

# Thermodynamic Cost of Quantum transfers

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## Research Article

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# Thermodynamic Cost of Quantum transfers

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In this work, we will study thermodynamic cost of dense coding. In this regard, a scheme is proposed for quantum channel where is induced from two initially uncorrelated thermal quantum systems. At first, the quantum Fisher information and spin squeezing is used to quantify the correlation dynamics over the system. The system reveals that the dynamics of quantum correlations depends crucially on specific energy and temperature. Also, they can be utilized as control parameters for optimal dense coding. Several interesting features of the variations of the energy cost and the dense coding capacity are obtained. It can keep its own valid capacity value in a broad range of temperature by increasing in the energy value of excited states. Also, we can identify valid dense coding with the help of calculating energy cost in the system. Using this approach, identifying a critical point of this model in dense coding capacity quality can be very effective.

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## I. INTRODUCTION

Studying the function of thermodynamics in information theory has been provided a good insight in quantum mechanics [1, 2]. A focus on feature of quantum correlations and entanglement conducted remarkably to the progress in our realization of the thermodynamics of quantum systems [2–4]. Up to now, the role and significance of thermodynamic on quantum information processing has fascinated the attention of many people [5].

The main objective of quantum mechanics is considered the quantum correlations, which not only contains deeper understanding into the principles of quantum mechanics, but it can also reveal an essential function in quantum information[6] and enables the facility of quantum cryptographic protocol[7], quantum teleportation [8], and quantum dense coding [9], and so forth. For example, in dense coding which is considered as an intriguing nonclassical implication, the sender can communicate two bits of classical information to the receiver by forwarding a single qubit if they mix a two-qubit maximally entangled state. Dense coding has been investigated not only theoretically [10–14] but also experimentally [15].

To address the fundamental limitations inflicted by thermodynamics on the growth of correlations and entanglement in bipartite and multipartite settings, some systems has been inspected[16, 17]. In Ref. [17], the authors study the the energy cost of the created correlations between the uncorrelated thermal quantum systems via performing such a global unitary operation such as quantum discord and local quantum uncertainty. So anyone can ask whether other quantum correlations such as quantum Fisher information and spin squeezing can show correlation in present thermodynamic limitations. Recently, it has been represented that quantum Fisher information provides a tool for understanding the phase sensitivity that systems can prepare in the imperfection

of quantum measurement devices [18–20]. As a document of multipartite entanglement, it is asserted that it specifies topological states [21] and non-Gaussian many-body entangled states [22]. Quantum Fisher information can be widely explored in some intent such as the connection between quantum coherence and quantum phase transition [23–25], quantum metrology [26], and quantum speedup limit time [27]. Recently, it is attractive to find that spin squeezing is connected to quantum entanglement and one can use spin squeezing to characterize entanglement. It was found that spin squeezing relates to the minimum spin fluctuation on the plane perpendicular to the mean spin direction [28–30].

We know that correlated states play a significant role in dense coding. For this purpose, we present a straightforward calculation of how quantum Fisher information and also spin squeezing of states can occur the fundamental limitations coming from initial temperature of two initially uncorrelated thermal quantum systems for building quantum correlations. The problem can be intrested: How we can distinguish connection between the dynamic properties of energy cost and dense coding capacity? Our results suggest that valid dense coding can be detect by using the dynamic energy cost in thermodynamic effects.

The road map of this paper is formed as follows. In section II, we compute the density matrix for a two initially uncorrelated two-dimensional thermal quantum systems, then we give out the analytical expression to the quantum Fisher information and spin squeezing and demonstrate how creating quantum correlations can be confined by the thermodynamics of the system. In Section III the super dense coding is discussed. We finish the paper with our main results and outlook. in section IV .

