Calculation of tooth thickness errors and its influence on meshing backlash of harmonic drive

Congbin Yang
Beijing University of Technology

Honglie Ma
Beijing University of Technology

Tao Zhang (✉️ zhangtao@bjut.edu.cn)
Beijing University of Technology

Jigui Zheng
Beijing Institute of Precise Mechatronics and Controls

Zhifeng Liu
Beijing University of Technology

Qiang Cheng
Beijing University of Technology

Research Article

Keywords: tooth thickness deviation, dimensions of over pins, backlash, radial deformation

Posted Date: March 29th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1468023/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License
Calculation of tooth thickness errors and its influence on meshing backlash of harmonic drive

Congbin Yang\textsuperscript{1,2}, Honglie Ma\textsuperscript{1,2}, Tao Zhang\textsuperscript{1,2,8}, Jigui Zheng\textsuperscript{4}, Zhifeng Liu\textsuperscript{1,3,9}, Qiang Cheng\textsuperscript{1,3}

1 Institute of Advanced Manufacturing and Intelligent Technology, Beijing University of Technology, Beijing 100124, Peoples R China.
2 Beijing Key Laboratory of Advanced Manufacturing Technology, Beijing University of Technology, Beijing 100124, Peoples R China.
3 Machinery Industry Key Laboratory of Heavy Machine Tool Digital Design and Testing Technology, Beijing University of Technology, Beijing 100124, Peoples R China.
4 Beijing Institute of Precise Mechatronics and Controls, Beijing 100076, Peoples R China.
# Corresponding Author / Email: zhangtao@bjut.edu.cn.

Abstract: Meshing backlash mainly determines the transmission accuracy of harmonic drive, which is an important index in design. In this paper, a design method of short tooth involute profile is proposed, which relies on positive modification to enhance the strength of dedendum. By intercepting part of tooth profile, the meshing interference of addendum can be effectively avoided. Based on the dimensions of over pins of circular spline and flexspline, a calculation method of tooth thickness deviation is proposed, and the positive and negative of tooth thickness deviation are defined. Involute functions need not be calculated by this method, which simplifies the solution process. By discretizing the teeth profile points of circular spline and flexspline, a calculation method of time-varying backlash is proposed, the variation of backlash in the process of meshing in and out is analyzed, and the influence law of tooth thickness deviation on backlash is studied. On this basis, an algorithm of adjusting the radial deformation to compensate the backlash is proposed. The results show that the excessive backlash in the meshing area can be reduced by increasing the radial deformation. When meshing interference occurs, the interference free meshing can be realized by decreasing the radial deformation.

Keywords: tooth thickness deviation, dimensions of over pins, backlash, radial deformation

1. Introduction

As a high-precision gear transmission mechanism, harmonic drive (HD) is usually composed of flexspline (FS), circular spline (CS) and wave generator (WG). Because of its large transmission ratio, strong bearing capacity and high precision, HD is widely used in aerospace, industrial robots and military equipment [1-4]. However, CS and FS are special small module gears, which are difficult to manufacture and prone to produce tooth thickness deviation. If the products with deviation are discarded, it will increase the cost and waste the materials. Therefore, in the actual machining process, how to compensate the tooth thickness deviation is of great significance for enterprises to save resources.

