

Observation of fractional spin textures and bulk-boundary correspondence

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Abstract

Recently a zoology of non-collinear chiral spin textures has been discovered, most of which, such as skyrmions and antiskyrmions, have integer topological charges that provides them with enhanced stability. Here we report the experimental real-space observation of the formation and stability of fractional antiskyrmions and fractional elliptical skyrmions in a Heusler material. These fractional objects appear, over a wide range of temperature and magnetic field, at the edges of a sample, whose interior is occupied by an array of nano-objects with integer topological charges, in agreement with our simulations. We explore the evolution of these objects in the presence of magnetic fields and show their interconversion to objects with integer topological charges. This means the topological charge can be varied continuously. These fractional spin textures are not just another type of skyrmion, but are essentially a new state of matter that emerges and lives only at the boundary of a magnetic system. Moreover, they form the basis for topological quantum effects arising from the magnonic excitations around a skyrmion lattice. The coexistence of both integer and fractionally charged spin textures in the same material makes the Heusler family of compounds unique for the manipulation of the real-space topology of spin textures and thus an exciting platform especially for the future exploration of the magnonic bulk-boundary correspondence.

Introduction

The mathematical concept of topology has proven to be highly relevant in several fields of natural sciences¹⁻⁴. In condensed matter physics, it is used to classify and explain the enhanced stability of distinct phases, and as a guiding principle for the design of materials and systems with novel, robust properties. When infinitely extended periodic systems exhibit non-trivial reciprocal-space topology, characterized by integer-valued topological invariants computed from the band structure of their collective excitations, topologically protected boundary states emerge⁵⁻⁸-the hallmark of the bulk-boundary correspondence^{9,10}. In magnetism, non-collinear spin textures can also be characterized by a topological invariant, albeit defined in real space. This ‘topological charge’ or ‘skyrmion number’ N_{sk} is ± 1 for most of the recently observed objects^{11,12}, including skyrmions and antiskyrmions, which are mesoscopic magnetic whirls that have been observed in chiral systems¹³⁻¹⁵. An integer value of N_{sk} is guaranteed under the assumption of a continuous spin texture and that the surface on which it resides (the magnetic unit cell or the whole sample depending on the periodicity of the texture) can be mapped to a sphere. As a counter-example, magnetic whirls called merons, stabilized in in-plane magnetized systems¹⁶, carry $N_{\text{sk}} = \pm 1/2$ as their sample surface cannot be mapped to a sphere. Objects with fractional topological charge can be realized in the interior of the sample, but they always maintain a net integer N_{sk} in the unit cell, as previously reported^{17,18}.

Under the assumption of a continuous spin density, skyrmions and antiskyrmions are topologically protected by an infinite energy barrier from transforming into a trivial state, with $N_{\text{sk}} = 0$, such as a collinear ferromagnet. In fact, in this model, a discontinuous change in the topological charge can only occur via the introduction of a Bloch point singularity¹⁹. However, real spin textures are defined on a lattice. Still, a discretized, integer-valued version of N_{sk} can be defined²⁰ and the now finite energy barriers between different topological sectors are considerable.

Another route that allows for non-integer topological charges is the introduction of an edge. Herein, we consider skyrmions and antiskyrmions that carry a fractional topological charge and emerge along the edge of the sample. They are fundamentally different from topological edge defects in finite in-plane magnetized systems, whose winding number takes fractional values²¹. We experimentally observe the formation, stability, and annihilation of fractional antiskyrmions and fractional elliptical skyrmions at the edges of the Heusler material $\text{Mn}_{1.4}\text{Pt}_{0.9}\text{Pd}_{0.1}\text{Sn}$. Our real space observations, conducted by Lorentz transmission electron microscopy (LTEM), are supported by micromagnetic simulations of the magnetic

texture to reveal the stabilizing mechanism of these objects. Using atomistic simulations, we show that fractional skyrmions should also exist in typical skyrmion-hosting materials, like the B20 material MnSi, but their observation is difficult, due to a tilting of the fractional skyrmion tubes, and would be facilitated by alternative imaging techniques. Furthermore, our observation of the elusive boundary fractional (anti)skyrmions is a stepping stone to realize the recently predicted magnonic bulk-boundary correspondence in (anti)skyrmion lattices²².

The LTEM images that we present in the following were acquired from a thin specimen prepared using a Ga⁺-ion dual beam focused ion beam method (Supplementary Figure 1). More details of the preparation method can be found elsewhere²³.

Edge twist and the metastability of fractional objects.

Before we present our LTEM measurements and simulations, we want to convey the general idea as to why fractional objects may emerge at sample edges. Non-collinear spin textures on a lattice are not stabilized by topology alone. Instead, magnetic interactions are responsible for their stability. In systems with broken inversion symmetry, the Dzyaloshinskii-Moriya interaction (DMI)^{24,25} is the most relevant. Depending on the type of symmetry breaking, the interaction can be isotropic, as in MnSi, stabilizing rotationally symmetric skyrmions¹⁴, or can be anisotropic, as in the Heusler material Mn_{1.4}Pt_{0.9}Pd_{0.1}Sn, stabilizing antiskyrmions¹⁵.

