A comprehensive numerical investigation of Carreau-Yasuda slime beneath complex bacterial wavy surface

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A comprehensive numerical investigation of Carreau-Yasuda slime beneath complex bacterial wavy surface

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Abstract:
Gliding motility often noticed in phylogenetically rod-shaped bacteria which locomote via dissipating their own energy. The said gliding motion via producing waves and secreting slime is generally witnessed in gram-negative bacteria. The influence of Inertia in slime layer beneath glider is also a significant feature to this mechanism. The propulsive microbe pushes the slime backwards, while reactive forces in the slime aids a forward movement of the propeller. Out of several motility modes the complex wavy gliding mechanism is considered here so one can approximate the glider’s surface by undulating two-dimensional sheet. Moreover, the non-Newtonian slime beneath the organism is taken as Carreau Yasuda fluid. Following a traditional approach of a fluid flow problem balance of mass and momentum is utilized. The x and y-component of momentum equation is combined and reduced into fourth order DE via lubrication and creeping flow assumption. For suitable values of rheological parameters, swimming gait and some initial values of gliding speed and flow rate, the BVP is solved via MATLAB build in routine bvp-5c. Modified Newton-Raphson algorithm is employed to simulate the unknowns present in the boundary conditions. Power required by the glider, velocity of the slime and level curves are also obtained with the aid of these realistic numerical pairs. The computed results are plotted in the latest available version of MATLAB (2021a) and discusses in detail.

Keywords: Carreau Yasuda model, gliding bacteria, complex wavy sheet, modified Newton-Raphson method.

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1. Introduction

Bacteria motile them self by utilizing various mechanisms namely, sliding, twitching, swarming, swimming and gliding. These unexplained motions are common in bacteria and have long piqued the interest of scientists. Certain bacteria have evolved a variety of cell
movement without the need of flagella. However, it was still unknown how bacteria motile without flagella and what machinery is involved in cell propulsion.

The mode of self-propulsion near a solid boundary which does not entail flagella, cilia, or pili generally refer as gliding motility. The said locomotion is significant to bacterial growing cycle as it affects the proliferating of nutrition when they are starved. The mysterious mechanism of motility fluctuates significantly from species-to-species. It was found that nearly all gliding bacteria create a trail of sticky liquid or slime as they travel across a surface by [1]. The surface layer is composed of glycoprotein, while slime layer consists of glycolipids, exopolysaccharides and glycoproteins. S-layer is crucial for increasing bacterial capacity to interact with macrophages, maintaining bacterial shape, molecular sieving, and enabling bacterial attachment to host molecules [2]. The four physical factors that contribute to prokaryotes gliding at the cellular level are slime secretion, surface-tension gradients, travelling waves and osmotic forces. For different gliding species, there are great reviews that detail each of these mechanism [3, 4, 5]. Few bacteria transport via spinning filaments that extend from the cell surface into the surrounding medium [6, 7]. Few studies related to Bacterial locomotion via rotatory flagella can be found in ref. [8, 9].

The physical principle behind gliding is still a mystery. Nan and Zusman [10] published an outstanding review study on the gliding processes for myxobacteria, flavobacteria, and mycoplasmas. They examined two possible gliding processes for myxobacteria, the helical rotor model and the retrograde surface wave model. The dynamics behind bacterial gliding locomotion on soft substrates has been unveiled by Tchoufag et al. [11]. They studied the mechanics of gliding locomotion and developed a mathematical model that predicts a two-regime organism speed. The interaction of physical forces to produce an active self-propulsion and effects of soft substrate were also explored by Tchoufag et al. [11]. On the other hand, rheology of slime present between organism and surface is another important factor to explore.

