Performance Analysis of NOMA Systems with Imperfect SIC in Cooperative Cognitive Radio Networks

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Performance Analysis of NOMA Systems with Imperfect SIC in Cooperative Cognitive Radio Networks

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Abstract This paper presents non-orthogonal multiple access (NOMA)-based spectrum sharing scheme for cooperative underlay cognitive radio (CR) wherein the cell-edge primary receiver (CE-PR) can decode the symbols in every time slot by exploiting cooperative multiplexing. In the proposed scheme, the secondary receiver is employed as a cooperative node to achieve a full-duplexing rate at the primary network. Due to this fact and the absence of interference from the primary network, the performance of the secondary network suffers only from self-loop interference (SI). On the other hand, since the secondary transmitter (ST) is not transmitting simultaneously with the primary transmitter, CE-PR can receive the symbol from the direct link without ST interference. In this paper, we derive the closed-form expression for computing the outage probability and the sum rate of the proposed system under imperfect successive interference cancellation (SIC). For comparison purposes, we present the rate and outage performance of the orthogonal multiple access (OMA)-based underlay CR.

Keywords Non-orthogonal multiple access (NOMA) · decode-and-forward (DF) · cooperative multiplexing · spectrum sharing system

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1 Introduction

Non-orthogonal multiple access (NOMA) promises to improve the channel utilization by allowing the base station to serve many users simultaneously on the same spectrum using power domain multiplexing and, hence, satisfy the high demand for massive connectivity requirements in future wireless communications. On the other hand, cooperative cognitive radio (CR) is emerging as a spectrum sharing scheme to improve spectral efficiency. In [5], the authors have investigated the outage performance of the secondary users in NOMA-based half-duplex (HD) cooperative underlay cognitive radio under a partial relay selection scheme. The authors of [10] have analyzed the end-to-end outage and throughput performance of the full-duplex (FD) cooperative underlay cognitive radio by considering the direct link as an interference at the destination. The authors of [6] have investigated the outage behavior and throughput performance of the energy harvesting (EH) HD cooperative CR-NOMA under imperfect successive interference cancellation (SIC). The outage performance of NOMA-based underlay CR networks, with randomly deployed users, was intensively studied in [13].

The authors of [1–4] have investigated the outage performance of the HD cooperative underlay CR-NOMA with multiple secondary users under imperfect channel state information (CSI) conditions. In [8], the authors have proposed the partial relay selection scheme for underlay cognitive networks and analyzed its performance with fixed gain relays. In [14], the authors have analyzed the outage behavior of the NOMA secondary users in the HD cooperative underlay CR network. The NOMA-based coordinated direct and relay transmission (CDRT) protocol for underlay CR was proposed in [15], wherein the authors have investigated the outage performance of both primary and secondary users. The relay selection scheme for the cooperative underlay cognitive radio was proposed in [16]. The authors of [18] have analyzed the outage performance of the energy harvesting cooperative NOMA-based CR networks with simultaneous wireless information and power transfer (SWIPT). In [11], the authors have examined the outage and rate performance of the NOMA-based HD cooperative underlay CR networks.

From the recent studies of NOMA-based full-duplex (FD) cooperative cognitive radio [9, 12, 17, 19], it can be observed that the primary system incurs a loss in capacity and outage performance due to self-loop interference (SI) and multi-user interference. Moreover, the primary user experiences an error-floor in the outage performance due to SI-dependent error propagation. On the other hand, the outage performance of the secondary network in the underlay cognitive radio experiences severe loss due to interference from the primary network. Inspired by these problems, we present a NOMA-based FD cooperative underlay CR protocol to achieve a full-duplexing rate at the cell-edge primary receiver (PR). In the proposed system, the secondary receiver (SR) assists the primary network to achieve full-rate by exploiting cooperative multiplexing. Although FD relaying protocol is employed, the outage performance of the primary network is not degraded due to SI since the relay node is not
transmitting simultaneously with the primary transmitter (PT). Besides, the performance of the secondary network is not influenced by the unknown interference from the primary network since PT is not transmitting while SR is receiving its symbol from the secondary transmitter (ST). Although the secondary network incurs a loss in the outage performance due to SI, it is not varying with PT’s transmit power. The main contribution of the proposed work is to avoid PT interference at the secondary receiver and ensure that the primary receiver can decode symbols from the direct link without ST interference. It is shown in the comparison results that the rate and outage performance of the proposed scheme outperforms the OMA-based underlay CR network.