Spins along the sample edge have missing neighbors leading to uncompensated DMI bonds that result in an edge twist of the texture^{26,27}. The sample surface, hosting such an edge-twisted texture, can no longer be mapped to a sphere, thus the topological charge is not restricted to take on integer values: Even for a trivial ferromagnetic or helical state, the twisted edge leads to a small $N_{\text{sk}} \neq 0$. Below a critical magnetic field, the twisted texture along the edge becomes unstable against the nucleation of incipient stripe domains²⁸. Therefore, as our measurements will reveal, (anti)skyrmions from the interior that are sufficiently close to the edge can further reduce their energy by transforming into such incipient stripe domains. The nucleated domains, repelled by a crystal of (anti)skyrmions, stabilize along the edge as fractional (anti)skyrmions.

In Fig. 1k fractional antiskyrmions are shown, obtained by micromagnetic simulations (details and parameters can be found in the Methods section). While the twist in the region surrounding the fractional antiskyrmions is into the vacuum direction, inside the antiskyrmions the twist is along the opposite direction, i.e. into the interior of the sample. If fractional antiskyrmions were pushed toward the edge of the sample, the twist would have to be overcome. Therefore, this edge twist brings about an energy barrier that protects fractional

antiskyrmions from spontaneous annihilation. Pushing fractional objects in the opposite direction would not be hindered by the edge twist, but by the repulsive interaction from the bulk antiskyrmions.

Results and Discussion

Experimental observation of fractional antiskyrmions.

We start by discussing the measured textures at room temperature. As was presented in our earlier study²³, at this temperature antiskyrmions form in the interior of the lamella. However, in these thin lamellae of the D_{2d} Heusler compound, a special protocol has to be used in order to stabilize them. If we simply apply an out-of-plane magnetic field, only a few antiskyrmions form and the phase diagram is dominated by the helical and the ferromagnetic phases. However, when an in-plane field is provided temporarily by reversibly tilting the sample in the TEM column by $\sim 40^\circ$, the nucleation of antiskyrmions is triggered. We start from a large field, so that the ferromagnetic phase is stabilized. Next, we reduce the field to a value B^* and reversibly tilt the sample once to provide the in-plane field component. Thereafter, the perpendicular magnetic field is reduced without tilting the sample.

B^* serves as a parameter to control the antiskyrmion density in our sample. Depending on the magnitude of B^* , a sparse or dense array of antiskyrmions is formed²³. In Fig. 1, we start from a rather dense array of antiskyrmions in the interior of the sample (B^* around 304 mT). Upon further decreasing the field to $B = 128$ mT, the antiskyrmion lattice remains stable; cf. Fig. 1a. The antiskyrmions have the same square-shaped contrast that we have analyzed in our previous study²³. This deformation is a signature of the dipole-dipole interaction. Due to the anisotropic DMI, the square-shaped antiskyrmions form a square lattice. This antiskyrmion lattice reaches close to the edge, which is an interface between the magnetic $\text{Mn}_{1.4}\text{Pt}_{0.9}\text{Pd}_{0.1}\text{Sn}$ and the non-magnetic PtC_x applied during the fabrication of the lamella.

In the following, we concentrate on this region of the sample (orange border; shown also in Fig. 1b) and discuss the texture upon decreasing the magnetic field. In Fig. 1c the field is 64 mT and the size of the square-shaped antiskyrmions has increased. At 0 mT, the antiskyrmions that are closest to the edge have turned into fractional objects (Fig. 1d). Upon applying negative magnetic fields in Figs. 1e-f, the bulk antiskyrmions become more and more square-shaped because their size would increase if they were not confined by the neighboring antiskyrmions. This pushes the fractional antiskyrmions, which now appear as

triangles, even closer to the edge: They are square-shaped antiskyrmions that have been ‘cut in half’. Upon further increasing the magnitude of the negative field, the fractional antiskyrmions merge among themselves forming a polarized region in the vicinity of the edge, while the bulk antiskyrmions also merge and turn more and more into helices; cf. Figs. 1g-i.

In Figs. 1j we have stabilized a square lattice of square-shaped antiskyrmions with fractional antiskyrmions at the edge in the micromagnetic framework used (see Methods for details). This texture is stable over a wide field range and resembles the experimentally observed system qualitatively well. In Fig. 1k a magnified view of the red highlighted region is shown. Just like in Fig. 1j, the color encodes the orientation of the magnetic moments but here their orientation is also visualized by arrows. It becomes apparent that the antiskyrmions are practically cut in half at the edges but also that they slightly deform due to the edge twist. As discussed above, this twist constitutes an energy barrier enabling the stability of the fractional objects. Also, it leads to a redistribution of topological charge density that is shown in Fig. 1l. An antiskyrmion that is stabilized in a positively magnetized background has an exclusively positive topological charge density that integrates to almost 1 (here it is 0.99 for bulk antiskyrmions due to the discussed lattice effects). However, directly at the edge, negative contributions to the topological charge arise. Integrating over a fractional antiskyrmion yields the topological charge 0.42 which is quite far from 0.5 which would be the expected value without the edge twist. This value depends on the magnetic interaction parameters, as well as the size of these objects.