Several researchers [12-17] have examined slime as a non-Newtonian fluid, namely third grade, Johnson-Segalman, power law, Careau model and micropolar models. Asghar et al. [18] presented an analysis of bacterial gliding motion with three different fluid models (SPTT, Rabinowitsch and FENE-P models). They used perturbative technique to compute cell speed which is accurate aimed at smaller values of rheological parameters and gliding gait. Recently, an implicit FDM is implemented by Ali et al. [19] to explore the dynamics of bacteria on an inclined substrate. They approximated slime layer by constitutive equations of Carreau and Bingham fluid. To the best of our knowledge Careau Yasuda fluid model is not yet integrated.
with undulating surface model to study the bacterial gliding mechanism. Therefore, we employ the constitutive equations of Careau Yasuda model to capture the slime rheology. Several authors have already utilized Careau Yasuda fluid model in various flow configurations [20–26]. There are five controlling parameters in Careau Yasuda fluid which are infinity and zero shear-rate viscosity, Carreau-Yasuda fluid parameter, time constant and Power law index.

The said analysis has underlined the diversity of fluid organism interaction with relatively complex rheological models. The importance of motile microbe applications in several disciplines is also a driving force behind this study. Some research in the field of microbe swimming is also worth to mention. Alouges et al. [27] utilized numerical simulations to show the usage of Riemannian geometry for complex swimmers. A generic technique to determine the maximum efficiency of swimming organism at low Reynolds number is also expounded by Alouges et al. [28]. Further, Maso et al. [29] discuss the mathematical features and constraints of self-propulsion mechanism. Guasto et al. [30], investigate the effect of fluid dynamic environment on bacterial adeptness and serving. Lauga. [31] studied the biomechanics of microorganism self-propulsion and apprehend future encounters. Recently, Dufrene and Persat [32] discussed how biophysics and microbiology can be integrated to improve our knowledge towards microbe mechanobiology. They addressed the utility of pressures to improve grip and motility of specialized cellular apparatus which govern a variety of phenotypes.

The current work is assembled in a following way: Section 2 describes the problem's geometry along with governing equations of Careau Yasuda fluid. Section 3 describes the modelling of the gliding problem under the lubrication notion. Section 4 contains formulae of forces generated. Section 5 discusses solution method. Section 6 depicts and explains the mechanical impacts of slime rheology via graphics. Section 7 outlines the key findings.

2. Geometry and governing equations

In this paper, a mathematical analysis of two-dimensional undulating sheet approximating the glider surface is explored. The organism is assumed to secrete slime on a solid substrate. We assume a Cartesian coordinate system (X, Y) where X-axis is along the surface parallel/beneath the glider and the Y-axis perpendicular to the surface. Further assume that $U$ and $V$ are velocity components along X- and Y- directions respectively. The undulating sheet model of the bacterial gliding phenomenon is shown in Fig. 1. Wave trail (with speed $c$) in the bacterial
surface pushes the fluid backward which grades forward movement of glider (with speed $V_g$). Thus, relative speed of the glider is $(V_g - \dot{c})$.

The mathematical form of the shape of undulating sheet can be written as:

$$h(X,t) = h_0 + b_1 \sin \left( \frac{2\pi}{\hat{\lambda}} (X - (\dot{c} - V_g)t) \right) + b_2 \sin \left( \frac{4\pi}{\hat{\lambda}} (X - (\dot{c} - V_g)t) \right),$$  \hspace{1cm} (1)

here $h_0$ is mean distance between undulating sheet to the solid substrate, $\hat{\lambda}$ is the wavelength, $t$ is the time while $b_1$ and $b_2$ are the amplitude.

The BC’s in the lab frame are:

$$U = 0 \text{ at } Y = 0, \quad U = -V_g \text{ at } Y = h.$$  \hspace{1cm} (2)

To transform the flow phenomena from lab frame into wave frame we utilize Galilean transformations which are given as follows:

$$X = x + (\dot{c} - V_g)t, \quad Y = y,$$

$$U = \dot{u}_1 + (\dot{c} - V_g), \quad V = \dot{u}_2$$  \hspace{1cm} (3)

where $\dot{u}_1, \dot{u}_2$ are the velocity components along the $x$- and $y$- direction respectively.