Recent years, in order to improve the performance of HD, many scholars have focused on the research of tooth profile and meshing performance analysis. Dong et al. [5,6] put forward the kinematic model of planar HD and revealed the relative motion law of CS and FS. Chen et al. [7] studied the application of double-circular-arc
common-tangent tooth profile in HD, and studied the influence law of tooth profile parameters on conjugate interval and conjugate tooth profile. Yang et al. [8] established three different tooth profiles (involute, double circular arc and double cycloid) in the same coordinate system, and analyzed the meshing performance between different tooth profiles. In order to accurately describe the neutral layer curve, Yu et al. [9] established a new FS deformation mathematical model, and proposed a new tooth profile design method based on the model. Song [10] et al. proposed a double circular arc tooth profile design method of HD based on curve mapping and bidirectional conjugate, which provides a new design method for the tooth profile design of HD. Dennis et al. [11] used orthogonal test to analyze the influence of tooth profile change on HD performance. Yang et al. [12] studied the non-uniformity of load distribution on the surface of FS teeth, and the selection of appropriate materials is helpful to reduce partial axial load. Yao et al. [13] approximated the FS tooth into a rack model, and studied the influence of gear tooth geometric parameters on the flange bending stiffness coefficient and stress concentration factor. The finite element simulation model is established by Hrcek et al [14], and the global sensitivity analysis of various design parameters affecting motion loss is carried out. Gao et al. [15] proposed a FS machining error measurement system, which provided a method for hobbing quality control. Ma et al. [16-18] proposed many new experimental devices and methods, which were used to measure and analyze the meshing characteristics of CS and FS at different speeds. Zhang et al. [19,20] analyzed the tribological characteristics of FS of different materials under dry friction and grease lubrication, which can provide a reference for selecting appropriate FS materials. Hu et al. [21] considered the influence of the uncertainty of dynamic parameters on the transmission accuracy, and proposed an interval algorithm to represent the value of dynamic parameters. The effectiveness of the algorithm is verified by experiments.

WG is an important factor affecting transmission performance, which has also been concerned by many researchers. The FS relies on the rotation of the WG to realize the transmission of force and torque. The WG is the direct source of radial deformation. Li et al. [22] proposed a contour optimization method of WG, which can reduce the maximum stress of FS after the assembly of WG. Yague et al. [23] established four WG models with different shapes, and compared and analyzed the influence on the stress of the FS after assembly through finite element simulation. Gravagno et al. [24] discussed the influence of the shape of the WG on the pure motion error of HD, and quantitatively evaluated it. Considering the influence of gear teeth on the position of equivalent neutral layer, Yao et al. [25] proposed a piecewise deformation calculation method of FS with cam WG. Jia et al. [26] used a special experimental device to verify and explain the pure motion error, and proposed a new structure of WG which can improve the transmission performance.

The preceding researches provide a lot of useful references for tooth profile optimization and WG shape improvement. Although the bearing capacity of double circular arc tooth profile is stronger, its tool is lack of universality. More different types of products can be processed by involute tooth cutter through modification. Considering the high cost of cutting tools and long production cycle, involute tooth profile (ITP) has
more advantages in this aspect. When manufacturing errors occur in CS and FS, how to make reasonable compensation is of great significance to further save costs and resources.

This paper mainly aims to study how to compensate the backlash by adjusting the radial deformation of the WG. The design process and method of a short tooth involute are introduced in Section 2. A calculation method of tooth thickness deviation based on the dimension of over pins is proposed in Section 3. The method for solving the backlash of ITP using discrete tooth profile points is proposed in Section 4. The backlash compensation scheme under different tooth thickness deviation is discussed in Section 5.

2. Design of involute short tooth profile

2.1 Modified involute profile model

ITP is formed by pure rolling of a straight line around the base circle. Positive addendum modification will lead to higher load capacity of each tooth [27]. As shown in Fig 1, the blue curve is used as the gear curve after modification, and the black curve is used the standard tooth profile curve. $o_i$ is the center point of the FS. $m$ is modulus. $h_{ai}$ is the addendum coefficient, $c_{ri}$ is the bottom clearance coefficient, $\alpha$ is the pressure angle of reference circle, $z_i$ is the number of FS teeth, $x_i$ is the modification coefficient.

![Fig. 1 Modified involute profile](image)

$r_b$, $r_a$ and $r_c$ are the radius of the base circle, the addendum circle and the dedendum circle respectively.
\[
\begin{align*}
    r_a &= mz_i \cos \alpha / 2 \\
    r_b &= \left( mz_i + 2h^* m + 2x_m \right) / 2 \\
    r_c &= \left( mz_i - 2h^* m - 2c^* m + 2x_m \right) / 2 \\
\end{align*}
\] (1)

Involute equation after modification:
\[
\begin{align*}
    x_i &= r_i \sin \varphi \\
    y_i &= r_i \cos \varphi \\
    \varphi &= \pi / 2z_i - \left( \text{inv} \alpha_i - \text{inv} \alpha \right) \\
\end{align*}
\] (2)