As mentioned above, we can tune the density of the bulk antiskyrmion lattice by the field B^* . For an intermediate density of bulk antiskyrmions, fractional antiskyrmions form as well but some of them extend far into the interior of the sample (Suppl. Fig. 2). As discussed before, the edge tilting only protects the fractional antiskyrmions from annihilation but not from extension into the interior. This extension can only be suppressed by the repulsive interaction with the bulk antiskyrmions. For a low density of bulk antiskyrmions, no fractional antiskyrmions have formed. The bulk antiskyrmions are essential for the formation of these objects at the edges. This becomes also apparent when we compare the presented results with our recent study on (anti)skyrmions in nano-strips²⁹. In that paper the sample had a width less than 500 nm and could host only two rows of antiskyrmions. Under no circumstances did we observe fractional objects in that sample. Apparently, this is because no real bulk antiskyrmion lattice can form in such a geometry.

It is to be noted that the role of PtC_x is to protect the sample during the sample processing and to produce an interface region that is free from Fresnel fringes, which

negatively affect LTEM measurements. However, adding PtC_x is not mandatory for the formation of fractional nano-objects. In Supplementary Figure 3 we show that fractional antiskyrmions also form at the interface of $\text{Mn}_{1.4}\text{Pt}_{0.9}\text{Pd}_{0.1}\text{Sn}$ with vacuum even though this interface is rougher due to the preparation process.

Experimental observation of fractional elliptical Bloch skyrmions.

When we start from an antiskyrmion lattice and field-cool the sample to low temperatures, the antiskyrmion lattice and the fractional antiskyrmions survive, as is shown in Supplementary Figure 4. However, at low temperatures the antiskyrmion is not the naturally stable type of texture^{23,30}. The dipole-dipole interaction, which is the dominating interaction at low temperatures, leads to the stabilization of Bloch skyrmions instead of antiskyrmions. Due to the anisotropic DMI, these skyrmions are elongated along the [100] direction for clock-wise Bloch skyrmions that we find here. Due to this elongation we have dubbed them ‘elliptical skyrmions’. In Fig. 2, it is shown that the stabilization of these objects at 150 K is very similar to the antiskyrmionic case. Again, we can control the density of non-collinear objects by tuning B^* as shown in Supplementary Figure 5. This time the dense starting lattice of elliptical skyrmions that was created by B^* near 384 mT remains stable also at 128 mT in Fig. 2b. Again, upon decreasing the perpendicularly applied magnetic field, these objects become increasingly larger until the skyrmions closest to the edge touch the edge and form fractional objects. Since these skyrmions do not form a square-shaped lattice but rather a hexagonal lattice, we have considered periodic boundary conditions along the horizontal direction in the simulations. This avoids finite-size related quenching effects of the skyrmions due to the square shaped geometry. At the top and the bottom of that simulated spin cluster, we find fractional elliptical skyrmions (Fig. 2j). Looking at the texture of such an object in detail, cf. Fig. 2k, we can again see the edge twist.

Conversion vs. annihilation mechanism.

Above, we have discussed the decreasing field behavior of the magnetic textures. Next, we present how fractional antiskyrmions and fractional elliptical skyrmions evolve when the field is increased. In both cases we start from a dense lattice of bulk (anti)skyrmions with fractional objects at the edge at -64 mT and 0 mT in Figs. 3a,f, respectively. When the field is increased, the size of the bulk objects shrinks. This brings about two counter-acting effects for the fractional (anti)skyrmions. The increased field favors a shrinking size of the fractional objects *per se* but since the bulk objects also shrink, their repulsive interaction decreases, which

allows for an elongation of the fractional (anti)skyrmion. The dominating behavior of these two counteracting trends depends strongly on the starting configuration. If we look at a small fractional skyrmion or antiskyrmion (red arrow in Figs. 3a,f), this object shrinks until it disappears. On the other hand, if we focus on a fractional object that is initially elongated, due to a lower density of bulk objects in its vicinity, the fractional (anti)skyrmion elongates, and finally nucleates a new bulk (anti)skyrmion (green arrow in Figs. 3a,f). The conversion-annihilation mechanism is further illuminated by performing atomistic spin dynamics simulations (see Supplementary Video 1).

For both objects and for both transformation mechanisms – annihilation and conversion – the topological charge density varies continuously. No fractional objects remain stable at increased fields. This points to the fact that bulk (anti)skyrmions are more stable than fractional (anti)skyrmions and that the latter disappear or transform before the bulk objects are affected. Since the number of topological objects has to decrease when the magnetic field is increased, the fractional objects do not remain stable at higher fields. This observation is in accordance with our prior observation that fractional (anti)skyrmions do not exist individually but always require a rather dense interior region for their formation in the decreasing field mode.

