A New Event-triggered Control Strategy for Multi-agent Systems

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A new event-triggered control strategy for multi-agent systems

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This paper studies the consensus tracking control of multi-agent systems with noise and external disturbances. Based on Lyapunov stability theory, a robust adaptive controller is proposed to compensate the uncertainty of system parameters and external disturbances. The main contribution of this paper is to ensure consensus in multi-agent systems with noise and external disturbances. Numerical simulations show the validity of the theoretical results.

1. Introduction

In recent years, due to the extensive application of multi-agent systems in the control of unmanned aerial vehicles, formation control of mobile robots, etc., the consensus issues of multi-agent systems have been extensively studied [1, 21, 22, 27]. Recently, adaptive consensus control of multi-agent systems with different uncertainties was proposed in [11]. In [19], the consensus problem of high-order nonlinear multi-agent systems with Brunovsky model is studied. In [18], based on neural networks, an adaptive consensus protocol was developed to solve control difficulties caused by non-affine dynamics. Therefore, in general, the issue of consensus is still an extremely active area in research.

On the other hand, compared to a periodic sampling controller, event-triggered control can balance limited bandwidth and control system performance. Therefore, for networked control systems, event-triggered control has been widely studied [8-13]. In these problems, the control signal is updated only at discrete time, and the control input remains constant between events.

Recently, there have been many consensus work on multi-agent systems for event-triggered control. For examples, the authors studied the centralized and distributed event-triggered consensus control of multi-agents connected to undirected

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A particularly interesting topic is the consensus with leaders on a multi-agent system, where leaders are special agents whose actions are independent of all other agents, so all other agents follow the leader. This kind of problem is often referred to as the problem of leader-following consensus. Recently, Wang et al. [25] investigated the leader-following consensus problem on a class of multi-agent systems with nonlinear dynamics and directed communication topology. Based on distributed observer, Liu et al. [12] studied the leader-following attitude consensus problem of multiple rigid-body systems under the jointly connected switching networks. In [26], for the multi-agent system with unknown parameter mismatch and uncertain external disturbances, the leader-following problem was solved. Garcia et al. [5] studied the problem of consensus tracking control for uncertain multi-agent systems and proposed an adaptive event-triggered consensus controller.

In the real world, the information exchange between multiple agents will be affected by noises from communication channels and external environment. Li and Ma et al. [14-16] the multi-agent system under measurement noise in networks with fixed topology and switched topology is studied. By assuming that the intensity of the noise is proportional to the absolute value of the relative state of neighboring agents, the problem of seeking consensus in a multi-agent system with multiplicative noise was discussed in [17]. In addition, the event-triggered consensus of multi-agent systems with noise was proposed in [6].

Based on the above discussion, this paper extends the research results of [5], and further studies the consensus tracking of multi-agent systems with noise and external disturbances through event-triggered adaptive control methods.

The rest of this paper is organized as follows. Section 2 introduces the background of graph theory. The multi-agent system tracking problem is described in Section 3. The main theoretical results are obtained in Section 4. The simulation results are given in Section 5. Finally, conclusions and discussions are given in Section 6.

2. Problem formulation

**Lemma 1.** [20]. Consider $N - 1$ followers labeled as $i = 2, \ldots, N$ and leader $i = 1$. Let $G$ be the directed graph of the $N - 1$ followers and let $\mathcal{L}$ be the corresponding Laplacian matrix. Define $\hat{\mathcal{L}} = \mathcal{L} + \text{diag}\{a_{i1}\}$ for $i = 2, \ldots, N$, where $a_{i1} > 0$ if the leader is a neighbor of agent $i$ and $a_{i1} = 0$ otherwise. Then, all
eigenvalues of $\hat{L}$ have positive real parts if and only if the leader has directed paths to all followers.