2 Proposed System

2.1 System Model

Consider a NOMA-based underlay CR system, as shown in Fig. 1, including primary transmitter (P), secondary transmitter (S), and two users (U_P and U_s), in which the secondary user U_s is equipped with two antennas to transmit and receive simultaneously. Besides, we consider U_s as a fixed decode-and-forward (DF) cooperative node which helps primary network to achieve full-duplexing rate via cooperative multiplexing. During the first time slot, the primary transmitter, P, superimposes its two consecutive symbols using superposition coding, such as \( \sqrt{a_1 P_p x_{p,1}} + \sqrt{a_2 P_p x_{p,2}} \), and broadcasts it to U_s (relay) and U_p, where \( x_{p,1} \) and \( x_{p,2} \) represent the modulation symbols corresponding to the primary information, and \( P_p \) denotes the transmit power of P. According to the NOMA protocol, the power allocation coefficients \( a_1 \) and \( a_2 \) satisfy \( a_1 + a_2 = 1 \). Assume that U_s is a strong user (near user) in a NOMA user pair (U_p, U_s) and, hence, it can decode both \( x_{p,1} \) and \( x_{p,2} \) using successive interference cancellation (SIC) technique. On the other hand, U_p can decode only \( x_{p,1} \) from the received superimposed signal. The proposed scheme exploits the knowledge of the symbol \( x_{p,2} \) available at U_s to achieve cooperative multiplexing.

During the second time slot, the secondary transmitter, S, transmits its modulation symbol \( x_s \) to U_s. The transmit power of S is restricted to maintain the interference power at U_p below the tolerable limit \( P_I \). Hence, the transmit power of S is given by \( P_S \leq \min \left\{ P_{\text{max}}^S, \frac{P_I}{|h_s U_p|^2} \right\} \), where \( P_{\text{max}}^S \) is the maximum available power of S and \( h_s U_p \) is the fading coefficient of the wireless link from S to U_p. At the same time slot, U_s also transmits the decoded primary user’s symbol \( \hat{x}_{p,2} \) using the same frequency resource while receiving the symbol \( x_s \). Consequently, in the second time slot, U_p and U_s can decode the symbols, \( x_{p,2} \) and \( x_s \), respectively. It is worth noting that, although FD relaying protocol is employed at U_s, the relay node is not transmitting simultaneously with P. As a result, the SINR received from the wireless link \( P \rightarrow U_s \rightarrow U_p \) is not degraded due to the residual SI. Further, it can be observed that the CE-PR can decode
the symbols $x_{p,1}$ and $x_{p,2}$, respectively, during the first and second time slots. Hence, the primary network can realize full-duplexing rate at CE-PR since it can decode a symbol in every time slot.

2.2 Channel Model

In the subsequent analysis, we consider the frequency-flat slowly varying Rayleigh fading channel. The complex fading coefficient of the wireless link between source $i$ and destination $j$ is represented by $h_{ij} \sim \mathcal{CN}(0, d_{ij}^{-\alpha})$, where $d_{ij}$ is the distance between $i$ and $j$, and $\alpha$ denotes the path-loss exponent. Besides, it is assumed that all the channel gains are exponentially distributed with a mean value of $\Omega_{ij} = \mathbb{E}[|h_{ij}|^2]$. We consider the distance between $P$ and $U_p$ is long, thus arrange the channel gains, such that $|h_{p,U_1}|^2 > |h_{p,U_2}|^2$.

During first time slot, $P$ broadcasts the superimposed signal, and the received signal can be formulated as $y_r = h_{p,r} \left( \sqrt{a_1} P_p x_{p,1} + \sqrt{a_2} P_p x_{p,2} \right) + n_r$ for $r \in \{U_p, U_s\}$, where the average signal power $\mathbb{E}(|x_{p,1}|^2) = \mathbb{E}(|x_{p,2}|^2) = 1$ and $n_r$ is a circularly symmetric additive white Gaussian noise (AWGN) that is distributed as $n_r \sim \mathcal{CN}(0, \sigma_r^2)$.