Comparison to B20 materials.

In this paper, we have discovered, for the first time, the existence of fractional skyrmions and antiskyrmions. The question arises why these objects have not yet been seen in typical skyrmion hosts, like the B20 material MnSi, despite the fact that these materials have been investigated far more than the antiskyrmion-hosting Heuslers.

Following from our atomistic simulations presented in Fig. 4, fractional Bloch skyrmions can indeed also be stabilized in materials with an isotropic DMI like in MnSi. However, one has to take into account the full three-dimensional texture to understand the difference between both material classes. The anisotropic DMI in Heusler materials is a two-dimensional interaction that acts layer-wise. Instead, the bulk DMI in B20 materials also has DMI vectors for bonds along the z direction. Consequently, the tubes of fractional antiskyrmions in Heusler materials are straight but the tubes of fractional Bloch skyrmions in B20 materials are tilted.

Like in the present study, typically, the real-space texture is experimentally measured by techniques that average the signal over the thickness of the sample. If we observe a straight tube, for which the magnetization does not change significantly throughout the layers, this is

unproblematic: the measured signal can be assumed to represent the magnetic texture of every single layer. However, for the tilted tubes in B20 systems this becomes problematic: In Fig. 4 we also show the averaged magnetization over all layers. While the bulk Bloch skyrmions can be observed without a problem, as has been done in many publications, the fractional skyrmions at the edge result in a smeared-out signal due to the tilted tubes. This signal is very similar to a signal that arises from a mere edge twist without the emergence of fractional objects as was assumed in FeGe³¹. In summary, fractional Bloch skyrmions can exist in B20 materials and could be observed by imaging techniques that average over the sample thickness as long as the thickness is comparable to the size of skyrmions. On the other hand, for thicker samples, three-dimensional imaging techniques should be utilized such as X-ray nanotomography³².

Fractional (anti)skyrmions and the magnonic bulk-boundary correspondence.

The magnonic excitations supported by (anti)skyrmion lattices can also be topologically protected. Recently, the magnonic quadrupole topological insulator, a novel symmetry-protected topological phase, was predicted to be realized in (anti)skyrmion lattices²². Its hallmark signatures, as dictated by the bulk-boundary correspondence, are robust magnonic states that emerge at the corners of finite-sized samples. The boundaries of the sample, however, tend to distort the magnetic texture and hence break the protecting symmetries. Boundary fractional (anti)skyrmions are required for restoring these protecting symmetries, thus allowing the emergence of the magnonic corner states, as sketched in Fig. 4c. Therefore, our observation of fractional (anti)skyrmions identifies Heuslers as suitable platforms for the realization of the magnonic bulk-boundary correspondence in higher-order topological phases in magnetic materials.

Conclusion

We have presented the experimental discovery of the formation of fractional antiskyrmions and fractional elliptical skyrmions in a material with an anisotropic DMI. These objects are stable in a finite field and temperature range and only when a considerably dense bulk (anti)skyrmion lattice is present. Fractional objects cannot exist on their own. While the bulk lattice hinders an elongation of the fractional (anti)skyrmions, the DMI-mediated edge twist protects them from spontaneous annihilation via the edge. By tuning the field, we can continuously control their topological charges because they either annihilate or convert to integer-charged objects. Furthermore, we have predicted by simulations that fractional objects

are not unique to materials with an anisotropic DMI like Heuslers. Fractional Bloch skyrmions can exist in materials with a three-dimensional DMI like the B20 material MnSi as well. However, it is hardly possible to observe them by techniques that integrate the signal over the whole thickness of the sample.

Our observation of fractional (anti)skyrmions brings about new possibilities for realizing fascinating fundamental effects and advantageous spintronic and magnonic applications. This includes the realization of the recently predicted magnonic quadrupole topological insulator, supported by (anti)skyrmion lattices²², in which fractional (anti)skyrmions restore the protecting symmetries that guarantee the emergence of robust magnonic corner states, hallmarks of the bulk-boundary correspondence of this higher-order topological phase. Boundary fractional (anti)skyrmion tubes could support magnonic modes similar to those recently reported in skyrmion tubes^{33,34}, thus they could act as robust, reconfigurable magnonic waveguides running along the lateral facets of the sample. The demonstrated topological charge annihilation and conversion processes identify Heuslers as unique platforms to investigate the emergent electrodynamics³⁵ arising from spin textures with continuously tunable topological charge.

Methods

Experiment.