Where, \( r_i \) is the radius of the arbitrary circle on the involute, \( r_i = r_i / \cos \alpha_i \).
\( \alpha_i \) is the pressure angle of arbitrary point on the involute, \( \alpha_c \leq \alpha_i \leq \alpha_a \),
\( \alpha_a = \arccos(r_r / r_i) \), \( \alpha_c = \arccos(r_r / r_c) \).
\( \text{inv} \alpha_i \) is the involute spread angle on the circle with radius \( r_i \), \( \text{inv} \alpha_i = \tan \alpha_i - \alpha_i \),
\( \text{inv} \alpha \) is the involute spread angle on the reference circle, \( \text{inv} \alpha = \tan \alpha - \alpha \).

After the gear is modified, the tooth thickness at the reference circle increases. In order to make the symmetry axis of the modified gear y-axis, the tooth profile is rotate, and the rotation transformation matrix is
\[
\begin{bmatrix}
    x_2 \\
    y_2 \\
    1
\end{bmatrix} =
\begin{bmatrix}
    \cos \beta & \sin \beta & 0 \\
    -\sin \beta & \cos \beta & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    y_1 \\
    1
\end{bmatrix}
\] (4)

Where, \( \theta = x_i m \tan \theta / r \)

### 2.2 Solution of conjugate tooth profile

Fig. 2(a) shows the structural parameters of the FS, \( \delta_i \) is the wall thickness of the FS, \( D \) is the inner wall diameter of the FS, and the neutral layer is located in the middle of the wall thickness. The specific parameters shown in Fig. 2(b) are defined according to the literature[10].

![Fig. 2 Relationship of meshing coordinate. (a) Structure of FS. (b) The motion frame of the meshing process of CS and FS.](image)
The FS coordinate system is established based on the neutral layer, so the coordinate system needs to be translated along the center line, and the translation amount is \( r_m \). The translation matrix is represented as follows:

\[
\begin{bmatrix}
x_f \\
y_f \\
1
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & -r_m \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_2 \\
y_2 \\
1
\end{bmatrix}
\] (5)

Where, \( r_m = \frac{D + \delta}{2} \)

According to references [7], the profile equation of CS can be expressed as:

\[
\begin{align*}
x_c &= x_f \cos(\beta) + y_f \sin(\beta) + \rho \sin(\Delta \varphi) \\
y_c &= -x_f \sin(\beta) + y_f \cos(\beta) + \rho \cos(\Delta \varphi)
\end{align*}
\] (6)

The basic conjugate equation of HD based on envelope conjugate theory is [8]:

\[
\frac{\partial x_g}{\partial x} \frac{\partial y_g}{\partial \varphi_H} - \frac{\partial x_g}{\partial \varphi_H} \frac{\partial y_g}{\partial x} = 0
\] (7)

Where, \( x \) is the FS abscissa variable and \( \varphi_H \) is the rotation angle of the WG. The conjugate tooth profile can be obtained by solving the Eq. 7.

2.3 Correction of CS and FS tooth profiles

For ITP, it is more susceptible to interference. In order to avoid interference and improve the bearing capacity, the CS and FS tooth profiles are treated accordingly. As shown in the Fig. 3, \( h^* \) is defined as the addendum removal coefficient. Part of the addendum of the FS is cut off, compared with the direct use of short tooth design, the dedendum thickness of the modified gear is greater and the load capacity is stronger.

Because the FS intercepts part, the short teeth can be used for CS without reserving too much bottom clearance. The fitting process of CS tooth profile is shown in the Fig. 4. Firstly, the completed profile parameters of FS are substituted into the conjugate equation for solution, the result is a series of discrete points. It is difficult to directly fit discrete points into involute equation. The modulus and pressure angle of the CS are the same as those of the FS. Therefore, by changing the modification coefficient of the
CS and comparing it with the conjugate tooth profile point set, the tooth profile parameters of the CS can be determined when the errors meet the requirements. It should be noted here that the FS tooth profile substituted into the envelope equation is complete without removing the addendum, so the CS tooth profile obtained by solving is also a long tooth. The directly established equation tooth profile of CS is short tooth.