The conservation of mass and momentum in mathematical form are as follows:
\[ \nabla \cdot \mathbf{\dot{u}} = 0, \quad (4) \]
\[ \rho \frac{d\mathbf{\dot{u}}}{dt} = \nabla p + \nabla \cdot \mathbf{S}, \quad (5) \]

where \( \mathbf{\dot{u}} \) is velocity, \( \rho \) is density, \( d/dt \) is the material derivative, \( p \) is pressure and \( \mathbf{S} \) is extra stress tensor. The value of \( \mathbf{S} \) for Carreau-Yasuda model is given by:

\[ \mathbf{S} = \left( \mu_x + (\mu_0 - \mu_x) \left[ 1 + (\Omega \zeta)^n \right] \right) \mathbf{D}. \quad (6) \]

where \( \mu_x, \mu_0, a, \zeta, n \) is the infinity and zero shear-rate viscosity, Carreau-Yasuda fluid parameter, time constant, Power law index and \( \mathbf{D} = \frac{1}{2} \left[ (\text{grad } \mathbf{\dot{u}}) + (\text{grad } \mathbf{\dot{u}})^T \right] \) with magnitude \( \Omega = \sqrt{2nD^2} \).

3. Problem formulation

The component form of Eqs. (4) and (5) are given as follows:

\[ \frac{\partial \mathbf{\dot{u}}_1}{\partial x} + \frac{\partial \mathbf{\dot{u}}_2}{\partial y} = 0, \quad (7) \]
\[ \frac{\partial p}{\partial x} + \rho \left( \mathbf{\dot{u}}_1 \frac{\partial}{\partial x} + \mathbf{\dot{u}}_2 \frac{\partial}{\partial y} \right) \mathbf{\dot{u}}_1 = \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{xy}}{\partial y}, \quad (8) \]
\[ \frac{\partial p}{\partial y} + \rho \left( \mathbf{\dot{u}}_1 \frac{\partial}{\partial x} + \mathbf{\dot{u}}_2 \frac{\partial}{\partial y} \right) \mathbf{\dot{u}}_2 = \frac{\partial S_{xy}}{\partial x} + \frac{\partial S_{xy}}{\partial y}, \quad (9) \]

where

\[ S_{xx} = 2 \left( \mu_x + (\mu_0 - \mu_x) \left[ 1 + \zeta^a \Omega^a \right] \right) \left( \frac{\partial \mathbf{\dot{u}}_2}{\partial x} \right), \quad (10) \]
\[ S_{xy} = \left( \mu_x + (\mu_0 - \mu_x) \left[ 1 + \zeta^a \Omega^a \right] \right) \left( \frac{\partial \mathbf{\dot{u}}_1}{\partial x} + \frac{\partial \mathbf{\dot{u}}_2}{\partial y} \right), \quad (11) \]
\[ S_{yy} = 2 \left( \mu_x + (\mu_0 - \mu_x) \left[ 1 + \zeta^a \Omega^a \right] \right) \left( \frac{\partial \mathbf{\dot{u}}_2}{\partial y} \right), \quad (12) \]
where
\[ \Omega = \sqrt{\left( \frac{\partial \hat{u}_1}{\partial x} \right)^2 + \left( \frac{\partial \hat{u}_2}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial \hat{u}_1}{\partial y} + \frac{\partial \hat{u}_2}{\partial x} \right)^2} \]  \hfill (13)

Using Eq. (3) in Eqs. (7)-(12) and then introducing the following dimensionless variables:
\[ \hat{h} = \frac{h(x)}{h_0}, \quad \hat{u} = \frac{\hat{u}_1}{\hat{c}}, \quad x' = \frac{2\pi}{\lambda} x, \quad \hat{u}_2 = \frac{\hat{u}_2}{\delta \hat{c}}, \quad y' = \frac{y}{h_0}, \quad \Omega' = \frac{h_0}{\hat{c}} \Omega, \]
\[ \delta = \frac{2\pi h_0}{\lambda}, \quad Re = \frac{\rho \hat{c} h_0}{\mu_0}, \quad p^* = \frac{2\pi h_0^2}{\lambda \mu_0 \hat{c}}, \quad We = \frac{\zeta \hat{c}}{h_0}, \quad \beta = \frac{\mu_e}{\mu_0}, \]
\[ \hat{u}_1 = \frac{\partial \psi}{\partial y'}, \quad \hat{u}_2 = -\frac{\partial \psi}{\partial x'}. \]  \hfill (14)