The dynamics of the leader is given by

$$x_1(t) = a_1 x_1(t) + r(t)$$

where $x_1 \in \mathbb{R}$ is the state of leader agent, $x_1(0) = x_{1_0}$, $r(t) \in \mathbb{R}$ is the input of leader agent, and the system parameter $a_1 < 0$ is unknown. The input signal is bounded, that is, $|r(t)| \leq \bar{r}$ for $t \geq 0$, where $\bar{r} > 0$ is a known constant parameter.

Consider $N-1$ follower agents. The $i$th follower has the following form:

$$x_i(t) = a_i x_i(t) + u_i(t) + \sigma_i(t)$$

for $i = 2, \ldots, N$, where $x_i \in \mathbb{R}$ is the state of agent $i$, $x_i(0) = x_{i_0}$, $u_i(t) \in \mathbb{R}$ is the control input, and $\sigma_i(t) \in \mathbb{R}$ is an unknown disturbance acting upon each agent. Let $\sigma(t) = [\sigma_2(t) \ldots \sigma_N(t)]^T$. It is assumed that there exists an unknown constant $\bar{\sigma} > 0$ such that $\|\sigma(t)\|_\infty \leq \bar{\sigma}$. In this paper, the infinite norm is used, and the subscript $\infty$ will be omitted next to the norm operators from this point forward unless it is necessary.

It is assumed that all system parameters $a_i$ are unknown. Note that each agent may have its own distinct dynamic. This is called non-identical or heterogeneous agent dynamics.

In this paper, each agent $i$ can only get its own states and it transmits these states to agent $j$ at some discrete time instants $t_k$ via using event-triggered strategies to schedule broadcasting instants and to reduce communication among agents. Fig. 1 shows a control block diagram of follower $i$. 
Figure 1. The control block diagram of follower.

Remark 1. It is worth pointing out that the existing results [5] consider the event-triggered consensus problem without noise and external disturbances, and we consider noise and external disturbances here. This work is significant because noise and external disturbances are unavoidable in the real world.

3. Main results

On the basis [5], the modified MAS controller is given by

$$
\dot{\phi}_i(t) = -\sum_{j=2}^{N} a_{ij}[\phi_j(t) - (\phi_i(t_{k_i}) + \delta(t))] - a_{ii}[\phi_j(t) - x_i(t)]
$$

(3)

where $\phi_i \in \mathbb{R}$ is the state of the MAS controller of follower $i$, for $i = 2, \ldots, N$, $\phi_i(0) = \phi_{in}$ denotes the initial state of the MAS controller, and $\delta(t) \in \mathbb{R}$ denotes the channel noises. Let $\Delta=[\delta(t) \ldots \delta(t)]^T \in \mathbb{R}^{N-1}$, $\|\Delta\| \leq \overline{\Delta}$. When an event is triggered at node $i$, the state $\phi_i(t_{k_i})$ is transmitted to its neighbor agents. The index $t_{k_i}$ represents the sequence of time instants at which agent $i$ generates its own events. Thus, the information that is exchanged by the followers is $\phi_i(t_{k_i})$. Fig. 1 shows that each follower has the MAS controller, adaptive controller and event detector. The discontinuous arrows outside the large block represent the agents’ capabilities of receiving $\phi_j(t_{k_j})$ for $j$ such that $a_{ij} > 0$ and transmitting $\phi_i(t_{k_i})$ which are generated by events. Let $z_i(t) = \phi_i(t_{k_i}) - \phi_i(t)$ be the MAS controller state error. This error will be
used to detect events and to decide when to broadcast the MAS controller state $\phi_i$. Let $z(t) = [z_2(t) \ldots z_N(t)]^T$. Let $\xi_i(t) = \phi_i(t) - x_i(t)$ be the error between the state of the MAS controller of agent $i$ and the leader’s state. Let $\xi(t) = [\xi_2(t) \ldots \xi_N(t)]^T$, and $\xi = \|\xi(0)\|$.

Finally, consider the tracking error $\zeta_i(t) = x_i(t) - \phi(t)$. Let $\zeta(t) = [\zeta_2(t) \ldots \zeta_N(t)]^T$. By Lemma 1, there exist $\hat{\beta}, \hat{\lambda} > 0$ such that $\|e^{-\beta t}\| \leq \hat{\beta} e^{-\hat{\lambda} t}$.