## II. CREATING QUANTUM CORRELATION BETWEEN TWO THERMAL QUBITS

Let us take a global system can be included by two basic separable  $d$ -dimensional quantum systems  $A$  and

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B. And they link with a hot heat bath at temperature  $T$ , the Hamiltonian of each of which may be expressed as  $H = \sum_{i=0}^{d-1} E_i |\psi_i\rangle \langle \psi_i|$ . The thermal state of this system at temperature  $T$  given by

$$\rho_T = \rho \otimes \rho \quad (1)$$

where  $\rho = \frac{e^{-\beta E_n}}{\text{Tr}(e^{-\beta E_n})}$ ,  $\beta = 1/kT$  ( $k = 1$ ) and  $k$  is the Boltzmann constant. When discussing qubits, we will stand for energy of the ground and excited state by  $E_0 = 0$  and  $E_1 = E$ , respectively. Also, the initial thermal state of a qubit can be given as  $\rho = p|0\rangle \langle 0| + (1-p)|1\rangle \langle 1|$ . The coefficient  $p$  sets the ground populations that can be written

$$p = \frac{1}{1+e^{-\beta E}} \quad (2)$$

As in Ref. [31] was showed the maximal entanglement of two qubit arises by optimal global unitary operations. The global unitary operator  $U$  which is made in this protocol can be easily represented as two unitary operators ( $U = V_2 V_1$ ), so we can obtain the final state as  $\rho_f = V_2 V_1 \rho_i V$ , where

$$V_1 = |00\rangle \langle 00| + |01\rangle \langle 01| + |11\rangle \langle 10| + |10\rangle \langle 11| \quad (3)$$

is a the action of the CNOT gate and

$$V_2 = |\varphi_{00}\rangle \langle 00| + |01\rangle \langle 01| + |10\rangle \langle 10| + |\varphi_{11}\rangle \langle 11| \quad (4)$$

is a rotation in the subspace spanned by  $|00\rangle, |11\rangle$  to maximally entangled states which are  $|\varphi_{00}\rangle = \frac{1}{\sqrt{2}}|00\rangle + |11\rangle$  and  $|\varphi_{11}\rangle = \frac{1}{\sqrt{2}}|00\rangle - |11\rangle$ . Hence, the density matrix of the final state in the basis  $|00\rangle, |10\rangle, |01\rangle, |11\rangle$  can be easily obtained as the following forms:

$$\rho_f = \begin{pmatrix} \frac{p}{2} & 0 & 0 & p^2 - \frac{p}{2} \\ 0 & p(1-p) & 0 & 0 \\ 0 & 0 & (1-p)^2 & 0 \\ p^2 - \frac{p}{2} & 0 & 0 & \frac{p}{2} \end{pmatrix}, \quad (5)$$

To address In the following, for obtaining quantum correlation we focus on the quantum Fisher information and spin squeezing.

### A. Fisher information Between Two Thermal Qubits

The quantum Fisher information ( $QFI$ ) is a widespread tool for explaining optimal validity in parameter estimation protocols [32–34]. An active area of research dedicated to evaluate the evolution of  $QFI$  to determine the relation between quantum entanglement and quantum metrology [35, 36]. It has been illustrated that in the unitary evolution, quantum entanglement leads to a remarkable promotion in precision of parameter estimation. For an arbitrary parametric state  $\rho_\theta$  that depends on  $\theta$ , the  $QFI$  is characterized as follows

$$F^2(\rho_\theta) = \frac{1}{4} \text{tr}(\rho_\theta L_\theta^2) \quad (6)$$

where  $L_\theta$  is the symmetric logarithmic derivative operator.  $L_\theta$  is defined as the solution of the equation

$$\frac{\partial \rho_\theta}{\partial \theta} = \frac{1}{2}(L_\theta \rho_\theta + \rho_\theta L_\theta) \quad (7)$$