The received signal-to-interference-plus-noise ratio (SINR) at a receiver $r$ to detect the symbol $x_{p,1}$ can be written as $\Gamma_{x_{p,1}}^{P,r} = \frac{a_1 P_p |h_{p,r}|^2}{\sigma_r^2 |h_{p,r}|^2 + \sigma_{p,r}^2}$, for $r \in \{U_p, U_s\}$. After SIC, the received SINR to detect the symbol $x_{p,2}$, at $U_s$, is given by $\Gamma_{x_{p,2}}^{P,U_s} = \frac{a_2 P_p |h_{p,U_s}|^2}{\sigma_{p,U_s}^2 |h_{p,U_s}|^2 + \sigma_{p,U_s}^2}$, where $\rho_p = \frac{P_p}{\sigma_r^2}$ is the transmit SNR per symbol and $g_{p,U_s} \sim \mathcal{CN}(0, \xi_{p,U_s} \Omega_{p,U_s})$. The parameter $\Omega_{p,U_s}$ denotes the level of residual interference due to SIC imperfection. For instance, $\xi_{p,U_s} = 1$ implies a poor SIC process, and $\zeta_{p,U_s} = 0$ refers to a perfect SIC.

During the second time slot, the signal received at $U_p$ and $U_s$, respectively, is given by $y_{U_p} = \sqrt{P_{U_p} h_{U_p,U_p}} x_{p,2} + \sqrt{P_{U_p} h_{U_p,U_s}} x_s + n_{U_p}$ and $y_{U_s} = \sqrt{P_{U_s} h_{U_p,U_s}} x_s + \sqrt{P_{U_s} h_{SI}} x_{p,2} + n_{U_s}$. Hence, the received SINR at $U_s$ to decode the symbol $x_s$.
can be written as $\Gamma_{x,p} = \frac{\rho_{p} |h_{p,U_s}|^2}{\rho_{s} |h_{s,U_p}|^2 + 1}$, and the received SNR at $U_p$ to decode the symbol $x_{p,2}$ is given by $\Gamma_{x,p,2} = \frac{\rho_{s} |h_{s,U_p}|^2}{\rho_{s} |h_{s,U_p}|^2 + 1}$, where $\rho_{s}$ and $\rho_{U_s}$ represent transmit SNR of $S$ and $U_s$, respectively, $r_{SI} \sim \mathcal{CN}(0, \rho_{U_s}^{-\xi})$ and the parameter $\xi$ represents SI cancellation quality.

### 3 Outage Probability Analysis

In this section, the outage probability of the proposed protocol over the Rayleigh fading channel is derived, and the outage performance of both the primary and secondary system are investigated.

#### 3.1 Outage Probability of the Primary Receiver

##### 3.1.1 Imperfect SIC (i-SIC)

According to the NOMA based system model, $U_p$ can detect the symbol $x_{p,1}$ during the first time slot, which considers the interference from the symbol $x_{p,2}$. From the above description, the outage probability expression for decoding the symbol $x_{p,1}$, at PR, can be written as

$$P_{out}^{x_{p,1}} = \Pr \left( \Gamma_{x,p,1} < \Gamma_{th,p} \right),$$

(1)

and for decoding the symbol $x_{p,2}$ can be written as

$$P_{out}^{x_{p,2}} = \Pr \left( \min \left( \Gamma_{p,2}^{x_{p,1}}, \Gamma_{p,2}^{x_{p,2}}, \Gamma_{s}^{x_{p,2}} \right) < \Gamma_{th,p} \right),$$

(2)

where $\Gamma_{th,p} = 2R_p - 1$ and $R_p$, in terms of bits per channel use (bpcu), is the target data rate of the primary system. Substituting $\Gamma_{x,p,1}^{p,U_p}$ in (1) and using CDF of the random variable (r.v) $|h_{p,U_p}|^2$, $P_{out}^{x_{p,1}}$ can be expressed as

$$P_{out}^{x_{p,1}} = 1 - e^{-\Delta_1 \Gamma_{th,p}},$$

(3)

where $\Delta_1 = \frac{\Gamma_{th,p}}{\rho_p (a_1 - a_2 \Gamma_{th,p})}$. Now, consider the computation of the outage probability, $P_{out}^{x_{p,2}}$, presented in (2).