The adaptive controller is designed as

$$u_i(t) = k_{x_i} x_i(t) + k_{\phi_i} \phi_i(t) + k_{\xi_i} \xi_i(t) - \text{sign}(\xi_i(t)) \bar{e}$$  \hspace{1cm} (4)

with the adaptive tuning laws

$$\dot{k}_{x_i}(t) = -g_{x_i} x_i(t) \xi_i(t)$$

$$\dot{k}_{\phi_i}(t) = -g_{\phi_i} \phi_i(t) \xi_i(t)$$

$$\dot{k}_{\xi_i}(t) = -g_{\xi_i} \xi_i(t) \xi_i(t)$$  \hspace{1cm} (5)

where $\text{sign}(\cdot)$ denotes signal function, $g_{x_i} > 0$, $g_{\phi_i} > 0$, and $g_{\xi_i} > 0$ are designed parameters, and

$$\xi_i(t) = x_i(t) - \phi(t)$$

The event-triggered instant is defined as

$$t_{e+1} = \min\{t > t_k \, | \, \|z_i(t)\| \geq \beta e^{-\beta t} + \gamma\}$$  \hspace{1cm} (6)

where $\beta > 0, \lambda > 0, \gamma > 0$.

**Theorem 1.** Consider the MAS given by (2), under the adaptive scheme (4) and (5) with the event trigger condition (6). Then tracking error is bounded by

$$\lim_{t \to +\infty} \|e(t)\| \leq \frac{\hat{B}}{\hat{\lambda}} [\bar{d}(\gamma + \bar{\lambda}) + 2\bar{r}]$$

**Proof:** In fact, we note that at the time when an event is triggered by agent $i$ the error $z_i$ is reset to zero, that is, $z_i(t_k) = 0$. Thus, from Eq. (6), the error $z_i$ satisfies $|z_i(t)| \leq \beta e^{-\beta t} + \gamma$, for $i = 2, \ldots, N$. Hence, $\|z(t)\| \leq \beta e^{-\beta t} + \gamma$ is obtained. The derivative of the error $\xi_i(t)$ is represented as follows:
\[
\dot{\xi}(t) = -\sum_{j=2}^{N} a_{ij} [\phi_i(t) - (\phi_j(t)) + \delta(t)] \\
- a_{ii} [\phi_i(t) - x_i(t)] - \dot{x}_i(t) \\
= -\sum_{j=2}^{N} a_{ij} [x_i(t) + \xi_i(t) - \phi_j(t) - z_j(t) - \delta(t)] \\
- a_{ii} \xi_i(t) - \dot{x}_i(t) \\
= -\sum_{j=2}^{N} a_{ij} [\xi_i(t) - \xi_j(t)] + \sum_{j=2}^{N} a_{ij} [z_j(t) + \delta(t)] \\
- a_{ii} \xi_i(t) - \dot{x}_i(t). 
\] (7)

For \( i = 2, \ldots, N \), Eq. (7) can be written in compact form as follows:

\[
\dot{\xi} = -\hat{L} \xi + \hat{A} (z + \Delta) - \dot{x}_i \cdot 1_{N-1} 
\] (8)

where \( \hat{A} \in \mathbb{R}^{(N-1) \times (N-1)} \) is the followers adjacency matrix. The matrix \( \hat{L} \) was defined in Lemma 1. From Eq. (8), the bounded of \( \xi(t) \) is given by

\[
\|\xi(t)\| = \|e^{-t} \xi(0) + \int_{t_0}^{t} e^{-\hat{L}(t-s)} [\hat{A}(z(s) + \Delta(s)) - x_i(s) \cdot 1_{N-1}] ds\| 
\] (9)

From Eq. (1), we can get

\[
|x_i(t)| \leq e^{a_{i0}} |x_i(0)| + \frac{r(1 - e^{a_{i0}})}{a_i}. 
\] (10)

