \mathbf{U}_{W,0}^T \mathbf{B}_{W,0} \mathbf{U}_{W,0} \mathbf{f}_{W,0}(t) = \mathbf{U}_{W,0}^T F_{W,0}(t)
\]

(22)

For milling of thin-walled workpiece, the modal parameters such as mass, stiffness and damping in different machining states are difficult to obtain accurately. Therefore, assuming that the mass, stiffness and damping are monotonic decreasing functions in milling, and the change of the parameters is the same value in each cutting step, we obtain

\[
\begin{align*}
\Delta \mathbf{A}_{W,m} &= \frac{(A_n - A_0)m}{l} \\
\Delta \mathbf{B}_{W,m} &= \frac{(B_n - B_0)m}{l}
\end{align*}
\] (23)

where \( l \) is the length of cutting path, \( A_0 \) and \( B_0 \) represent initial system information, \( A_n \) and \( B_n \) denote system information after milling.

Thus, the characteristic equation \( \mathbf{G}_{W,m} \) of Eq.(23) can be given by

\[
\{ \lambda \left[ \text{diag}(a \ a^*) + \mathbf{U}_{W,0}^T \Delta \mathbf{A}_{W,m} \mathbf{U}_{W,0} + \mathbf{U}_{W,0}^T \Delta \mathbf{A}_{i,m} \mathbf{U}_{W,0} \right] + \left[ \text{diag}(b \ b^*) + \mathbf{U}_{W,0}^T \Delta \mathbf{B}_{W,m} \mathbf{U}_{W,0} + \mathbf{U}_{W,0}^T \Delta \mathbf{B}_{i,m} \mathbf{U}_{W,0} \right] \} \mathbf{\Psi} = \mathbf{0}
\] (24)

For \( \mathbf{A}_{i,m} \) and \( \mathbf{B}_{i,m} \) caused by MR dampers, the modified Bouc-Wen model is adopted to describe the dynamic characteristics of the MR dampers, and the damping force can be written as

\[
F_{B-W} = C_i \dot{q} + K_i q + P_i q
\] (25)

where the model parameters \( C_i, K_i, P_i \) are related with control current \( i \) of MR dampers, and based on the measured data of MR damper, the cubic polynomial is used to expressed the element of the model parameters of the damper, and
\[
\begin{align*}
C_{i,m}(i) &= \alpha_1 i^3 + \alpha_2 i^2 + \alpha_3 i + \alpha_4 \\
K_{i,m}(i) &= \beta_1 i^3 + \beta_2 i^2 + \beta_3 i + \beta_4 \\
P_{i,m}(i) &= \gamma_1 i^3 + \gamma_2 i^2 + \gamma_3 i + \gamma_4
\end{align*}
\]

where \( \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \beta_3, \gamma_1, \gamma_2, \gamma_3, \gamma_4 \) are calibrated by the measured force and displacement data of the MR dampers.

Substituting Eq.(25) and Eq.(26) into Eq.(24), the complex eigenvalues and complex eigenvectors of Eq.(24) can be calculated and given by

\[
\Lambda_{W,m}^* = \text{diag}\left(\Lambda \Lambda^*\right) = \text{diag}\left(\Lambda_1^* \Lambda_2^* L \Lambda_n^* \Lambda_1^* \Lambda_2^* L \Lambda_n^*\right)
\]

Subsequently, it is found that from Eq.(24), the relationship of modal displacement at present moment and that at previous moment, which can be given by

\[
\Gamma_{W,0} = \Phi_{W,m} \Gamma_{W,m}
\]

Substituting Eq.(29) into Eq.(18), we obtain the relationship between present physical space coordinates and modal coordinates

\[
x_W(t) = U_{W,m} \Gamma_{W,m}(t)
\]

where \( U_{W,m} \) denotes modal shape matrix at present moment, which can be expressed as

\[
U_{W,m} = U_{W,0} \Phi_{W,m}
\]

Therefore, according to Eq.(24), the natural frequencies, damping ratio and stiffness of the dynamic milling system in the \( m \)-th machining states can be obtained by:

\[
\begin{align*}
\omega_{W,m,n} &= \sqrt{\text{Re}(\lambda_n) + \text{Im}(\lambda_n)} \\
\zeta_{W,m,n} &= \frac{\text{Re}(\lambda_n)}{\omega_{W,m,n}} \\
K_{W,m,n} &= \left[\text{Re}(\lambda_n) + \text{Im}(\lambda_n)\right] \psi_m^T \left[M_{W,0} + \Delta M_{W,m}\right] \psi_m
\end{align*}
\]

Similarly, based on the parameters given in Eq.(32), the modal parameters at any moment in milling of thin-walled workpiece can be calculated.