![Diagram](image)

**Fig. 4 Fitting process of CS tooth profile**

3. Calculation model of thickness error of CS and FS

The tooth thickness error of CS and FS is an important factor affecting the transmission accuracy. In engineering practice, the method of measuring over pins (M value) is generally used to gauge the tooth thickness deviation. However, the conventional calculation process for the over pins distance is not convenient. This is mainly due to the computation complexity of involute function included within the calculation[28]. The tooth number of CS and FS is usually even, as shown in Fig. 5 and the calculation of M value is

\[ M = D_s \pm d_p \]  \hspace{1cm} (8)

Where, "+" and "-" represent the M value of CS and FS respectively, 

$D_s$ is twice the distance from the measuring pin center to the gear center, 

$d_p$ is diameter of pin used for measurement.
3.1 Calculation of thickness error of FS

The FS is an external meshing gear. As shown in Fig. 5, the measuring pin and the tooth space of the FS are tangent to point $q_i$. According to the principle of ITP forming, $p_i q_i$ is the normal of the gear and tangent to the base circle, therefore $o_i$, $q_i$, and $p_i$ are collinear. During measurement, the two pins with the same diameter selected by modulus are placed in the two opposite tooth space, and the M dimension of the outermost end (maximum value) is measured by micrometer. The coordinates of $o_i$ is $(0, (M_1 - d_p)/2)$. According to the geometric relationship in the Fig. 6(a), the coordinates of point $q_i$ can be expressed as follows:

\[
\begin{align*}
\frac{M_1 - d_p}{2\sqrt{k^2 + 1}} &= r_i \\
x_{q_i} &= \frac{-d_p k^{-1}}{2 \sqrt{k^2 + 1}}, k > 0 \\
y_{q_i} &= \frac{M_1 - d_p}{2} - \frac{d_p k}{2 \sqrt{k^2 + 1}}
\end{align*}
\]

(9)

$E_{sup}$ and $E_{sub}$ are the upper and lower deviation of tooth thickness respectively. As shown in Fig. 6(b), the FS tooth profile is discretized into multiple points. $(x_i, y_i)$ and $(x_{i+1}, y_{i+1})$ are continuous two points on the FS tooth profile, $R_{q_i}$ and $R_{q_{i+1}}$ are the distances from these two points to the center of the circle respectively. $R_{q_i}$ is the distance from $q_i$ to the center of the circle. When $R_{i} < R_{q_{i}}$ and $R_{q_{i}} < R_{q_{i+1}}$ are satisfied, $i$ and $i+1$ points on the tooth profile can be determined. Point $t_i$ is the intersection of a circle with radius $R_{q_i}$ and the tooth profile. When there are enough discrete points of the tooth profile, point $t_i$ can be obtained by solution of the following equations:

\[
\begin{align*}
\frac{y_{i+1} - y_i}{x_{i+1} - x_i} &= \frac{x - x_i}{y - y_i} \\
y &= \sqrt{R_{q_i}^2 - x^2}
\end{align*}
\]

(10)

Where, the first formula represents the linear equation of $i$ and $i+1$, the second
formula represents the circular equation with radius $R_q$.

The tooth thickness deviation $\Delta E_m$ is

$$\Delta E_m = \pm \sqrt{(x_{q_1} - x_{i_1})^2 + (y_{q_1} - t_{i_1})^2}$$

(11)

Where, $x_{q_1} > x_{i_1}$, $\Delta E_m$ is positive, otherwise it is negative.

![Fig. 6 Calculation of thickness deviation of FS. (a) Position relationship between FS and pin. (b) Definition of positive and negative deviation.](image)

### 3.2 Calculation of thickness error of CS

The CS is an internal meshing gear. As shown in Fig. 7(a), the pin and the tooth space of the CS are tangent to point $q_2$. Similarly, three points $o_2$, $q_2$ and $p_2$ are collinear. The coordinates of $o_2$ is $\left(0, \frac{M_2 + d_p}{2}\right)$. The coordinates of point $q_2$ can be expressed as follows:

$$\begin{align*}
M_2 + d_p \left(2k^2 + 1\right) = r_s \\
x_{q_2} &= \frac{d_p - 1}{2 \sqrt{k^2 + 1}}, k < 0 \\
y_{q_2} &= \frac{M_2 + d_p - k}{2} + \frac{d_p - k}{2 \sqrt{k^2 + 1}}
\end{align*}$$