$$P_{out}^{x_{p,2}} = 1 - \Pr \left( \frac{a_1 \rho_p |h_{p,U_s}|^2}{a_2 \rho_p |h_{p,U_s}|^2 + 1} > \Gamma_{th,p}, \frac{a_2 \rho_p |h_{p,U_s}|^2}{a_1 \rho_p |g_{p,U_s}|^2 + 1} > \Gamma_{th,p}, \frac{\rho_{c} |h_{s,U_p}|^2}{\rho_{s} |h_{s,U_p}|^2 + 1} > \Gamma_{th,p} \right).$$

(4)
After some mathematical manipulation, the outage probability can be expressed as

\[ P_{x,p,2}^{\text{out}} = 1 - \Pr \left( |h_{p,U_s}|^2 > \Delta_1, |h_{p,U_t}|^2 > (a_1 \rho_p |g_{p,U_t}|^2 + 1) \Delta_2, \\
|h_{U_s,U_t}|^2 > \frac{\Gamma_{\text{th},p}(\rho_s|h_{s,U_t}|^2 + 1)}{\rho_s'} \right), \tag{5} \]

where \( \Delta_2 = \frac{\Gamma_{\text{th},p}}{\rho_p' \pi^2} \). Considering the cases \( \rho_s^{\text{max}} \geq \frac{\rho}{|h_{s,U_t}|^2} \), where \( \rho_1 = \frac{\rho}{\pi^2} \), the outage probability can be written as

\[ P_{x,p,2}^{\text{out}} = 1 - \left( \int_0^{\rho_1} \int_0^{\pi} \int_0^{\Delta_1} f(w, x, y, z) \, dw \, dy \, dz \\
+ \int_0^{\rho_1} \int_0^{\pi} \int_0^{\Delta_2} f(w, x, y, z) \, dw \, dy \, dz \right), \tag{6} \]

for \( \Delta_2 \geq \Delta_1 \). Note that, for \( \Delta_2 < \Delta_1 \), (5) can be expressed by considering the cases \( (a_1 \rho_p |g_{p,U_t}|^2 + 1) \Delta_2 \geq \Delta_1 \) and conditioning on \( |g_{p,U_t}| \). Therefore, the outage probability can be written as

\[ P_{x,p,2}^{\text{out}} = 1 - \left( \int_0^{\rho_1} \int_0^{\pi} \int_0^{\Delta_1} f(w, x, y, z) \, dw \, dy \, dz \\
+ \int_0^{\rho_1} \int_0^{\pi} \int_0^{\Delta_2} f(w, x, y, z) \, dw \, dy \, dz \right) \tag{7} \]

for \( \Delta_2 < \Delta_1 \), where \( f(w, x, y, z) = \frac{1}{\rho_{p,U_s\Omega_{U_s,U_t}\Omega_{U_t,U_p}\Omega_{U_s,U_t}}} e^{-\frac{w}{\rho_{p,U_s\Omega_{U_s,U_t}\Omega_{U_t,U_p}\Omega_{U_s,U_t}}} - \frac{x}{\rho_{p,U_t}\pi^2}} \). \( e^{-\frac{w}{\rho_{p,U_s\Omega_{U_s,U_t}\Omega_{U_t,U_p}\Omega_{U_s,U_t}}} - \frac{x}{\rho_{p,U_t}\pi^2}} \) represents the joint PDF of \( |h_{p,U_s}|^2, |g_{p,U_t}|^2, |h_{U_s,U_t}|^2 \) and \( |h_{s,U_t}|^2 \), \( g_1(x) = a_1 \rho_p x + 1, g_1(z) = \Gamma_{\text{th},p}(\rho_s z + 1) \), and \( \tau = \frac{\Delta_1 - \Delta_2}{a_1 \Delta_2 \rho_p} \). After some mathematical simplification, we obtain (6) and (7) as