The effect of the unitary evolution  $U_\theta = e^{-iH\theta}$  on  $\rho$ ,  $\rho_\theta = U_\theta \rho U_\theta^\dagger$ , can be lead to parametric state  $\rho_\theta$ . For a defined quantum state  $\rho = \sum_l |\lambda_l\rangle \langle \lambda_l|$  with  $\lambda_l \geq 0$ ,  $\sum_l \lambda_l = 1$ . Here, we stand for  $F^2(\rho_\theta)$  with  $F^2(\rho, H)$ , which can also be expressed by [37]

$$F(\rho, H) = \frac{1}{2} \sum_{l \neq n} \frac{(\lambda_l - \lambda_n)^2}{\lambda_l + \lambda_n} |\langle l|H|n\rangle|^2 \quad (8)$$

According to Eq. (5),  $QFI$  can easily obtain as follows:

$$F(\rho, H) = \frac{(1-e^{-\beta E})^2}{2} \left\{ \frac{1}{(1-e^{\beta E})^3} - \frac{1}{(1+e^{2\beta E})} \right\} \quad (9)$$

Obviously if  $T \rightarrow 0$ ,  $F(\rho, H)$  is equal to zero. Also, the maximal amount of the  $QFI$  can be obtained in the limit  $T \rightarrow \infty$ , which is equal to 0.5. In Fig. 1(a), we plot the  $QFI$  given in Eq. (9) to study the quantum correlations behavior with respect to initial temperature of the system. As can be observed for any specific energy  $E$  due to thermal fluctuations  $QFI$  is decreased by increasing temperature. Also, we see that by enhancing the energy value of excited states  $E$ , the threshold temperature is raised and therefore the  $QFI$  can be existed in a vast limit of temperature. It should be noted that the maximal value of the  $QFI$  is equal to 0.5 which is obtained at  $T = 0$ .

### B. Spin squeezing Between Two Thermal Qubits

Following Kitagawa and Ueda's criterion of spin squeezing, we briefly review the definition of the spin squeezing parameter for a collection of  $N$  qubits with components  $S^\alpha = \sum_{i=1}^N \frac{\sigma_i^\alpha}{2}$ , ( $\alpha \in x, y, z$ ) as

$$\xi^2 = \frac{2(\Delta S_{\vec{n}_\perp})_{min}^2}{J} = \frac{4(\Delta S_{\vec{n}_\perp})_{min}^2}{N} \quad (10)$$

Here the subscript  $\vec{n}_\perp$  refers to an axis perpendicular to angular momentum operator.  $S = (S_x, S_y, S_z)$  denotes the angular momentum operator of an ensemble of spin-1/2 particles. Where  $(\Delta S_{\vec{n}_\perp})_{min}^2$  is the minimal spin fluctuation in a plane perpendicular to the mean spin, and  $J = \frac{N}{2}$ , and  $S_{\vec{n}_\perp} = \vec{S} \cdot \vec{n}_\perp$ . The noncorrelated limit yields  $\xi^2 = 1$ ; while the inequality  $\xi^2 < 1$  indicates that the system is spin squeezed and entangled. In Ref [38] was indicated for the mean spin along the  $z$  direction spin squeezing can be written as

$$\xi^2 = 1 + \frac{N}{2} - \frac{2}{N} [\langle S_z^2 \rangle + |\langle S_+^2 \rangle|^2] \quad (11)$$

where  $S_\pm = S_x \pm S_y$  are the ladder operators. From Eq. (11) we see that the squeezing parameter is determined by a sum of two expectation values  $\langle S_z^2 \rangle$  and  $\langle S_+^2 \rangle$  hence