Next, from section 2, the initial axial limit depth of cut \( a_{p,\text{lim}} \) can be obtained from Eq.(10). Then, when the material removal increases with the machining, and the \( A_{i,m} \) and \( B_{i,m} \) are viewed as zero matrix, according to the Eq.(10), Eq.(16)-Eq.(32), the natural frequencies \( \omega_{W,m} \), damping ratio \( \zeta_{W,m} \) and stiffness \( K_{W,m} \) of the dynamic milling system in the any machining state and the corresponding axial limit depth of cut \( a_{p,\text{lim}} \) can be obtained. To obtain the required current of system dynamic characteristics reconstruction for stable machining, the above calculation process is iterative until the requirements are met, and the calculated process of the system dynamic characteristic reconstruction current is illustrated in Fig. 4.
During the iteration process, considering the dynamic characteristics of the MR dampers and the saturation magnetization of MR fluids, optimal selection range of control current parameters for MR damping fixture is \([0, k_i]\), and the calculation accuracy \(|i_m - i_n| \leq 10^{-4}\), \(m, m+1 \in [0, i_i]\). Dynamic characteristics reconstruction scheme satisfied the stable machining is determined. After continuous calculation, control currents of the MR damping fixture-workpiece system are obtained and shown in Fig. 5. From Fig. 5, with the increase of current, the growth of the limit depth of cut \(\Delta a_{plim}^{i}\) is. When the control current increase to \(i_i\), the maximum limit depth of cut is obtained. While the magnetic field intensity exceeds the maximum value, the performances of the MR dampers reduces and the depth of cut decreases. From here, we see that under the magnetic saturation, the damping and stiffness properties of the fixture-workpiece system can be improved using the MR dampers with the material removal, and the machining is stable for a long time.
4. Experimental verification and discussion

A series of experiments using four MR dampers are conducted to verify the effectiveness of the proposed method in milling of the thin-walled plate. A MR damping fixture is designed to provide the extra damping and stiffness characteristics for the thin-walled plate. Hence, both the impact experiments and the machining experiments are implemented on the three-axis milling center (YHVT8507).

4.1 Dynamic properties of MR damper

For MR damper in the fixture, its dynamic characteristics are measured and the measured devices are shown in Fig. 6(a) and Fig. 6(b). In low frequency vibration, the damping forces are measured under the amplitudes (0.01 mm, 0.05 mm, 0.1 mm, 0.2 mm, 0.3 mm, 0.5 mm), frequency (1 Hz, 2 Hz, 3 Hz, 5 Hz, 6 Hz, 10 Hz) and currents (0 A, 0.25 A, 0.5 A, 1 A, 1.5 A, 2 A) using Instron8871 material testing machine. Then, under frequency 2 Hz, 3 Hz, the relationships of vibration amplitude and damping force under vibration amplitude 0.2 mm are shown in Fig. 6(c) and Fig. 6(d). For high frequency vibration, The device contains steady state exciter (JZK-2), power amplifier (YE1311E), dynamometer (Kistler9255B), acceleration sensor (Dytran3325F1-16845), data acquisition system (DEWEsoft-SIRIUS) et al. Then, the force and displacement signals are obtained under different excited frequencies (500 Hz, 600 Hz, 800 Hz, 900 Hz, 1000 Hz et al.) and excited currents (0 A, 0.5 A, 1 A, 1.5 A, 2 A et al.). Subsequently, the force-displacement hysteresis curve of MR damper in different currents under excited frequency 1000Hz and the force-displacement hysteresis curve of MR damper in different excited frequencies under excited currents 2A are as shown in Fig. 6(e) and Fig. 6(f). Finally, based on Eq.(25) and Eq.(26), the model parameters can be calibrated by genetic algorithm using the measured data, and \(\alpha_1 = 0.3324, \alpha_2 = -0.9768, \alpha_3 = 1.6430, \alpha_4 = 0.5883,\ 
\beta_1 = -0.1029, \beta_2 = 0.0764, \beta_3 = 1.047, \beta_4 = 2.1050, \gamma_1 = 0.1831, \gamma_2 = -0.7090, \gamma_3 = 1.2630, \gamma_4 = 0.0197.\
Fig. 6 Damper performances test devices and dynamic characteristics of MR damper under different conditions. (a) Low frequency vibration performances test devices; (b) High frequency vibration performances test devices; (c) Relationship of vibration amplitude and damping force under vibration amplitude 0.2 mm and frequency 2 Hz, the blue line 0 A current, the red line 0.5 A current, the yellow line 1 A current, the purple line 1.5 A current, the green line 2 A current; (d) Relationship of vibration amplitude and damping force under vibration amplitude 0.2 mm and frequency 2 Hz, the blue line 0 A current, the red line 0.5 A current, the yellow line 1 A current, the purple line 1.5 A current, the green line 3 A current; (e) Force-displacement hysteresis curve of MR damper under excited frequency 1000 Hz and different currents; (f) Force-displacement hysteresis curve of MR damper under excited and excited currents 2 A and different frequencies.
4.2 Impact tests