\[ P_{x,p,2}^{\text{out}} = \frac{1}{1 + a_1 \rho_p |g_{p,U_t}|^2 \Delta_2 \rho_s' \pi^2} \left( 1 - \frac{\rho_s' \Omega_{U_s,U_p}}{\rho_s' \Omega_{U_s,U_p} + \rho_s^{\text{max}} \Omega_{U_s,U_p} \Gamma_{\text{th},p}} \right) \times \left( \frac{\rho_s' \Omega_{U_s,U_p}}{\rho_s' \Omega_{U_s,U_p} + \rho_s^{\text{max}} \Omega_{U_s,U_p} \Gamma_{\text{th},p}} \right), \tag{8} \]
During the second time slot, the user

\[ P_{\text{out}}^{x_{p2}} = 1 - \left\{ \left( e^{-\frac{\alpha}{\rho_{\text{U}}^{\text{U}_{p}}}} - \frac{\rho_{\text{th},p}}{\rho_{\text{U}}^{\text{U}_{p}}} \right) \left( -\frac{\rho_{\text{U}}^{\text{U}_{p}}}{\rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p}} \right) \right\} \left( 1 - e^{-\frac{\alpha}{\rho_{\text{U}}^{\text{U}_{p}}}} \left( \frac{\rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p}}{\rho_{\text{U}}^{\text{U}_{p}} + \rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p} \Gamma_{\text{th},p}} \right) \right) \]

where

\[ \rho_{\text{U}}^{\text{U}_{p}} = \rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p} \]

and

\[ \Gamma_{\text{th},p} = \frac{\rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p} \Gamma_{\text{th},p}}{\rho_{\text{U}}^{\text{U}_{p}} + \rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p} \Gamma_{\text{th},p}} \]

Using (3), (8), and (9), the outage probability of the primary network can be obtained in closed-form.

### 3.1.2 Perfect SIC (\( p\)-SIC)

Substituting \( \Gamma_{\text{th},p} = a_{2} \rho_{p} |h_{p, U_{s}}|^{2} \) into (4), the outage probability under perfect SIC can be computed as

\[ P_{\text{out}}^{x_{p2}} = 1 - \Pr \left( |h_{p, U_{s}}|^{2} > \Delta_{1}, |h_{p, U_{s}}|^{2} > \Delta_{2}, \frac{|h_{p, U_{s}}|^{2}}{\rho_{s}} > \frac{\Gamma_{\text{th},p} \rho_{s} |h_{s, U_{p}}|^{2} + 1}{\rho_{s}} \right) \]

(10)

After simplification, \( P_{\text{out}}^{x_{p2}} \) can be expressed as

\[ P_{\text{out}}^{x_{p2}} = 1 - e^{-\frac{\alpha}{\rho_{\text{U}}^{\text{U}_{p}}}} - \frac{\rho_{\text{th},p}}{\rho_{\text{U}}^{\text{U}_{p}}} \left( 1 - e^{-\frac{\alpha}{\rho_{\text{U}}^{\text{U}_{p}}}} \left( \frac{\rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p}}{\rho_{\text{U}}^{\text{U}_{p}} + \rho_{\text{U}}^{\text{U}_{p}} \rho_{\text{th},p} \Gamma_{\text{th},p}} \right) \right) \]

(11)

where \( \Delta = \max (\Delta_{1}, \Delta_{2}) \). Substituting (11) into (1), we can directly compute the outage probability under perfect SIC.

### 3.2 Outage Probability of the Secondary Receiver

During the second time slot, the user \( U_{s} \) can decode its own symbols in the presence of self-loop interference. Based on this description, the outage probability expression can be formulated as

\[ P_{\text{out}}^{x_{s}} = \Pr \left( \frac{\rho_{s} |h_{s, U_{s}}|^{2}}{\rho_{s} |h_{s}|^{2} + 1} < \Gamma_{\text{th},s} \right) \]