Eq. (8) and (9) takes the following form:
\[ \frac{\partial p}{\partial x} + \delta Re \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} \right) \left( \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial y} \right) \right] = \delta \frac{\partial}{\partial x} S_{xy} + \frac{\partial}{\partial y} S_{xy}, \]  \hfill (15)
\[ \frac{\partial p}{\partial y} - \delta Re \left[ \left( \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} \right) \left( \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial x} \right) \right] = \delta \frac{\partial}{\partial x} S_{xy} + \delta \frac{\partial}{\partial y} S_{xy}, \]  \hfill (16)

where Eq. (10)-(13) are given as:
\[ S_{xx} = 2\delta \left( \beta + (1-\beta) \left[ 1 + We^a \Omega^a \right]^{\frac{n-1}{a}} \right) \left( \frac{\partial^2 \psi}{\partial x \partial y} \right), \]  \hfill (17)
\[ S_{xy} = \left( \beta + (1-\beta) \left[ 1 + We^a \Omega^a \right]^{\frac{n-1}{a}} \right) \left( \frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right), \]  \hfill (18)
\[ S_{yy} = -2\delta^2 \left( \beta + (1-\beta) \left[ 1 + We^a \Omega^a \right]^{\frac{n-1}{a}} \right) \left( \frac{\partial^2 \psi}{\partial x \partial y} \right), \]  \hfill (19)
\[ \Omega = \sqrt{\left[ \frac{\partial^2 \psi}{\partial x \partial y} \right]^2 + \frac{1}{2} \left( \frac{\partial^2 \psi}{\partial y^2} - \delta^2 \frac{\partial^2 \psi}{\partial x^2} \right)^2}. \]  \hfill (20)
For convenience superscript * has been removed from the above equations. The dimensionless expressions of Weissenberg number \((We)\), Reynolds number \((Re)\), ratio of infinite to zero shear rate viscosity \((\beta)\) and wave number \((\delta)\) are already defined in Eq. (14).

The present work is based on creeping flow \((Re \ll 1)\) and long wavelength \((\delta \ll 1)\) assumption [12-21]. Employing this approximation on Eqs. (15)-(20) and replacing the reduced value of \(S_{xy}\) in Eq. (15), one finally arrives at the following fourth order DE:

\[
\frac{\partial^2}{\partial y^2} \left[ \beta \left( 1 - \beta \right) \left( 1 + We^a \left( \frac{\partial^2 \psi}{\partial y^2} \right)^a \right)^{n-1} \left( \frac{\partial^2 \psi}{\partial y^2} \right) \right] = 0, \tag{21}
\]

with pressure gradient (from Eq. (15)):

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left[ \beta \left( 1 - \beta \right) \left( 1 + We^a \left( \frac{\partial^2 \psi}{\partial y^2} \right)^a \right)^{n-1} \left( \frac{\partial^2 \psi}{\partial y^2} \right) \right]. \tag{22}
\]

A similar procedure (via utilizing Galilean transformations/ dimensionless variable and stream function) is adopted to reduce the BC’s.

BC’s corresponding to Eq. (21) are as follows:

\[
\psi = 0, \quad \frac{\partial \psi}{\partial y} = V_b \quad \text{at} \quad y = 0,
\]

\[
\psi = Q, \quad \frac{\partial \psi}{\partial y} = -1 \quad \text{at} \quad y = h = 1 + \phi_1 \sin x + \phi_2 \sin 2x, \tag{23}
\]

here \(Q = \Theta + V_b\) is the flow rate in wave frame, \(V_b \left( = \frac{V_s}{C} - 1 \right)\) is bacterial speed and \(\phi_1 \left( = \frac{b_1}{h_0} \right)\) is wave amplitude.