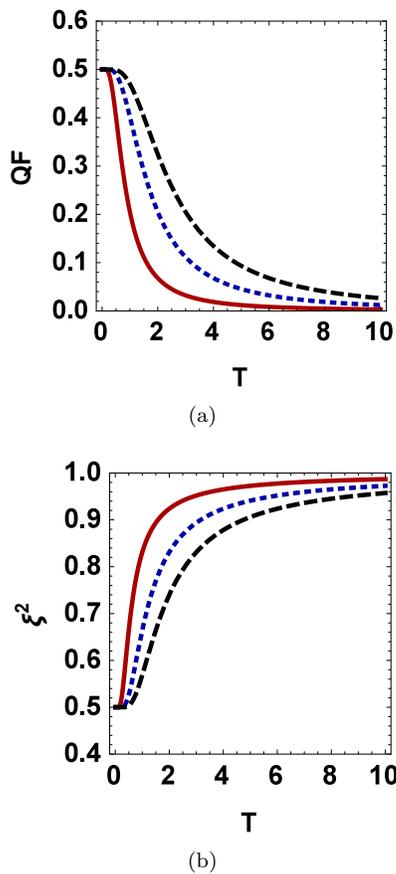


FIG. 1. (a)  $QFI$  and (b) spin squeezing versus  $T$  for various values of  $E$  ( $E = 1$  (red, solid line),  $E = 2$  (blue, dotted line),  $E = 3$  (black, dashed line)). It is seen that the quantum correlations vanishes completely after a threshold temperature for any specific energy.

the calculations are greatly simplified. For this density matrix Eq.(5), the associated spin squeezing is given by

$$\xi^2 = 1 + \frac{1 - e^{-\beta E}}{4(1 + \cosh \beta E)} \quad (12)$$

According to Eq. (12), it is clear that the maximal value of the spin squeezing is equal to 1, which is obtained at  $T \rightarrow \infty$ . Also, the minimal amount of the infity can be obtained in the limit  $T \rightarrow 0$ . Variations of spin squeezing is given in Fig. 1(b) for for any specific energy  $E$ . We can see the spin squeezing undergoes to value one with increasing temperature of the system for any specific energy. It is also known in which all the specific energy is supplied at a low temperature has an spin squeezing efficienc. Therefor, low temperature can maintain squeezing.

### III. SUPERDENSE CODING

Now, we carry out the thermal optimal dense coding in a global system consisted of two initially uncorrelated

$d$ -dimensional quantum systems A and B as a quantum channel. For this purpose, the set of mutually orthogonal unitary transformations is necessary to be made. The set of mutually orthogonal unitary transformations for two-qubit are given as follows [39]:

$$\begin{aligned} U_{00}|j\rangle &= |j\rangle \\ U_{01}|j\rangle &= |j + 1(\text{mod}2)\rangle \\ U_{10}|j\rangle &= e^{\sqrt{-1}(2\pi/2)j}|j\rangle \\ U_{11}|j\rangle &= e^{\sqrt{-1}(2\pi/2)j}|j + 1(\text{mod}2)\rangle \end{aligned} \quad (13)$$

where  $|j\rangle$  is the single qubit computational basis ( $|j\rangle = |0\rangle, |1\rangle$ ). The average state of the ensemble of signal states generated by the unitary transformations Eq.(13) given by:

$$\bar{\rho}^* = \frac{1}{4} \sum_{i=0}^3 (U_i \otimes I_2) \rho (U_i^\dagger \otimes I_2) \quad (14)$$

where 0 stands for 00, 1 for 01, 2 for 10, 3 for 11, and  $\rho$  is the density matrix of the quantum channel. Eq. (13) represents the operations that Alice(sender) performs on the shared entangled state  $\rho$ . If the sender does the set of mutually orthogonal unitary transformations, the maximum dense coding capacity  $\chi$  can be obtained by