(12)
where \( \rho_s = \min \left( \rho_s^{\max}, \frac{\rho_t}{|h_{s,U'|^2}} \right) \), \( \Gamma_{th,s} = 2^{2R_s} - 1 \) and \( R_s \) is the target data rate of the secondary system. Applying the transmit power constraint of ST in (12), and considering the cases \( \rho_s^{\max} \geq \frac{\rho_t}{|h_{s,U'|^2}} \), \( P_{x_s}^{out} \) can be formulated as

\[
P_{x_s}^{out} = \Pr \left( \rho_s^{\max} < \frac{\rho_t}{|h_{s,U'|^2}} \right)^2 < \left( \frac{\rho_t|h_{s}|^2 + 1}{\rho_s^{\max}} \right) \\
+ \Pr \left( \rho_s^{\max} > \frac{\rho_t}{|h_{s,U'|^2}} \right)^2 < \left( \frac{|h_{s,U'|^2} (\rho_t|h_{s}|^2 + 1)^{\Gamma_{th,s}}}{\rho_t} \right).
\]

(13)

Applying algebraic manipulation to (13) yields

\[
P_{x_s}^{out} = \int_0^{\rho_t/\rho_s^{\max}} \int_0^\infty \int_0^{\min(\left| h_{s,U'} \right|^2, \left| h_{s,U'} \right|^2)} f(u, v, w) \, du \, dv \, dw \\
+ \int_0^\infty \int_0^\infty \int_0^{\min(\left| h_{s,U'} \right|^2, \left| h_{s,U'} \right|^2)} f(u, v, w) \, du \, dv \, dw,
\]

(14)

where \( f(u, v, w) = \frac{1}{|h_{s,U'}| \sigma^4} e^{-\frac{u}{|h_{s,U'}|^2}} \) is the joint PDF of \( |h_{s,U'}|^2 \), \( |h_{s}|^2 \) and \( |h_{s,U'}|^2 \). Using the identity \([7, (3.352.4)]\) and after some mathematical simplification, the outage probability of SR can be obtained as

\[
P_{x_s}^{out} = 1 - \Psi_1 (1 - e^{-\phi_1}) e^{-\phi_2} + \Psi_2 e^{-\phi_1 \phi_4} E_1(\psi_1 \phi_3 - \psi_1 \phi_4),
\]

(15)

where \( E_1(\cdot) \) is the exponential integral function, \( \Psi_1 = \frac{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \), \( \phi_1 = \frac{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \), \( \phi_2 = \frac{\Gamma_{th,s}}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \), \( \phi_3 = \frac{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \), \( \phi_4 = \frac{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \), \( \psi_1 = \frac{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \), and \( \Psi_2 = \frac{\phi_4}{\phi_3} \).

4 Ergodic Rate Analysis

4.1 Ergodic Rate of the Primary System

The ergodic rate of the primary system can be computed from

\[
R_p = \mathbb{E} \left[ \frac{1}{2} \log_2 \left( 1 + W \right) \right] + \mathbb{E} \left[ \frac{1}{2} \log_2 \left( 1 + Z \right) \right],
\]

(16)

where \( W = \min(U, V), \ Z = \min(X, Y), \ U = \Gamma_{x,p,1}^{U_s, U_p}, \ V = \Gamma_{x,p,1}^{U_s, U_p}, \ X = \Gamma_{x,p,2}^{U_s, U_p} \) and \( Y = \Gamma_{x,p,2}^{U_s, U_p} \). By using the relationship \( F_W(w) = 1 - \frac{F_U(w) F_V(w)}{\sigma^2}, \) the CDF of the r.v \( W \) can be expressed as, \( F_W(w) = 1 - e^{-\frac{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \), where \( \sigma = \frac{1}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} \). Similarly, \( F_Z(z) \) can be obtained as \( F_Z(z) = 1 - \frac{p}{\rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}} e^{-n z}, \) where \( p = a_2 p_\rho \rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}, \) \( q = a_1 p_\rho \rho_t^{\max} \rho_t^{\max} \rho_t^{\max} \rho_t^{\max}, \) and \( n = \frac{1}{p} \). Using the relation
\[
\int_0^\infty \log_2(1 + w) f_w(w) \, dw = \frac{1}{\ln 2} \int_0^\infty \frac{1 - F_w(w) - w}{1 + w} \, dw,
\]
the first term in (16) can be expressed as
\[
R_{p,1} = \frac{1}{2\ln 2} \int_0^{\pi a_1^a} \frac{1}{1 + w} e^{-\frac{\pi}{a_1} \frac{w}{\mu^2}} \, dw.
\]
(17)