An impact experiment is carried out in this section for obtaining the modal parameters of the MR damping fixture-workpiece system. In experiments, the dimension of the thin-walled plate made from stainless steel is $300 \times 100$ mm with the thickness of 3 mm, the Young’s modulus of the material is 195 Gpa, the Poisson’s ratio is 0.25, and the density of the material is 7930 kg/m$^3$. The impact test system contains model hammer (Kistler, 500 N), acceleration sensors (Dytran, 3325F1-16847, Ref. sensitivity 10.25 mV/g.), SIRIUS ACC data acquisition instrument (DEWESoft-SIRIUS i series), and computer.

In order to validate the effectiveness of the MR damping fixture for dynamic properties reconstruction, the impact tests and machining setup are shown in Fig. 7(a), it is well known that different impact points on the thin-walled plate can generate different responses. Considering the four MR dampers, the two edges of the thin-walled plate along the long direction are fixed on the fixture, and four MR dampers are located in the fixture, which can provide support force for the thin-walled plate, and the impact measured point 1, 2, 3 are distributed on the thin-walled plate as shown in Fig. 7(b). We found that point 1 and point 3 are symmetric with respect to point 2, while the point 2 is located on the middle of the thin-walled plate. Subsequently, all the vibration responses are obtained and analyzed in different measured points. Experimental results are shown that the vibration responses at point 1 are equal to that at point 3 due to the measured point symmetry on the plate. Then, the frequency response functions on point 1, 2, 3 are obtained and shown in Fig. 8(a). From Fig. 8(a), we found that the natural frequencies on point 1, 2, 3 are equal, but the amplitude on point 2 is smaller than that on point 1, 3, therefore, the representative stable point 2 are selected for measuring responses, which is shown in Fig. 7(c). Then, the frequency response functions (FRFs) before and after MR damping activation under different currents are described in Fig. 8(b), and the identified modal parameters are calculated and listed in Table 1. From Fig. 8(b) and Table 1, we can see the dynamic properties of the workpiece-fixture system can be adjusted by MR dampers, the natural frequency, damping ratio and normal stiffness are improved with the increasing of currents, which is the reason for designing a MR damping fixture in milling of thin-walled workpiece.

Fig. 7 Setups in impact tests/machining and measured point 1, 2, 3 on the thin-walled plate for impact tests.
To investigate the effect of MR dampers on the dynamic characteristics of the fixture-workpiece system, cutting force coefficients should be calibrated. Slot milling was conducted and the machining parameters are spindle speed 2000 r/min, axial depth of cut 0.3 mm, feed rate 120 mm/min, 240 mm/min, 360 mm/min, 480 mm/min and 600 mm/min. The cutter used was a two-flute end milling cutter with a diameter of 12 mm and a helix angle of 30°. Then, based on Altintas [29], the cutting coefficients are calibrated as $K_{tc} = 207.5 \text{ N/mm}^2$, $K_{rc} = 2080 \text{ N/mm}^2$, $K_{ae} = 390.7 \text{ N/mm}^2$, $K_{te} = 63.69 \text{ N/mm}$, $K_{re} = 47.6 \text{ N/mm}$, $K_{ae} = 20.397 \text{ N/mm}$. In addition, based on the test system modal parameters in table 1, when no material removal is, stability lobes in different control currents of the MR damping fixture for the machining system are shown in Fig. 9.