Also

\[
|\dot{x}_i(t)| \leq |a_i| |x_i(t)| + r, \\
\leq |a_i| \bar{x}_{i0} e^{a_{i0}} + 2r - \bar{r} e^{a_{i0}}, 
\] (11)

where \( \bar{x}_{i0} = |x_i(0)|. \) From Eqs. (9)-(11), we can get

\[
\|\xi(t)\| \leq \hat{\beta} \bar{x}_{i0} e^{-\lambda t} + \hat{\beta} \hat{\beta} \hat{d} \int_{0}^{t} e^{-\lambda(t-s)} e^{-a_{i0}} ds + \hat{\beta} (d_0 (\gamma + \bar{\Delta}) + 2r) \int_{0}^{t} e^{-\lambda(t-s)} ds \\
+ \hat{\beta} (|a_i| \bar{x}_{i0} - r) \int_{0}^{t} e^{-\lambda(t-s)} e^{a_{i0}} ds, \\
\leq \hat{\beta} \bar{x}_{i0} e^{-\lambda t} + \frac{\hat{\beta} \hat{d} \bar{d}}{\lambda - \lambda} (e^{-\lambda t} - e^{-\lambda s}) + \frac{\hat{\beta}}{\lambda} \hat{d} (\gamma + \bar{\Delta}) (1 - e^{-\lambda t}) \\
+ \frac{\hat{\beta}}{\lambda} (|a_i| \bar{x}_{i0} - r) (e^{a_{i0}} - e^{-\lambda t}). 
\] (12)

From Eq. (12), we can get

\[
\lim_{t \to \infty} \|\xi(t)\| \leq \frac{\hat{\beta}}{\lambda} \hat{d} (\gamma + \bar{\Delta} + 2r). 
\] (13)
A model reference adaptive controller is implemented, and the controller 'ideal gains' are given by
\[ k^*_\phi = -(a_i + k^*_u), k^*_\psi = 1. \] (14)

Let \( \tilde{k}_u(t) = k_u(t) - k^*_u \), \( \tilde{k}_\phi(t) = k_\phi(t) - k^*_\phi \), \( \tilde{k}_\psi(t) = k_\psi(t) - k^*_\psi \) be the adaptive gain errors. Using Eqs. (4) and (14)

\[ \dot{\varepsilon}_i(t) = a_i x_i(t) + \tilde{k}_u x_i(t) + k_\phi \phi_i(t) + k_\psi \psi_i(t) - \text{sign}(\varepsilon_i(t))\bar{\sigma} + \sigma_i(t) - \varphi_i(t) \]
\[ = a_i x_i(t) + (k^*_u + \tilde{k}_u(t)) x_i(t) + (k^*_\phi + \tilde{k}_\phi(t)) \phi_i(t) + (k^*_\psi + \tilde{k}_\psi(t)) \psi_i(t) - \text{sign}(\varepsilon_i(t))\bar{\sigma} + \sigma_i(t) - \varphi_i(t) \]
\[ = a^*_i \varepsilon_i(t) + \tilde{k}_u x_i(t) + \tilde{k}_\phi(t) \phi_i(t) + \tilde{k}_\psi(t) \psi_i(t) + \sigma_i(t) - \text{sign}(\varepsilon_i(t))\bar{\sigma}. \] (15)

where \( a^*_i = a_i + k^*_u \).

Consider the Lyapunov function candidate
\[ V_i = \frac{1}{2} \varepsilon_i^2(t) + \frac{1}{2g_u} \tilde{k}_u^2(t) + \frac{1}{2g_\phi} \tilde{k}_\phi^2(t) + \frac{1}{2g_\psi} \tilde{k}_\psi^2(t) \] (16)