(12)

As shown in Fig. 7(b), similarly, the CS tooth profile is discretized into multiple points. $(x_j, y_j)$ and $(x_{j+1}, y_{j+1})$ are continuous two points on the CS tooth profile, $R_j$ and $R_{j+1}$ are the distances from these two points to the center of the circle respectively. $R_{q_2}$ is the distance from $q_2$ to the center of the circle. When $R_j < R_q$ and $R_q < R_{q+1}$ are satisfied, $j$ and $j+1$ points on the tooth profile can be determined. Point $t_2$ can be obtained by solution of the following equations:
In this case, the tooth thickness deviation $\Delta E_m$ is

$$\Delta E_m = \pm \sqrt{(x_{q2} - x_{i2})^2 + (y_{q2} - t_{i2})^2}$$

(14)

Where, $x_{q2} > x_{i2}$, $\Delta E_m$ is positive, otherwise it is negative.

4. Calculation model of backlash

The backlash will be changed by the tooth thickness error, and then the performance of HD will be affected. The impact between teeth will be caused when the backlash is too large, which will affect the stability of gear transmission.

The instantaneous backlash of a point on the tooth profile is usually represented by circumferential backlash. According to the transmission principle, when the CS is fixed, the WG rotates counterclockwise, and the FS rotates clockwise. The left tooth profile of the FS is meshed with the CS. In order to simplify the calculation process, the FS tooth profile is discretized into $i$ points, and the CS tooth profile is discretized into $j$ points. The CS is represented in the form of tooth space (please see Fig. 8(b)).
Fig. 9 Calculation of meshing backlash

For a point on the FS tooth profile, through the coordinate transformation of Eq. 6, the point $N$ is represented in the CS coordinate system, an arc is drawn with the distance from the point to the center of the circle as the radius, the intersection point of arc and working tooth profile of CS is $G$ (as shown in Fig 9). $(x_m, y_m)$ and $(x_{m+1}, y_{m+1})$ are two consecutive points on the CS tooth profile. $L_m$ and $L_{m+1}$ are the distances from these two points to the center of the circle respectively. When $L_m < R$ and $R < L_{m+1}$ are satisfied, points $m$ and $m+1$ can be determined.

The coordinates of point $G$ can be approximately solved by the following formula:

$$
\begin{align*}
  y &= \frac{y_{m+1} - y_m}{x_{m+1} - x_m} (x - x_m) + y_m \\
  y &= \sqrt{R^2 - x^2}
\end{align*}
$$

The backlash can be expressed as:

$$
\delta_{GN} = \pm \sqrt{(x_G - x_N)^2 + (y_G - y_N)^2}
$$

Where, $x_N > x_G$, $\delta_{GN}$ is positive. $x_N < x_G$, $\delta_{GN}$ is negative, which represents the meshing interference amount.

The backlash is meaningful only when the $G$-point exists. The rotation angle of the WG is discretized into $k$ parts. With the rotation of the WG, the backlash matrix $C_{ij}$ of the point in a meshing period can be obtained. When changing the points on the FS tooth profile, different backlash matrices can be obtained. The total backlash matrix is

$$
C_{all} = \begin{bmatrix}
  C_{11} & C_{21} & L & C_{1k} \\
  C_{22} & C_{32} & L & C_{2k} \\
  M & M & O & M \\
  C_{i2} & C_{i3} & L & C_{ik}
\end{bmatrix}
$$

The minimum value of each column is the time-varying meshing backlash,
5. Result and discussion