**4. Required forces and energy dissipation**

The forces \(F_x\) and \(F_y\) produced by the glider on the horizontal and vertical surfaces are define by [12-19]:
The set of components of stress vector \( \tau \) are defined as follows:

\[
\tau = \begin{pmatrix} \tau_x \\ \tau_y \end{pmatrix} = \begin{pmatrix} \frac{\partial h}{\partial x} + S_{xy} \\ -\frac{1}{\delta} p + S_{yy} \end{pmatrix}.
\] (25)

Using Eq. (25) in Eq. (24), we get:

\[
F_x = \int_0^{2\pi} \left( -h \frac{\partial p}{\partial x} + S_{xy} \right) dx = \int_0^{2\pi} \left( p \frac{\partial h}{\partial x} + S_{xy} \right) dx,
\] (26)

\[F_y = 0.\] (27)

The power dissipation in non-dimensional form is given by:

\[
P = -\int_0^{2\pi} p \frac{dh}{dx} dx = \int_0^{2\pi} \frac{dp}{dx}.\] (28)

For smooth gliding motion over a solid substrate \( F_x, F_y \) and \( \Delta p_x \) must be zero. This fact will lead to the following equilibrium conditions:

\[
\Delta p_x (Q,V_b) = \int_0^{2\pi} \frac{dp}{dx} (Q,V_b;x) dx = 0,
\] (29)

\[F_x (Q,V_b) = \int_0^{2\pi} \left( -\frac{h}{x} \frac{dp}{dx} (Q,V_b;x) + S_{xy} (Q,V_b;x) \right)_{y=-h} dx = 0.\] (30)

5. Problem solving methodology

The governing equation (Eq. (21)) of complex rheological slime is highly nonlinear and one faces exertion to obtain the solution (valid for large values of parameters) via an analytical approach. The ultimate goal is to calculate the suitable values of bacterial speed and flow rate which satisfy the dynamic equilibrium conditions. To fulfill the said purpose the BVP is
numerically solved for some fixed pairs of $\phi, n, \beta, a$ and $We$ with an appropriate initial guess of $Q$ and $V_b$. The solution of BVP (stream function) is then tested via Eqs. (29) and (30). It is obvious that the crude guesses of $Q$ and $V_b$ are not well refined to satisfy the equilibrium conditions. To tackle this fact, we need a root finding algorithm to modify $Q$ and $V_b$ which satisfies Eqs. (29) and (30). In the present work we utilize fifth-order method (MATLAB build in solver bvp5c) to solve BVP (Eq. (21) and (23)). This algorithm is a finite difference code that employs four-stage Lobatto IIIa formula. After achieving the convergent solution of stream function, $Q$ and $V_b$ are iterated via modified Newton Raphson technique.

6. Results and discussion

The numerical procedure is implemented in the latest version of MATLAB (MATLAB R2021a). In Figs. (2)-(6), the computed results of cell speed, flow rate, energy expenditure, slime velocity and level curves are plotted.