The bulk polycrystalline Heusler compound $\text{Mn}_{1.4}\text{Pt}_{0.9}\text{Pd}_{0.1}\text{Sn}$ was prepared by an arc-melting method, as described elsewhere¹⁵. The electron backscattering diffraction technique was performed in a TESCAN GAIA 3 (Quantax, Bruker) system to realize [001]-oriented grains. The single crystalline lamellae with parallel surfaces were fabricated from single [001] oriented grains by a Ga^+ ion-based dual beam focused ion beam system [FEI Nova Nanolab 600 SEM/FIB] operating at an accelerating voltage of 30 kV. Low energy polishing at 5 kV and then at 2 kV were performed to remove any damaged surface material. An aberration-corrected high-resolution transmission electron microscope [FEI TITAN 800-300] was used to observe the magnetic contrast in the LTEM mode. The magnetic field was varied from -0.28 T to 2.3 T along the microscope axis by partially exciting the objective lens. The magnetic field values were calibrated by a Hall bar sensor. The double tilt liquid nitrogen holder was used to vary the temperature of the specimen. The in-plane field components were realized at the specimen surface by tilting the sample holder up to $\pm 40^\circ$. The defocus value for the LTEM imaging is ~ 0.9 nm.

Simulations

Micromagnetic simulations.

For the micromagnetic simulations we started from an analytically constructed seed and propagated the magnetic texture towards the nearest energy minimum using mumax3^{36,37}. The propagation is according to the Landau-Lifshitz-Gilbert equation³⁸

$$\dot{\mathbf{m}} = -\gamma_e \mathbf{m} \times \mathbf{B}_{\text{eff}} + \alpha \mathbf{m} \times \dot{\mathbf{m}}.$$

where $\mathbf{B}_{\text{eff}} = -\frac{\delta F}{M_S \delta \mathbf{m}}$ is the effective magnetic field that is computed from the free energy F .

This energy includes the exchange interaction A , anisotropic DMI D , an easy-axis anisotropy K , interaction with a magnetic field \mathbf{B} and dipolar interactions. The parameters have been taken from our recent publication on nanostripes of $\text{Mn}_{1.4}\text{Pt}_{0.9}\text{Pd}_{0.1}\text{Sn}$ ²⁹: $D = 0.003 \text{ J/m}^2$, $A_{\text{ex}} = 0.5 \times 10^{-10} \text{ J/m}$, $K_u = 135 \text{ kJ/m}^3$, Gilbert damping $\alpha = 0.3$, saturation magnetization $M_S = 445 \text{ kA/m}$. These parameters allowed for the metastability of fractional antiskyrmions, as shown in Fig. 1 and Suppl. Fig. S2, in a finite geometry of size $162 \text{ nm} \times 162 \text{ nm} \times 280 \text{ nm}$ with a cell size $5 \text{ nm} \times 5 \text{ nm} \times 10 \text{ nm}$. For the fractional skyrmions in Fig. 2 we had to account for an anisotropy gradient close to the edge due to the PtC_x : we used a 50% increased K_u in the bulk region. Note that this is only required for the present parameters and is not generally

needed as our atomistic simulations establish. Furthermore, we used periodic boundary conditions along the x direction to reduce finite size effects. We simulated a region $240 \text{ nm} \times 290 \text{ nm} \times 280 \text{ nm}$. While antiskyrmions form a square lattice compatible with a square-shaped geometry, the skyrmions form a hexagonal lattice. We used a magnetic field of 0 mT in Fig. 1 and Suppl. Fig. S2, and 180 mT in Fig. 2.

Atomistic spin simulations.

We use a cubic spin lattice model with the following Hamiltonian

$$H = \frac{1}{2} \sum_{r,r'} (-J_{r,r'} \mathbf{S}_r \cdot \mathbf{S}_{r'} + \mathbf{D}_{r,r'} \cdot \mathbf{S}_r \times \mathbf{S}_{r'}) - \sum_r g \mu_B B_z \mathbf{S}_r \cdot \hat{\mathbf{z}} - \sum_r K (\mathbf{S}_r \cdot \hat{\mathbf{z}})^2 + \frac{I_{dp}}{2} \sum_{r,r'} \left[\frac{\mathbf{S}_r \cdot \mathbf{S}_{r'}}{|\mathbf{r} - \mathbf{r}'|^3} - \frac{3\{\mathbf{S}_r \cdot (\mathbf{r} - \mathbf{r}')\}\{\mathbf{S}_{r'} \cdot (\mathbf{r} - \mathbf{r}')\}}{|\mathbf{r} - \mathbf{r}'|^5} \right],$$

where \mathbf{S}_r is a spin vector defined at $\mathbf{r} = (x, y, z)$ with lattice constant $a = 1$. The first two terms are nearest neighbor interactions, including the ferromagnetic exchange coupling $J_{r,r'} = J(\delta_{\mathbf{r}-\mathbf{r}' \pm \hat{x}} + \delta_{\mathbf{r}-\mathbf{r}' \pm \hat{y}} + \delta_{\mathbf{r}-\mathbf{r}' \pm \hat{z}})$ with $J > 0$ and the anisotropic Dzyaloshinskii–Moriya interaction $\mathbf{D}_{r,r'} = D(\mp \hat{x} \delta_{\mathbf{r}-\mathbf{r}' \pm \hat{x}} \pm \hat{y} \delta_{\mathbf{r}-\mathbf{r}' \pm \hat{y}})$. The third term accounts for the external magnetic field $B_z \hat{\mathbf{z}}$, where g and μ_B denote the g-factor and Bohr magneton, respectively. The model also includes easy-axis anisotropy ($K > 0$) and long-range dipolar interaction with coupling strength I_{dp} . Energetically stable magnetic configurations are obtained by Monte Carlo simulated annealing.