Fig. 8 Frequency response functions (FRFs) under different conditions. (a) Frequency response functions on point 1, 2, 3, the blue line in point 1, the green line in point 2, the yellow line in point 3; (b) Frequency response functions before and after MR damping activation, the green line 0A current, the yellow line 0.5 A current, the purple line 1A current, the green line 2 A current.

From Table 1 and Fig. 9, it is obviously seen that in different control current 0 A, 0.5 A, 1 A, 2 A, the stability lobes with on material removal are markedly different. With the increase of control current of the MR damping
fixture-workpiece system, the natural frequency of the MR damping fixture-workpiece system increases significantly from 168.86 Hz to 269.68 Hz, and the damping factor and the normal stiffness of thin-walled workpiece improve simultaneously. More importantly, the stability lobes move up and right directions in different current 0 A, 0.5 A, 1 A, 2 A, and the stable regions enlarge greatly. Therefore, it can be seen that the damping and the stiffness characteristics of the fixture-workpiece system can be changed by the MR dampers. Therefore, the feasibility of the MR dampers in reconstructing the dynamic characteristics of milling system is validated in modifying the dynamic characteristics of the fixture-workpiece system.

Table 1
Test system modal parameters.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Natural frequency $\omega$ (Hz) (First order)</th>
<th>Damping ratio $\zeta$</th>
<th>Normal stiffness $k$ (N·m$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>168.86</td>
<td>0.025</td>
<td>$7.2848 \times 10^4$</td>
</tr>
<tr>
<td>0.5</td>
<td>197.60</td>
<td>0.032</td>
<td>$7.8793 \times 10^4$</td>
</tr>
<tr>
<td>1</td>
<td>225.78</td>
<td>0.0368</td>
<td>$1.4079 \times 10^5$</td>
</tr>
<tr>
<td>2</td>
<td>269.68</td>
<td>0.0443</td>
<td>$2.7471 \times 10^5$</td>
</tr>
</tbody>
</table>

Fig. 9 Stability lobes for the MR damping fixture-workpiece system with parameters in Table 1. The blue line 0 A current, the purple line 0.5 A current, the green line 1 A current, the red line 2 A current.

4.3 Effects of MR damping on the MR fixture-workpiece system in milling

In order to investigate the effect of the MR dampers on the vibration suppression of the fixture-workpiece system, several machining tests are carried out. According to the stability lobes of the fixture-workpiece system at different currents as shown in Fig. 9, in milling of thin-walled plate, several experiments at spindle speed 1000 r/min, 1500 r/min, 2000 r/min, 3000 r/min, 6000 r/min and axial depth of cut 0.2 mm, 0.5 mm, 0.8 mm, 1 mm are conducted and four sample plates are used for experiments under currents 0 A, 0.5 A, 1 A, 2 A. While the machining
region is shown in Fig. 7(d), and the acceleration sensor is arranged in the middle of the workpiece face relative to machining region. Then, the acceleration response of the system and the surface state at spindle speed 2000 r/min, feed speed 320 mm/min, axial depth of cut 0.5 mm are obtained in different current 0 A, 0.5 A, 1 A, 2 A, which are shown in Fig. 10.

From Fig. 10, when no control current is applied to the MR damping fixture-workpiece system, the vibration acceleration signal fluctuates violently, the maximum amplitude of vibration acceleration is 60 g, and the machined surface is rough and has obvious vibration marks, which shows that chatter occurs. When the control current 0.5 A is exerted on the machining system, the amplitude of the vibration acceleration signal reduces obviously to 40 g, but the fluctuation state is sharp, and the machined surface quality has improved, but chatter also occurs. While the control current 1 A and 2 A are exerted to the machining system, the value of the vibration acceleration is very low, the maximum acceleration value is less than 20 g, and the signal status is stable, the machined surface is smooth without vibration marks. The results can further validate that the stability of the machining system can be effectively improved by reasonably adopting the dynamic characteristics of the fixture-workpiece system by the MR dampers.