5.1 Numerical results

In order to analyze the influence of tooth thickness error on backlash, the parameters of a certain type of harmonic reducer are adopted. The detailed design parameters are shown in Table 1. The modulus and pressure angle of CS and FS are the same.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
<th>Symbol</th>
<th>Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>Modulus</td>
<td>0.5mm</td>
<td>α</td>
<td>Pressure angle</td>
<td>20°</td>
</tr>
<tr>
<td>z₁</td>
<td>Tooth number of FS</td>
<td>100</td>
<td>z₂</td>
<td>Tooth number of CS</td>
<td>102</td>
</tr>
<tr>
<td>w₀</td>
<td>Radial deformation</td>
<td>0.5mm</td>
<td>δ₁</td>
<td>Wall thickness of the FS</td>
<td>0.36mm</td>
</tr>
<tr>
<td>D</td>
<td>Inner diameter of FS</td>
<td>50mm</td>
<td>dₛ</td>
<td>Diameter of pin</td>
<td>0.866mm</td>
</tr>
<tr>
<td>hₛ₁⁺</td>
<td>Addendum coefficient of FS</td>
<td>1</td>
<td>hₛ₂⁺</td>
<td>Addendum coefficient of CS</td>
<td>1</td>
</tr>
<tr>
<td>cₛ₁⁺</td>
<td>Coefficient of bottom</td>
<td>0.25</td>
<td>cₛ₂⁺</td>
<td>Coefficient of bottom</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>clearance of FS</td>
<td></td>
<td></td>
<td>clearance of CS</td>
<td></td>
</tr>
<tr>
<td>x₁</td>
<td>Modification coefficient</td>
<td>2.3647</td>
<td>x₂</td>
<td>Modification coefficient</td>
<td>2.3765</td>
</tr>
<tr>
<td></td>
<td>of FS</td>
<td></td>
<td></td>
<td>of CS</td>
<td></td>
</tr>
<tr>
<td>hₜ⁺</td>
<td>Addendum removal coefficient of FS</td>
<td>0.8</td>
<td>hₜ⁺</td>
<td>Addendum removal coefficient of CS</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The CS and FS tooth profiles without tooth thickness error are shown in the Fig. 10. In order to better show the meshing, the CS is represented in the form of tooth space.

The motion simulation analysis, which is mainly used to judge the relative position of FS teeth and CS teeth in the assembly state. When the WG is rotating through the interval $[- \pi/2, \pi/2]$, the single tooth envelope process is shown in Fig. 11, there is a good envelope process for FS. Moreover, it can be seen from Fig. 12 that the backlash is composed of three segments: engage-in segment, engaged segment and engage-out segment. It can be clearly seen that the backlash in the engage-in and engage-out segments are very large, and the minimum backlash appears in the engaged segment. Conjugate contact will not occur during engage-in and engage-out segments. Therefore, reducing the backlash of the engaged section is the key to ensure the accuracy.

$$C = \min\{C_{all}\}$$ (18)
Fig. 11 Envelope process of theoretical tooth profile

The multiple-tooth conjugated state of the CS and FS at a certain time is shown in the Fig. 13. It can be clearly seen that the backlash of the FS in the engage-in and engage-out regions are very large, and the minimum backlash appears in the engaged region. However, the angle range of the engaged region is limited, meaning that the number of simultaneously meshing teeth is smaller, therefore the bearing capacity of the ITP is not as good as that of the double arc tooth profile.

Fig. 12 Backlash distribution of different WG angles
5.2 Discussion on error compensation scheme of different tooth thickness

When the tooth making of CS and FS is completed, its structure cannot be changed. In order to better improve the utilization rate of defective products, taking the backlash as the evaluation index, a method of adjusting the radial deformation of cam is proposed. In the actual production process, the cam production process is relatively simple.

The adjustment process of radial deformation is shown in Fig. 14. After inputting the basic parameters, the $M$ value is converted into the tooth thickness deviation, a new tooth profile equation is established, and the meshing backlash is calculated. Then, the radial deformation is continuously adjusted until the minimum value of backlash is below 1μm. The iteration variation is set to ± 0.001 mm.

![Adjustment algorithm of radial deformation](image)

5.2.1 Adjustment scheme for excessive backlash

The minimum backlash of HD used in high-precision transmission is often close
to zero. Excessive backlash includes the following three conditions: The first is that the tooth thickness of CS increases and FS decreases, the reduction of FS is greater. The second is that the tooth thickness of FS increases and CS decreases, the reduction of CS is greater. The third is that the tooth thickness of CS and FS decrease at the same time.