Substituting \( y = \frac{1}{a_1 - a_2} \) into (17) and after some mathematical simplification, we obtain
\[
R_{p,1} = \frac{1}{2\ln 2} e^{\frac{\pi}{a_2^a}} \int_4^{\infty} \frac{1}{y(y - 1)} e^{-\mu y} \, dy,
\]
(18)

where \( \mu = \frac{\pi a_1}{a_2^a} \). With the aid of [7, 3.352.2] and after some algebraic manipulation, we obtain the following equation:
\[
R_{p,1} = \frac{1}{2\ln 2} e^{\frac{\pi}{a_2^a}} \left( \text{Ei}\left( -\frac{\mu}{a_1} \right) - e^{-\mu} \text{Ei}\left( -\frac{\mu}{a_1} + \mu \right) \right)
\]
(19)

Using CDF of \( Z \), the second term in (16) can be written as
\[
\bar{R}_{p,2} = \frac{1}{2\ln 2} \int_0^\infty \frac{p}{1 + z} \frac{e^{-nz}}{(p + qz)} \, dz.
\]
(20)

After some algebraic manipulation, (20) can be written as
\[
\bar{R}_{p,2} = \frac{1}{2\ln 2} \left( \int_0^\infty \frac{p}{1 + z} e^{-nz} \, dz + \int_0^\infty \frac{Q}{p + qz} e^{-nz} \, dz \right),
\]
(21)

where \( P = \frac{p}{(p-q)} \) and \( Q = \frac{pq}{(q-p)} \). With the help of identity [7, 3.352.4], we can write the ergodic rate expression as
\[
R_{p,2} = \frac{P}{2\ln 2} \left( -e^n \text{Ei}(-n) + e^{\frac{np}{q}} \text{Ei}\left( -\frac{np}{q} \right) \right).
\]
(22)

In the high SNR region, the asymptotic expression of (19) and (22) can be obtained by using \( \text{Ei}(-x) \approx E_e + \ln x \) and \( e^x \approx 1 + x \) when \( x \) is sufficiently small, where \( E_e \) is an Euler’s constant. Substituting (19) and (22) into (16), we can compute the ergodic rate of the primary system. Moreover, the ergodic rate under perfect SIC conditions can be directly obtained by substituting \( \zeta_{p,U} = 0 \) in (20), which can be expressed as
\[
\bar{R}_{p,2} = -\frac{1}{2\ln 2} e^n \text{Ei}(-n).
\]
(23)

Substituting (23) into (16), we can compute the ergodic rate under perfect SIC.
4.2 Ergodic Rate of the Secondary System

The ergodic rate of the secondary system can be computed from

\[ \bar{R}_s = \mathbb{E} \left[ \frac{1}{2} \log_2 (1 + X) \right] \]

where the r.v \( X = \Gamma_s U_s \). The CDF of the r.v \( X \) must be derived to compute the ergodic capacity. We first express the CDF of \( X \) as

\[ F_X(x) = \Pr \left( \frac{\rho_s |h_s U_s|^2}{\rho_s |h_{SI}|^2 + 1} < x \right) \]

After some mathematical simplification, we obtain

\[ F_X(x) = \frac{\rho_s \Omega_{s,U_s}}{\rho_s \Omega_{s,U_s} + \rho_s \Omega_{SI}} e^{-\frac{\rho_s |h_{s,U_s}|^2}{\rho_s |h_{SI}|^2 + 1}} \]

Substituting (26) into

\[ \frac{1}{2 \ln 2} \int_0^{\infty} \frac{1-F_X(x)}{1+x} \, dx \]

and with the aid of some mathematical simplification, we obtain the following equation:

\[ \bar{R}_s = \frac{A}{2 \ln 2} \left( -e^k \text{Ei}(-k) + e^{\frac{k}{d}} \text{Ei} \left( \frac{-k}{d} \right) \right), \]

where \( A = \frac{c}{1-c}, \quad c = \rho_s \Omega_{s,U_s}, \quad d = \rho_s \Omega_{SI}, \) and \( k = \frac{1}{c} \).