$$\chi = S(\bar{\rho}^*) - S(\rho) \quad (15)$$

where  $S(\bar{\rho}^*)$  is an von Neumann entropy for the average state of ensemble of signal states  $\bar{\rho}^*$ , and  $S(\rho)$  is the von Neumann entropy of the quantum channel. If  $\chi > 1$  dense coding is valid, and for optimal dense coding  $\chi$  must be the maximum, i.e.  $\chi_{max} = 2$ . We are now ready to discuss the validity of generated quantum correlation a global system consisted of two initially uncorrelated  $d$ -dimensional quantum systems A and B in contact with a heat bath at temperature  $T$  as a quantum channel to study the optimal dense coding. By considering the density matrix Eq. (1), we first focus on the thermal dense coding capacity. By numerical calculation we can plot dense coding capacity as a function  $T$  for different specific energy values. In Fig. 2(a), we observe valid dense coding for any specific energy in low temprature. It is reduced from the maximum to zero in a short interval of temperature and disappears abruptly for the large amount of  $T$ . The increase in the energy value of excited states help dense coding capacity can be alive in a broad range of temperature. Now we start to explore a work cost  $W$  to the optimal dense coding. Thermodynamic cost of dense coding  $W$  is given by

$$W = Tr(H_{tot}\rho^*) - Tr(H_{tot}\rho) \quad (16)$$

where  $\rho^*$  is the final state Eq. (14) and  $H_{tot} = \sum_i H^{(i)}$  is the total Hamiltonian as

$$H_{tot} = H \otimes I_2 + I_2 \otimes H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & E & 0 & 0 \\ 0 & 0 & E & 0 \\ 0 & 0 & 0 & 2E \end{pmatrix}, \quad (17)$$

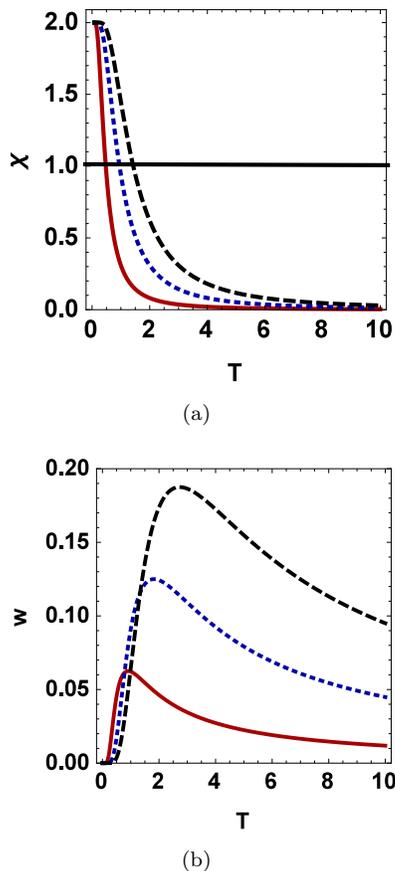


FIG. 2. (a) dense coding capacity and (b) Energy cost of dense coding versus  $T$  for various values of  $E$  ( $E = 1$  (red , solid line),  $E = 2$  (blue, dotted line),  $E = 3$  (black, dashed line)).

By substituting Eq.(1) and Eq. (14) in Eq. (16), work cost can be obtained as

$$W = \frac{1 - e^{\beta E}}{2(1 + e^{\beta E})^2} E \quad (18)$$

We can find that the energy cost is dependent of temperature and specific energy. According to Eq. (18), it is easy to prove that value of the energy cost can be maximal in the critical temperature  $T_c = 0.91E$  which the dense coding is not valid, i.e  $\chi < 1$  (see Fig. 2(a)). In Fig. 2(b), the energy cost of dense coding is plotted as a

function of temperature for any specific energy. It can be seen when the temperature is smaller than  $T_c$ , the energy cost increases until its maximum value with raising temperature. But, it for  $T < T_c$  specific energy exhibits a collapse as temperature increases. Moreover, the range of  $T$  for energy cost is also broadened as specific energy increases in this interval. Furthermore, the height of peaks energy cost changes manifestly. It approach to the higher peak by increasing specific energy. Comparing dynamic energy cost Fig. 2(b) and dense coding capacity Fig. 2(a) leads to an interesting outcome, there is no optimal dense coding when energy cost is decreasing, i.e after the phase transition  $T > T_c$ . We notice that, when  $T < T_c$ , until the energy cost reaches its maximum value we can obtain optimal dense coding. With these results at hand, one can detect optimal dense coding via the behavior of energy cost. Therefore, it is helpful to investigate the energy cost as indicators of valid dense coding in every system.