6.1. Bacterial speed, flow rate and energy expenditure

Fig. (2) illustrates a comparison of the bacterial gliding speed, slime flow rate, and power consumption in simple and complex wavy sheets for three different Carreau Yasuda parameter i.e. $a = 2$, $a = 4$ and $a = 6$ against $We$. Keeping $\phi_1$ and $\beta$ fixed, Figs. 2 (a), (c) and (e) are plotted for simple wavy sheet, where shear thinning and shear thickening cases are explored in these figures. While on the other hand by fixing $\phi_1, \phi_2$ and $\beta$, Figs. 2 (b), (d) and (f) are plotted for complex wavy sheet, by investigated shear thinning and shear thickening cases. The results of Carreau fluid model can be retained for $a = 2$. For shear-thinning case ($n < 1$), it is depicted in Fig. 2(a) (simple wavy sheet) that organism reaches its maximum speed near $We = 1$, while on the other hand it is observed in Fig. 2(b) (complex wavy sheet) that maximum speed is achieved by the glider for $We < 1$ in shear-thinning scenario. Further increase in Weissenberg number tends to slow down the glider in both panel. It is also observed in simple wavy sheet that Carreau slime aids the organism to glide faster as compared to Carreau Yasuda slime for comparatively smaller values of Weissenberg number ($We < 2.5$). Though for large values of $We$, Carreau Yasuda slime (say $a = 6$) is not suitable for efficient self-propulsion. While in complex sheet it is found that Carreau Yasuda slime (say $a = 6$) as compared to carreau slime is favorable scenario for bacterial gliding. An opposite trend of cell speed is expounded in both panel (a) and (b) for shear-thickening scenario ($n > 1$). It can be seen in simple and complex
wavy sheets that bacterial speed reduces, achieve minimum value then increases back to approach an asymptotic value. Impact of Carreau Yasuda parameter is also opposite in shear-thickening case. In simple sinusoidal sheet Carreau Yasuda slime with larger values of We is favorable physical condition for gliding mechanism while on the other hand in complex wavy sheet for \((a < 6)\) is suitable scenario for cell’s speed. It is also investigated from both cases that gliding speed of the bacteria as a function of emerging parameters is raised in complex sheet as compared to simple sheet. For simple and complex sheets flow rate of the slime as a function of power law index \((n)\), Weissenberg number \((We)\) and Carreau Yasuda parameter \((a)\) is plotted in Fig. 2(c) and (d). For shear-thickening slime the magnitude of \(Q\) is a decreasing function of \(We\) and \(a\), in panel (c) and (d), while an opposite behavior of flow rate is witnessed for shear-thinning slime. Similarly, for simple and complex sheet Variation of power dissipation against Weissenberg number, power law index and Carreau Yasuda parameter is displayed in Fig. 2(e) and (f). It is observed that Bacteria deprives less power in case of large values of Weissenberg number and shear-thinning slime. The impact of Carreau Yasuda parameter on power dissipation is negligible in Fig. 2(e) while on the other hand opposite trend is witnessed in Fig. 2(f). Moreover, the comparison of shear thinning and shear thickening effects of the simple and complex wavy sheets has also been shown through table and bar chart.
Fig. 2. (a)-(f) Gliding speed, slime’s flow rate and power expended by the glider as a function of $n$, $\beta$, $\text{We}$ and $a$. 
<table>
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<tr>
<th><strong>Rheological Conditions</strong> (a=4)</th>
<th>Weissenberg number</th>
<th>$\beta$ Viscosity ratio</th>
<th><strong>Simple Wave</strong> ($\phi_1 = 0.3$)</th>
<th></th>
<th><strong>Complex Wave</strong> ($\phi_1 = 0.2, \phi_2 = 0.3$)</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$Vg /c$</td>
<td>$Q$</td>
<td>$P$</td>
<td>$Vg /c$</td>
</tr>
<tr>
<td><strong>Shear-thinning</strong> ($n&lt;1$)</td>
<td>0</td>
<td>0.3</td>
<td>0.2288</td>
<td>-0.7712</td>
<td>3.014</td>
<td>0.3776</td>
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<td>3</td>
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<td>0.3866</td>
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</table>

**Table. 1:** Comparison of Rheological conditions in simple and complex wavy sheet on Carreau Yasuda slime.
Fig. 3.(a)-(c) Comparison of simple and complex wavy sheet through bar graph of gliding speed, slime’s flow rate and work done by the glider.

6.2. Streamlines and slime velocity

Figs. (4)-(7) depicts the velocity profiles and streamline patterns of Carreau Yasuda fluid between the undulating sheet and solid substrate. Four sets of figures are plotted against the rheological parameters $We$, $\beta$, $\phi$ and $n$ and the variables $Vg/c$ and $Q$ where fig.4 and fig. (5) is the comparison of streamlines and velocity profiles between the simple sinusoidal sheet and complex wavy sheet while rest of the figures (velocity with streamlines) i.e., (6) and (7) are displayed only for complex wavy. The velocity profiles are plotted at seven distinct positions (shown by thin horizontal blue lines), i.e., $x = -6, -4, -2, 0, 2, 4$ and $6$ below the undulating sheet. The streamlines near the substrate are usually straight lines, while in the middle region
few zones are generated (depending upon the rheological parameters and gliding gait). The streamlines near the gilder adopted its undulating shape. However, slime velocity at the organism surface is fixed (i.e. $u_1 = -1$ at $y = h$) and at the substrate it depends upon the gliding speed (i.e. $u_i = 1 + V_g / c$ at $y = 0$).