Figure 4 in the main text shows the classical ground-state magnetic texture for $60 \times 60 \times 5$ spins with free boundary conditions, $D/J = 1.0$, $g\mu_B B_z/J S = 0.3$, and $K = I_{dp} = 0.0$, at zero temperature. The magnetic texture was further relaxed employing the Landau-Lifshitz-Gilbert equation. A study of the magnetic configurations that result from the competition among the dipolar interaction, Dzyaloshinskii–Moriya interaction, and easy-axis anisotropy, at a finite temperature, can be found in the SI.

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Author Contributions

J.J. and S.P. conceived the project. J.J. carried out the Lorentz TEM observations and analyzed the Lorentz TEM data. The bulk material was synthesized by V.K. under supervision of C.F.. J.J. prepared the lamellae. B.G. performed the micromagnetic simulations with supervision from I.M.. To.H. performed atomistic simulation with the help of Ta.H. and S.D. under supervision of D.L.. J.J., B.G., S.D., To.H., D.L., and S.P. wrote the manuscript with substantial contributions from all authors. S.P. supervised the project. All authors discussed the results.

Competing Interests: The authors declare that they have no competing interests.

Figures

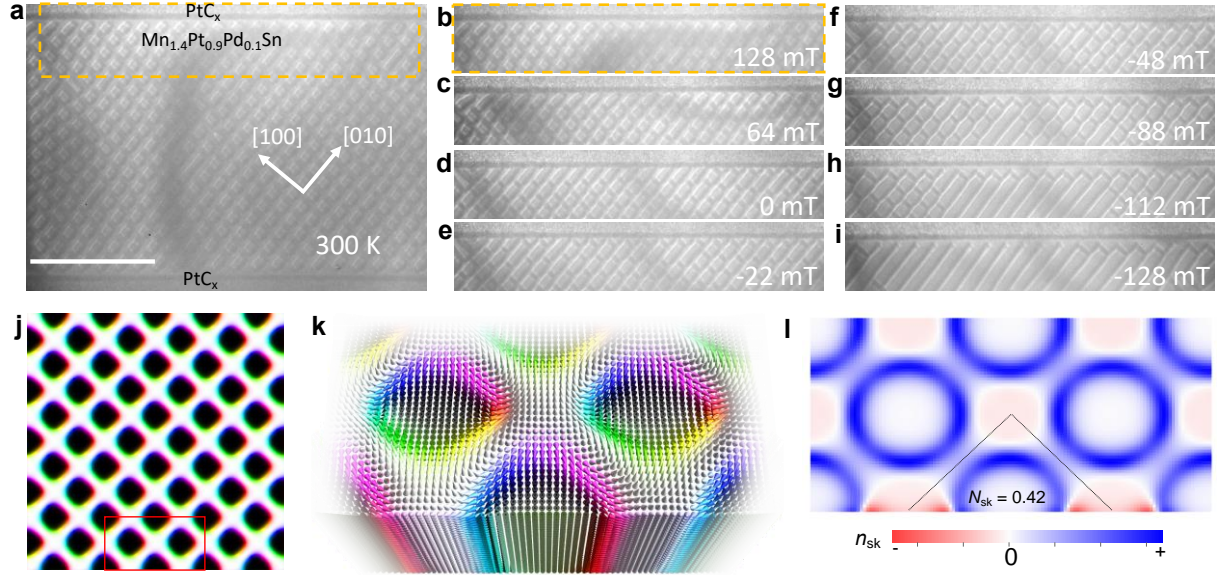


Figure 1. Formation of fractional antiskyrmions at an interface of $\text{Mn}_{1.4}\text{Pt}_{0.9}\text{Pd}_{0.1}\text{Sn}$. (a) The lamella is filled with a dense lattice of antiskyrmions and a few short helices at 128 mT and 300 K. The region marked by the dashed orange rectangle in (a) is shown for various magnetic fields in (b-i). Note that (b) is the same image as in (a). (c) Image in a field of 64 mT. (d) Fractional antiskyrmions that form at the interface region with PtCx are elongated in the absence of a magnetic field. (e-f) When a negative magnetic field is applied, the fractional antiskyrmions become smaller in size. (h, i) At fields beyond -100 mT, the antiskyrmions in the interior region first join with each other and form helices and then these helices join with the fractional objects so that a helical state is also found at the interface. The scale bar in (a) corresponds to 2 μm . (j) Simulated antiskyrmion lattice with fractional antiskyrmions at the edge. The color code corresponds to the orientation of the magnetic moments. (k) A magnified view of the red area in (j) with arrows indicating the orientations of the magnetic moments. (l) The topological charge density of the spin texture in (k).