5 Numerical Results and Discussion

This section presents analytical and simulation results of outage probability with SIC imperfection. The positions of \( S, P, U_p, \) and \( U_s \) in 2D network topology is assumed as \((0, 0), (0, 150), (120, 150), \) and \((70, 100)\) respectively, where the coordinates are expressed in meters and the reference distance is considered as \( d_0 = 100 \) m. Accordingly, the normalized distances are calculated as \( d_{p,U_p} = 1.2, d_{p,U_s} = 0.86, d_{U_s,U_p} = 0.7, d_{s,U_p} = 1.92 \) and \( d_{s,U_s} = 1.11 \). It is assumed that the power allocation coefficients \( a_2 = 0.1 \) and \( a_1 = 0.9, \) and the path-loss exponent \( \alpha = 3. \) The computer simulation results are presented to validate the analytical results.

Fig. 2 depicts the outage probability of the primary network under p-SIC and i-SIC conditions, where the impact of SIC imperfection and the interference threshold limit is investigated. We can observe that, as anticipated, the outage probability of the primary network decreases with SIC imperfection. On the other hand, the outage performance improves when the interference threshold limit decreases. Besides, it can be noted that the outage behavior of the primary network experiences the error floor due to the SIC imperfection. Fig. 3 illustrates the outage behavior of the primary network of the proposed system and the OMA-based underlay CR. The proposed scheme requires two and three time-slots for FD-NOMA and HD-NOMA, and OMA-based underlay CR requires five time-slots to complete end-to-end transmission. However,
Fig. 2 Outage probability versus transmit SNR under imperfect SIC for the target data rate of $R_p = 0.5$ bpcu.

Fig. 3 Outage probability versus transmit SNR under imperfect SIC for the target data rate of $R_p = 0.5$ bpcu.

the mathematical framework for outage probability and ergodic sum rate of OMA-based underlay CR system is omitted. We can observe from Fig. 3 that the FD-NOMA scheme outperforms the other protocols for low and medium SNR. However, its performance is inferior in the high SNR region since SIC imperfection and ST interference cause the error-floor at high SNR.

Fig. 4 investigates the impact of SI cancellation quality on the outage performance of the secondary receiver. It can be observed that the outage
performance of the proposed system with FD-NOMA protocol is comparable to HD-NOMA in the low SNR regime. However, it is worth noting that the outage probability experiences an error floor in the high SNR region due to the constraint on ST transmit power to satisfy the interference threshold at PR. On the other hand, since the interference threshold is not a dominant factor in the low to medium SNR region, the outage performance of the proposed system outperforms the OMA-based underlay CR system.

Fig. 5 illustrates the maximum achievable sum rate of the proposed system under i-SIC and p-SIC conditions. It can be observed that the sum rate of the proposed system with FD-NOMA protocol outperforms its counterpart.
HD-NOMA protocol and the OMA-based CR system for the low to medium SNR. However, FD-NOMA incurs a loss in the high SNR region since the ST interference on the relay link is dominant. Besides, the asymptotic result of the sum rate is closely matched to the analytical results in the high SNR region.

6 Conclusion

In this paper, a NOMA-based cooperative CR protocol is proposed where CE-PR can receive symbols in each time slot, and hence it can realize a full-duplexing data rate by exploiting cooperative multiplexing. It can be observed from the results that, since the FD relay is not transmitting during the time slot of PT, the outage performance of FD-NOMA is superior to other protocols for low to medium SNR. Further, since SR is employed as a cooperative relay, the secondary transmitter can access all the alternate time slots and, in turn, it can improve its spectrum accessing opportunity. It can be concluded that, although the proposed system achieves the full-duplexing rate, as shown in the results, it can achieve a better outage performance compared to the OMA-based cooperative underlay CR scheme.

References


Statements & Declarations

The authors declare that the manuscript entitled "Performance Analysis of NOMA Systems with Imperfect SIC in Cooperative Cognitive Radio Networks" is original, has not been full or partly published before, and is not currently being considered for publication elsewhere.

Competing Interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Author Contributions

Chitra M (First Author): Investigation, Methodology, Resources, Software, Supervision, Validation, Visualization, Writing – review & editing.

Senthilkumar Dhanasekaran (Corresponding Author): Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Validation, Visualization, Writing - original draft, Writing - review & editing.
Data Availability

The authors declare that the manuscript has not used any data sets for the analysis.
Supplementary Files

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