Fig. 4(a)-(f) shows the result of level curves for three different values of $We$ i.e., $= 0, 3$ and $5$ and for fixed values of $\phi, \beta, \alpha$ and $\eta$. It can be seen in simple sheet i.e., panel (a), (c) and (e) that increasing the value of $We$ enhances circulating zones in the middle region. While on the other hand in panel (b), (d) and (f) (which is plotted for complex sheet) that as the value of $We$ increased circulating zone reduces.

Fig. 5 shows the velocity profiles of Carreau Yasuda fluid between the undulating sheet and solid surface. Panel (a), (c) and (e) are displayed for simple sheet while rest of the panel i.e., (b), (d) and (f) are sketched for complex wavy sheet in order to illustrate the effects of $We$ on their velocities profiles in term of other emerging parameters. It is founded in simple sheet that as the values of $We$ increases in velocity profiles the reverse flow trend decreases while on the other hand it can be noted in complex wavy sheet that reverse flow trend is occurred at point $x = 6$ which become slightly decrease when $We = 5$.

In Fig. 6 the panel (a), (c) and (e) shows the variation of fluid velocity while (b), (d) and (f) denote the streamlines pattern of Carreau Yasuda fluid between the complex wavy sheet and solid surface. Keeping $We, \beta, \alpha$ and $\phi$ fixed the flow rate and gliding speed is altered by varying power law index $n$ (i.e., $n = 0.5, 1$ and $1.5$) as shown in Fig. 5. It is founded that when rheology fluctuates from shear-thickening to shear-thinning slime size of trapped zone reduces while a very small variation in fluid velocity is observed.

In impact of different values of Careau Yasuda parameter ($\alpha$) on slime velocity and level curves are expounded in Fig. 7. A better slime flow is developed, and size of trapped circulating zones enhances due to larger values of $\alpha$. 
Fig. 4. (a)-(f) Streamlines comparison of simple and complex wavy locomotion on Careau Yasuda slime ($a = 4$) by varying Weissenberg number ($We$) and for different pairs of $\beta$, $n$, $V_g/c$ and $Q$. 
Fig. 5. (a)-(f) velocity comparison of simple and complex wavy locomotion on Careau Yasuda slime ($a = 4$) by varying Weissenberg number ($We$) and for different pairs of $n, \beta, Vg/c$ and $Q$. 
Fig. 6. (a)-(f) velocities and streamlines comparison of complex wavy locomotion on Careau Yasuda slime ($a = 6$) by varying power law index i.e., $n$ and for different pairs of $We$, $Vg/c$, and $Q$. 
Fig. 7. (a)-(f) velocities and streamlines comparison of complex wavy locomotion on Careau Yasuda slime by varying the Careau Yasuda parameter i.e. $a$ and for different pairs of $We$, $n$, $\beta$, $Vg/c$ and $Q$. 
7. Conclusion

The hydrodynamics of gliding bacteria in the form of simple and complex wavy sheet combined with Carreau Yasuda fluid model is presented. The gliding problem is reduced into fourth order BVP under lubrication and creeping flow assumption. A hybrid numerical technique (bvp-5c integrated with modified Newton Raphson) is employed in MATLAB 2021a. The glider's speed, slime flow rate, and power required for propulsion near the solid wall is computed for the simple sinusoidal and complex wavy sheet. For various rheological conditions and wave amplitudes, cell speed, flow rate, work done, streamlines and slime velocity are also plotted and discussed briefly. Few key points are as follows:

- Shear-thickening slime with high values of $We$ and shear-thinning slime with low values of $We$ are favorable scenario in the simple and complex wavy sheets.
- Gliding speed of the organism in simple wavy sheet is slower as compared to complex wavy sheet under the influence of emerging parameter.
- Bacterial speed and energy expended is an increasing function of occlusion parameter.
- As compared to Newtonian slime Carreau Yasuda slime is more versatile to alter gliding mechanism with five controlling parameters.
- The impact of organism wave on gliding mechanism is dominant as compared to rheological parameter.
- An efficient glider pushes the slime with greater speed and alters the streamlines.

Compliance with ethical standards

Conflict of interests: The authors declare that there is no conflict of interests regarding the publication of this article.

Ethical standard: The authors state that this research complies with ethical standards. This research does not involve either human participants or animals.

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References