The $M_2$ value of the CS is 51.904mm, and the tooth thickness increases 0.005mm. The $M_1$ values of FS are 53.196mm, 53.154mm and 53.071mm respectively, and the corresponding tooth thicknesses decrease by 0.010mm, 0.020mm and 0.040mm. Although the tooth thickness error is based on the circumferential direction, the error magnitude is very small, for convenience, the CS and FS tooth profile is translated left and right according to the error value. As shown in Fig. 15, the backlash increases with the M1 value of FS decreases. With the increase of radial deformation, the backlash in the engaged-in region increases, the backlash decreases significantly in the $[-10^\circ,10^\circ]$ interval, and the minimum value of backlash is close to zero, which is the most likely contact area.

![Fig. 15 Adjustment of radial deformation when the tooth thickness of FS decreases. (a) Before adjustment. (b) After adjustment.](image1)

In order to analyze the second case, the FS $M_1$ value is set to 53.257mm, the tooth thickness increases by 0.005mm, the CS $M_2$ values are 51.969mm, 52.011mm and 52.096mm respectively, and the corresponding tooth thicknesses decrease by 0.010mm, 0.020mm and 0.040mm. As shown in Fig. 16, compared with the former case, the radial deformation does not change.

![Fig. 16 Adjustment of radial deformation when the tooth thickness of CS decreases. (a) Before adjustment. (b) After adjustment.](image2)
In order to analyze the relationship between the three cases, the parameters of the third case are set as follows: the FS $M_1$ value is set to 53.216mm, the corresponding tooth thickness decrease 0.005mm, the CS $M_2$ value is 51.925mm, 51.968mm and 52.053mm respectively, and the corresponding tooth thicknesses decrease by 0mm, 0.01mm and 0.03mm.

![Fig. 17 Adjustment of radial deformation when the tooth thickness of CS and FS decreases at the same time. (a) Before adjustment. (b) After adjustment.](image)

As shown in Figure 17, compared with the first two cases, the amount of radial deformation to be adjusted is the same.

It is defined that the tooth thickness increases positively and decreases to negative. It is concluded that for the above three cases, when the sum of thickness teeth deviations of CS and FS are the same, the radial deformation required to be adjusted is the same.

### 5.2.2 Adjustment scheme for meshing interference

If meshing interference occurs under no-load, tooth wear will be intensified under load, resulting in poor transmission performance. Similarly, meshing interference also includes the following three cases: the first is that the tooth thickness of CS decreases, but the tooth thickness of FS increases more. The second is that the tooth thickness of FS decreases, but the increase amplitude of CS tooth thickness is greater. The third is that the tooth thickness of CS and FS increases at the same time.

The $M_2$ value of the CS is set to 51.947mm, and the corresponding tooth thickness decreases 0.005mm. The $M_1$ values of FS are 53.277mm, 53.298mm and 53.318mm, and the corresponding tooth thicknesses increase by 0.01mm, 0.015mm and 0.02mm respectively.

As can be seen from Fig. 18, with the increase of the tooth thickness of FS, there are different degrees of meshing interference, and the radial deformation need to be decreased more. As shown in Fig. 19(a), as the radial deformation decreases, the maximum meshing depth decreases, and the meshing trajectory swings outward, so that the FS keeps approaching the CS during meshing, and the backlash of the engaged-in region decreases. However, the tooth thickness deviation cannot be too large, which is easy to cause meshing interference as shown in Fig. 19 (b).
In order to analyze the second case, the FS $M_1$ value is set to 53.216mm and the tooth thickness decreases 0.005mm. The $M_2$ values of the CS are 51.882mm, 51.861mm and 51.839mm respectively, and the corresponding tooth thicknesses increase by 0.010mm, 0.015mm and 0.020mm. As shown in Fig. 20, the radial deformation does not change compared with the previous case.

Fig. 20 Adjustment of radial deformation when the tooth thickness of CS increases. (a) Before adjustment. (b) After adjustment.
The third case is that the tooth thickness of CS and FS increases at the same time. In order to analyze the relationship between the three cases, the parameters of the third case are set as follows: the FS $M_1$ value is set to 53.257mm, the corresponding tooth thickness increases by 0.005mm, the CS $M_2$ values are 51.925mm, 51.904mm and 51.882mm respectively, and the corresponding tooth thicknesses increase by 0mm, 0.005mm and 0.01mm.