4.4 Validation of the proposed method considering the effect of MR dampers

In order to verify the effectiveness of the proposed method, the same material and size of the thin-walled plate as the previous experiments are used. Row material removal form in the middle of the plates is selected as shown in Fig. 11(a). L1 is initial state of workpiece, L2 denotes the first cut, L3 and L4 represent the second and third cut. Based on Siemens NX and experimental modal analysis method, the modal responses of workpiece are obtained and analyzed, the first order natural frequency (173.7 Hz, 162.5 Hz, 151.8 Hz, 131.1 Hz) and corresponding stiffness
(1.191×10^6 N/m, 1.110×10^6 N/m, 0.910×10^6 N/m, 0.679×10^6 N/m) are obtained in different material removal states. Subsequently, under different material removal states, the changes of stability lobes are shown in Fig. 11(b). From Fig. 11(b), it can be seen that the row material removal method removes a large amount of material, the dynamic parameters of the machining system vary greatly, the system stability is obviously affected and the stability limit cutting depth is reduced.

![Fig. 11 Dynamic characteristics Evolution process of machining system under different condition. (a) Material removal form; (b) Evolutionary process of system stability lobes under row material removal form, the blue line is no cut, the green line denotes the first cut, the purple line represents the second cut and the red line is the third cut.](image)

In order to overcome the disadvantage of vibration suppression only relying on cutting parameter optimization, a novel fixture using MR dampers is designed to reconstruct the dynamic characteristics of the machining system to keep stable cutting state for a long time. For the row material removal form, the stability lobes under no material removal, the first cut and the second cut are shown in Fig. 12. Then, several machining experiments at spindle speed 1500 r/min, 1800 r/min, 2000 r/min, 2400 r/min, 2600 r/min, 3200 r/min, 3600 r/min under the same depth of cut 0.6 mm and at the depth of cut 0.2 mm, 0.3 mm, 0.5 mm, 0.6 mm, 1.0 mm, 1.2 mm, 1.5 mm, 2.0 mm under the same spindle speed 5000 r/min are carried out to analyze the machining system dynamic properties. Subsequently, the cutting parameters A (spindle speed 3600 r/min, axial depth of cut 0.6 mm) and B (spindle speed 5000 r/min, axial depth of cut 0.6 mm) are marked in Fig. 12. We can see that A is located in stable region, with the material removal, the stability boundary moves to left, but the cutting parameter in point A is always in the absolute stable region until exceeding the right boundary. Compared with the cutting parameters at point A, the cutting parameters at point B is relatively conservative, when there is no material removal, it is located in the absolute stability region, while after the
first cut, the stability boundary reduces, but the machining system is still stable. However, after the second cut, the point B is located in unstable region. In this case, to ensure machining efficiency and do not change the initial fixture layout, the dynamic properties of the machining system should be improved to guarantee the stable machining process. Therefore, to obtain the stable machining state, MR damping support fixture is designed and the dynamic properties reconstruction algorithm of the fixture-workpiece system is proposed in the paper.

Based on the reconstruction algorithm proposed in section 2 and section 3, from Fig. 12, in point B (spindle speed 5000 r/min, axial depth of cut 0.6 mm), the depth of cut corresponding to the first cut and the second cut are $a_{p1} = 1.5$ mm, $a_{p2} = 0.35$ mm, so the change of the depth of cut is $\Delta a_p = a_{p1} - a_{p2} = 1.15$ mm. According to Eq.(10), Eq.(16)-Eq.(32), the control current $i$ of the MR damping fixture-workpiece system is calculated and $i = 0.7953$ A. Considering the limitation of adjusting the DC regulated power supply, the control current $i = 1$ A is selected approximately. Subsequently, under the control current 1 A, the damping ratio and stiffness value of the MR damping support fixture-workpiece system are calculated, and damping ratio 0.0363, stiffness value $1.635 \times 10^4$ N/m. Then, the stability lobe reconstructed is shown in Fig. 13. It can be seen that the point B is still located in stable region, and the stable region is enlarged by adjusting the properties of the MR dampers, which can keep the machining system stable for a long time.