Then
\[ \dot{V}_i = \varepsilon_i(t) \dot{\varepsilon}_i(t) + \frac{1}{g_u} \tilde{k}_u \dot{x}_i(t) + \frac{1}{g_\phi} \tilde{k}_\phi \dot{\phi}_i(t) + \frac{1}{g_\psi} \tilde{k}_\psi \dot{\psi}_i(t) - \text{sign}(\varepsilon_i(t))\bar{\sigma} + \sigma_i(t) \]
\[ = a^*_i \varepsilon_i^2(t) - \varepsilon_i(t) \text{sign}(\varepsilon_i(t))\bar{\sigma} + \varepsilon_i(t) \sigma_i(t) \]
\[ \leq a^*_i \varepsilon_i^2(t) - \varepsilon_i(t) \text{sign}(\varepsilon_i(t))\bar{\sigma} + |\varepsilon_i(t)||\sigma_i(t)| \]
\[ = a^*_i \varepsilon_i^2(t) \leq 0. \] (17)

where \( \dot{\tilde{k}}_u(t) = \dot{k}_u(t) \), \( \dot{\tilde{k}}_\phi(t) = \dot{k}_\phi(t) \), and \( \dot{\tilde{k}}_\psi(t) = \dot{k}_\psi(t) \).

Then, using Barbalat’s lemma, it holds that
\[ \lim_{t \to \infty} \| \varepsilon(t) \| = 0, \]

where \( \varepsilon(t) = [\varepsilon_1(t) \ldots \varepsilon_N(t)]^T \).

Finally, we can get
\[ \zeta(t) = x(t) - x_i(t) \cdot 1_{N-1}, \]
\[ = \varepsilon(t) + \phi(t) - x_i(t) \cdot 1_{N-1}, \] (18)

and the following holds
\[
\lim_{t \to \infty} \|\xi(t)\| \leq \lim_{t \to \infty} \|\varepsilon(t)\| + \lim_{t \to \infty} \|\hat{\xi}(t)\| \leq \frac{\hat{\beta}}{\lambda} [\bar{d}(\gamma + \bar{\Delta}) + 2\bar{r}]
\]

(19)

Hence, the tracking error is ultimately bounded.

4. Simulations

In this section, for illustrating the aforementioned theoretical analysis clearly, some simulation results are given. The dynamics of the leader is given by

\[
\dot{x}_1(t) = a_1 x_1(t) + r(t).
\]

(20)

The followers dynamics are given by

\[
\dot{x}_i(t) = a_i x_i(t) + u_i(t) + \sigma_i(t).
\]

(21)

for \(i = 2, \ldots, 9\). The communication graph is depicted in Fig. 2, where the leader is agent 1.

![Communication graph](image)

Figure 2. Communication graph.

The initial state of the leader is \(x_1(0) = 10\), and \(r(t) = 15\). The followers’ initial states are \(x_i = [4, 4, 2, 6, -1, -3, -7, -5]\).

The parameter of the leader is \(a_1 = -3\). The parameters of followers are \(a_i = [3, 1, 2, -2, 7, 4, 5, 6]\). Random disturbance are given at the \(i\)th follower by \(\sigma_i(t) = \text{rand}(1) \cdot \cos(t)\), here \(\text{rand}()\) denotes random function. Channel noise is chosen as \(\delta(t) = 0.5 \cdot \text{rand}(1) \cdot \cos(t)\). Consider the control protocol of Theorem 1. The event-triggered control parameters are \(\beta = 0.5\), \(\lambda = 0.2\), and \(\gamma = 0.05\). Fig. 3 shows the agent states reach consensus. Fig. 4 shows the norm of error \(\xi\). The triggered events are shown in Fig. 5.
Figure 3. Evolution of the states for leader and followers.

Figure 4. The norm of error $\zeta$. 

5 Conclusions

The event-triggered adaptive consensus tracking control problem of multi-agents with noise and disturbances is studied. Finally, simulation results prove the effectiveness of the protocol. In the future, we will study the problem of consensus under time-varying topology or switching topology.

References


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Author contributions

Z.F. and L.D. wrote and reviewed the manuscript, L.D. conducted the simulations, S.Z. checked the manuscript. All authors have read and agreed to the published version of the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request. Correspondence and requests for materials should be addressed to Z.F.