#### IV. CONCLUSION

In conclusion, we have prospected the amount of quantum correlations that can be made between two initially uncorrelated thermal quantum systems via an optimal protocol which maximizes the quantum correlated. we inspected the dynamics of quantum correlations such as quantum Fisher information and spin squeezing in this system. It is illustrated that increasing the initial equilibrium temperature leads to diminishing the amount of quantum correlations that can be caused between two thermal states. Quantum correlations undergoes a abrupt death at the critical temperature. Moreover, we have investigated the quantum dense coding via this model. We have found appealing results as the existence of a relationship between dense coding and energy cost that will certainly lead to a better comprehension of the nonequilibrium systems. Our analyses have indicated by enhancing the value of temperature, the value of energy cost tends to its maximum. Interestingly, after crossing the critical temperature, when no optimal dense coding exists in the system, the energy cost of dense coding is become spoiled. We offer these features as a tool to detect optimal dense coding.

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# Figures

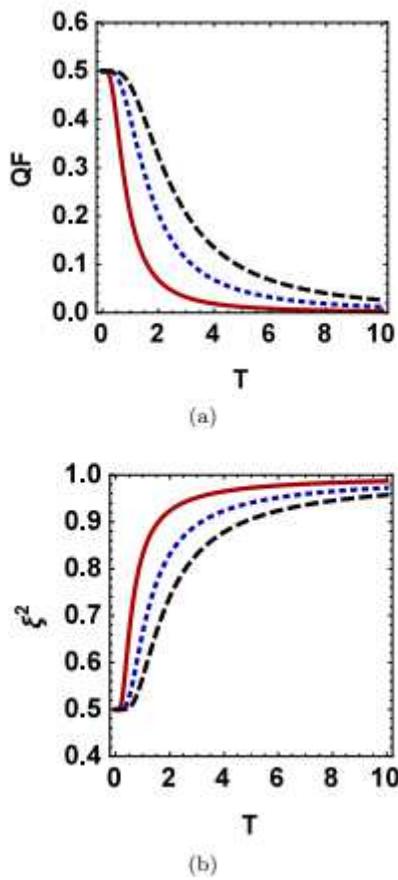
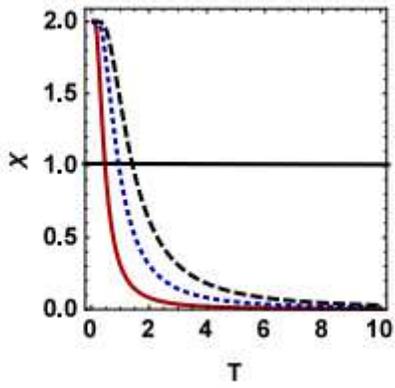
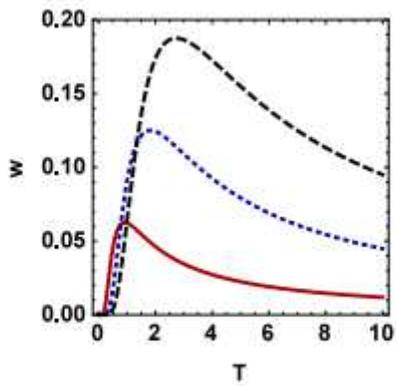


Figure 1

(a) QFI and (b) spin squeezing versus  $T$  for various values of  $E$  ( $E = 1$  (red , solid line),  $E = 2$  (blue, dotted line),  $E = 3$  (black, dashed line)). It is seen that the quantum correlations vanishes completely after a threshold temperature for any specific energy.



(a)



(b)

**Figure 2**

(a) dense coding capacity and (b) Energy cost of dense coding versus  $T$  for various values of  $E$  ( $E = 1$  (red, solid line),  $E = 2$  (blue, dotted line),  $E = 3$  (black, dashed line)).