In order to directly verify the effectiveness of the proposed reconstruction algorithm, the selected cutting parameters (spindle speed 5000 r/min, axial depth of cut 0.6 mm) are chosen, under no control current and 1 A control current, the milling of the thin-walled plate is conducted. Then, the normal forces in Z direction and the machined surface are obtained, and the cutting force signal is transformed by fast Fourier transform. The experimental results are shown in Fig. 14. It can be seen from Fig. 14 that when no control current is exerted to the machining system, the dynamic cutting force fluctuates greatly and the amplitude of it is between ± 200 N. In addition, from the cutting force spectrum, integral multiple of tool passing frequency (167.4 Hz, 500.2 Hz et al.)
appear, and the chatter frequencies (654.7 Hz, 738.4 Hz et al.) are also found, which indicates that chatter occurs during milling, and the spiral marks on the machined surface also prove that chatter occurs in machining. When the control current 1 A is exerted to the machining system, the cutting force signal is stable, the amplitude of cutting force is much lower than that without current control. In cutting force spectrum, only the integral multiple of tool passing frequency (167.4 Hz et al.) appear and the amplitude very small, the machined surface is smooth, which can be shown that the dynamic properties of the fixture-workpiece system are improved by regulating the MR damping fixture and the machining chatter is suppressed effectively.

Fig. 13 Stability lobes before and after dynamic characteristic reconstruction. The green line denotes the first cut, the purple line represents the second cut and the red line is the second cut after system reconstruction.

Fig. 14 Cutting force and its spectrum and machined surface before and after fixture-workpiece system reconstruction. (a) Z-direction cutting force and its spectrum in the second cut; (b) Z-direction cutting force and its spectrum in the second cut after system reconstruction; (c) Machined surface during the second cutting before and after fixture-workpiece system reconstruction.
5. Conclusion

MR damping fixture has the controllable damping and stiffness characteristics, which can reconstruct the dynamic characteristics of the thin-walled workpiece fixture system without changing the original fixture layout. Therefore, to realize the milling system dynamic characteristics reconstruction, an active control method based on MR damping fixture is proposed, besides the feasibility and efficiency of the proposed method are validated by experiments. Then, the critical conclusions were drawn from this work as follows:

(1) Compared with the conventional methods of process parameters optimization, uncontrollable process stiffness enhancement for suppressing machining vibration, the MR dampers of the MR damping fixture can regulate the local damping and stiffness characteristics of thin-walled workpiece under the determined cutting process parameters and fixture layout. Therefore, the dynamic characteristics reconstruction scheme of MR damping support fixture workpiece system is proposed, which are used to improve the system dynamic characteristics of machining system considering the effect of material removal.

(2) Based on the stability prediction model founded, the fixture-workpiece system dynamic characteristics reconstruction model is established, and the time-varying modal parameters in different conditions are iteratively identified and the stable depth of cut is obtained in any moment. Based on this, the control currents of MR damping fixture are calculated to offset the change of the damping and stiffness properties of the milling system caused by material removal.

(3) The feasibility and effectiveness of the proposed method are verified by the modal and machining tests with different control current. Experimental results reveal that the dynamic properties of the fixture-workpiece system are improved by regulating the MR damping support fixture, and after the dynamic characteristics of the fixture-workpiece system are reconstructed, the acceleration values and the dynamic cutting force in milling are reduced obviously, and the vibration is suppressed effectively.

Authors' contributions Junjin Ma: Conceptualization, Methodology, Formal analysis, Investigation, Writing - Original Draft; Yunfei Li: Data curation, Experimental verification, Methodology, Investigation; Dinghua Zhang: Writing - review & editing, Supervision; Bo Zhao: Validation, Writing - review & editing, Supervision; Xiaoyan Pang: Data processing, Visualization, Investigation.

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Code availability The codes that support the findings of this study are available from corresponding author upon reasonable request.

Ethics approval Not applicable.
Consent to participate All authors agree to participate.

Consent for publication All authors agree to publish.

Conflicts of interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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