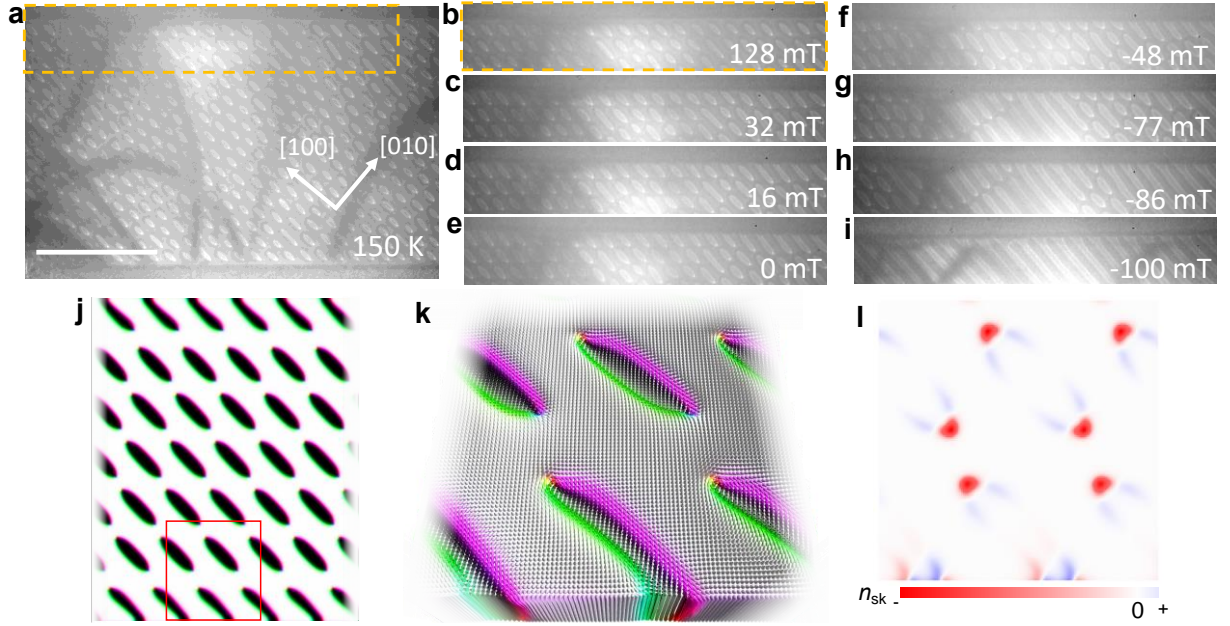


Figure 2. Formation of fractional elliptical Bloch skyrmions. (a) A lattice of elliptical Bloch skyrmions at 128 mT and 150 K. (b) The region identified by the dashed orange rectangle in (a) is shown in (b-i). (c) At 32 mT, fractional Bloch skyrmions start to form at the interface with PtC_x. (d, e) The number of fractional nano-objects increases when the elliptical Bloch skyrmions enlarge and join the interface. (f) In negative fields, the size of the elliptical skyrmions can decrease because the bulk elliptical skyrmions enlarge. (g, h) Upon further increasing the negative field, the fractional nano-objects start to merge with the bulk elliptical Bloch skyrmions and form helices. (i) A helical state is found everywhere including the interface region. The scale bar in (a) corresponds to 2 μm . (j) Simulated elliptical skyrmion lattice with fractional objects at the top and bottom edges. Periodic boundary conditions have been applied along the horizontal direction. The color code corresponds to the orientation of the magnetic moments. (k) A magnified view of the red area with arrows indicating the orientation of the magnetic moments. (l) The topological charge density of the spin texture in (k).