![Fig. 21](image-url) Adjustments of radial deformation when the tooth thickness of CS and FS increases at the same time. (a) Before adjustment. (b) After adjustment.

It is defined that the increase of tooth thickness is positive and the decrease is negative. As can be seen from Fig. 21, it is concluded that for the above three cases of meshing interference, when the sum of thickness teeth deviations of CS and FS are the same, the radial deformation required to be adjusted is the same.

6. Conclusions

In this study, a method of tooth thickness error compensation for HD is put forward. This method is realized by adjusting the radial deformation of the WG, which can improve the utilization of defective products. Firstly, considering that the involute is prone to tooth tip interference, a short tooth ITP design method is proposed. A calculation method of tooth thickness deviation is proposed in this paper, which does not need to solve the involute function and simplifies the calculation process. On this basis, a time-varying meshing backlash solution algorithm is proposed, which can calculate the magnitude of backlash and tooth profile interference. When the backlash becomes larger, the backlash in the engaged region can be reduced by increasing the radial deformation, thereby the meshing teeth are increased. When meshing interference occurs, in order to realize non-interference meshing of CS and FS, the radial deformation is necessary to be reduced. However, when the positive deviation of tooth thickness is too large, the envelope trajectory of FS is changed by reducing the radial deformation, meaning that meshing interference is easy to occur in the engaged region, which cannot be eliminated. Consequently, when the tooth thickness deviation is within a certain range, adjusting the radial deformation is still an effective method to improve the accuracy.

Acknowledgments

Not applicable.
Authors’ contributions
TZ was in charge of the whole trial and revised the manuscript; CY and HM wrote the original manuscript. HM and JZ were in charge of analysis and interpretation of data. ZL and QC provided rigorous theoretical guidance and revised the manuscript. All authors read and approved the final manuscript.

Funding
The authors would like to thank the National Natural Science Foundation of China No. 52175447 and National Key Research and Development Program of China No. 2020YFB2008200 for supporting the research.

Availability of Data and Materials
The datasets supporting the conclusions of this article are included within the article.

Competing interest
The authors declare no competing financial interests.

Authors’ Information
Congbin Yang received Ph.D. degree in School of Mechanical Engineering from Beijing Institute of Technology, Beijing, China, in 2015. Presently, he is an associate professor in the Institute of Advanced Manufacturing and Intelligent Technology, Beijing University of Technology. His current research interests include dynamics analysis, and Intelligent manufacturing.

Honglie Ma is currently a graduate student at the Institute of Advanced Manufacturing and Intelligent Technology, Beijing University of Technology, China. His research interests focus on the harmonic drive.

Tao Zhang received Ph.D. degrees from Beijing University of Technology, China, in 2020. He is a research assistant in the Institute of Advanced Manufacturing and Intelligent Technology, Beijing University of Technology. His research interests include optimization design and meshing characteristics analysis of harmonic drive.

Jigui Zheng received the Ph.D. degree from the Harbin Institute of Technology, Harbin, China, in 2018. Since 2001, he has been with the R&D Center, Beijing Institute of Precise Mechatronics and Controls, where he has been a Professor, since 2016. His current research interests include space electromechanical servo system technology, machine, and control.

Zhifeng Liu received Ph.D. degrees from Northeastern University, China, in 2001. Presently, he is a Professor in the Institute of Advanced Manufacturing and Intelligent Technology, Beijing University of Technology. His research interests include machine tool precision design, digital design and manufacture, advanced manufacturing technology, and automation. He was selected as a Chang Jiang Scholars in 2021.

Qiang Cheng received Ph.D. degree from Huazhong University of Science & Technology, China, in 2009. Presently, he is an associate professor in the Beijing Key Laboratory of Advanced Manufacturing Technology, Beijing University of Technology, China. His research interests include: adaptable design, modular design, and accuracy design, etc.
References

[17] D. Ma, J. Wu, S. Yan, A method for detection and quantification of meshing