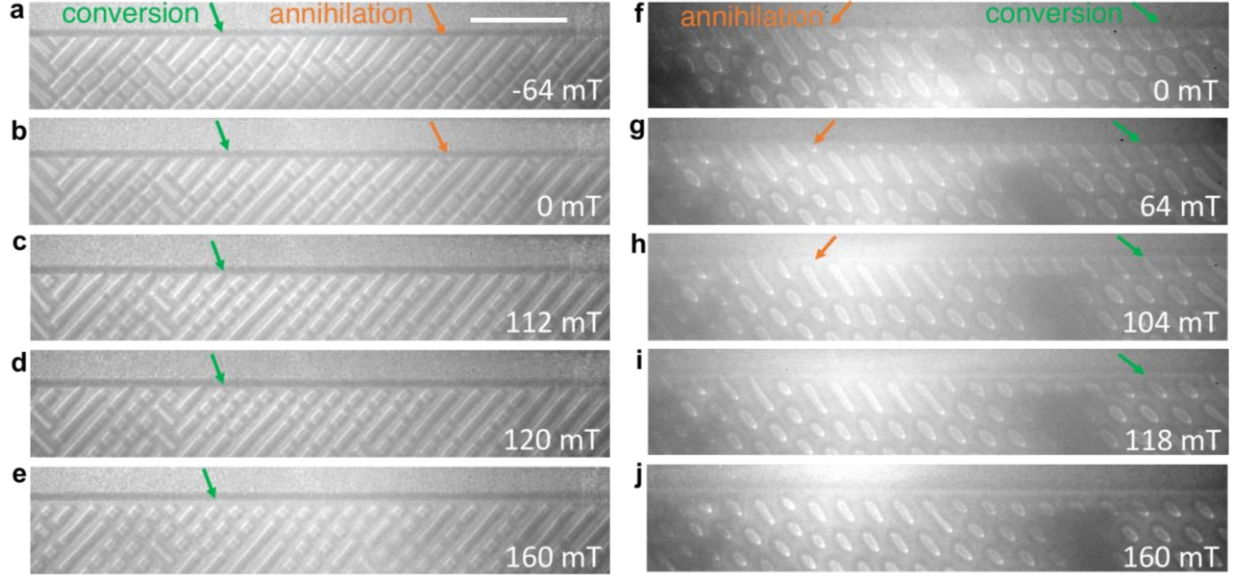


Figure 3. Conversion mechanisms of fractional nano-objects. (a) Fractional antiskyrmions of various sizes are observed at -64 mT and 300 K. The magnetic field is increased from this state and the corresponding LTEM images are shown in (b-e). (b) The fractional antiskyrmions that are shorter decrease in size. (c) At 112 mT, the short fractional nano-objects (e.g. marked by the orange arrow) vanish and the longer fractional nano-objects (e.g. marked by the green arrow) are pinched off at the edge. (d, e) Upon increasing the magnetic field, the longer fractional antiskyrmions have transformed to bulk antiskyrmions. No fractional antiskyrmions are present. Similar mechanisms are observed for fractional elliptical Bloch skyrmions at 150 K as illustrated in (f-j). The zero-field state in (f) was obtained by the procedure given in Fig. 2. Depending on their initial size, the fractional elliptical Bloch skyrmions can annihilate (orange arrow) or can form complete elliptical skyrmions in the bulk (green arrow). The scale bar in (a) corresponds to 1 μm .

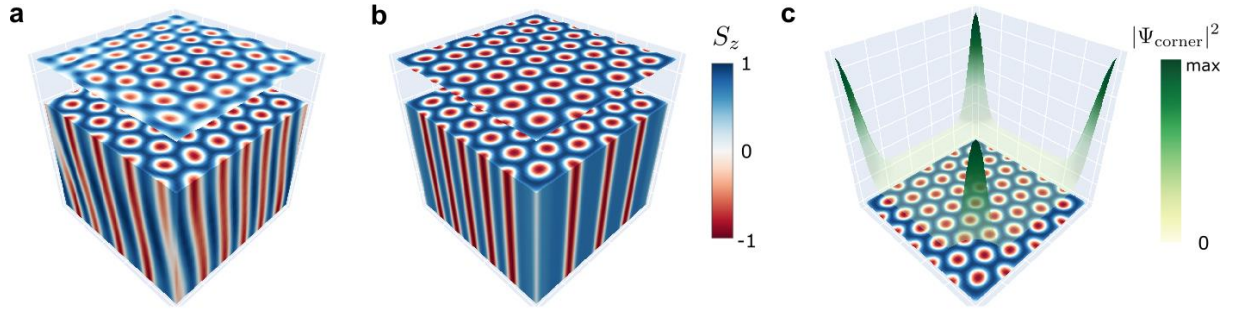


Figure 4. Three-dimensional profile of boundary fractional (anti)skyrmions and their role in the magnonic bulk-boundary correspondence. Three-dimensional magnetic configuration of confined skyrmion lattices obtained by atomistic spin Monte Carlo simulations with **(a)** bulk DMI, characteristic of B20 materials, and **(b)** with D_{2d} DMI, typical of Heuslers. The average of the out-of-plane spin components over the sample thickness is displayed atop the skyrmion lattices. Skyrmionic objects exhibit a vertical tubular structure in Heuslers allowing their clear observation via imaging techniques such as LTEM **(b)**. The DMI in B20 materials, however, favors tilted skyrmionic tubes resulting in blurry images along the sample edge **(a)**. This could explain why fractional skyrmions have not been detected in B20 materials by LTEM or similar imaging techniques. The dipolar interaction was not included. In the presence of the dipolar interaction, an equivalent result is obtained with a less pronounced tilting of the tubes. **(c)** Boundary fractional (anti)skyrmions restore symmetries of the skyrmion lattice that, via the bulk-boundary correspondence, guarantee the emergence and topological protection of magnonic corner states with probability density $|\Psi_{\text{corner}}|^2$.

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