Optimization of empty container allocation for inland freight stations considering stochastic demand

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Research Article

Keywords: empty container allocation, stochastic demand, empty container management, empty container leasing, differential evolution

Posted Date: February 9th, 2022

DOI: https://doi.org/10.21203/rs.3.rs-1338856/v1

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Optimization of empty container allocation for inland freight stations considering stochastic demand

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Research Highlights

- A novel model for container inventory management is developed.
- The probability distribution function of stochastic empty container demand is not necessary in our model.
- A resource allocation approach for empty containers, namely the largest-debt-first policy, is proposed.
- A binary search and differential evolution algorithm is designed to solve the problem.
- Numerical experiments demonstrate the effectiveness and efficiency of the proposed method.
Optimization of empty container allocation for inland freight stations considering stochastic demand

To meet the fluctuating demand for empty containers at inland freight stations, liner companies often adopt the method of a combination of self-owned and leased empty containers. Therefore, estimating the amount of self-owned and leased empty containers that need to be allocated to each inland freight station in a specific area becomes a critical issue for liner companies. However, owing to the high degree of unpredictability of the demand and the limited flexibility of empty container relocation, the abovementioned issue has not been fully addressed. This paper provides a model for empty container allocation without knowing the probability distribution function of empty container demand in advance. The abovementioned model can jointly optimize the quantities of self-owned empty containers and leased containers allocated to each inland freight station. To solve the abovementioned model, a largest-debt-first policy is adopted to simplify the complicated model, and a differential evolutionary algorithm is developed to solve the simplified model. Numerical experiments are conducted to validate the model. In comparison to conventional way such as overstock, our method significantly reduces the self-owned empty container storage level, improves the ability of liner companies to deal with fluctuations in empty container demand, and offers necessary decision support for liner companies’ daily container management.

Keywords: empty container allocation; stochastic demand; empty container management; empty container leasing; differential evolution

1. Introduction

As carriers of transportation objects, liner companies are always concerned with the management of empty containers. Scientifically carrying out the inventory management and transfer of empty containers is the key to the liner company’s profitability. Motivated by this realistic management background, our paper focuses on an inventory and transport problem in the Inland Empty container storage and Transhipment System (IETS). An IETS (Figure 1) is typically developed and operated by a liner company and consists of a container yard near a port and multiple inland freight stations. The role of a container yard is to store and tranship
empty containers. Inland freight stations are used to provide containerizing and unpacking services to shippers. IETS can be classified into two groups based on their functional characteristics: output type and input type. The former category includes IETS required to export empty containers for an extended period of time, such as those in North America or Western Europe, while the latter category includes IETS required to import empty containers for an extended period of time, such as those in China or Southeast Asia. In this paper, we focus on input type IETS to carry out specific research.

Figure 1 A typical inland empty container storage and transshipment system

One of the key features of input IETS is the necessity to have a certain number of empty containers on stock to deal with the uncertainty demand of empty containers. This storage is required to improve transit reliability, but it comes with a significant storage and administrative cost (Shintani, Konings, and Imai 2019; Griffiths, Connolly, and Brown 2020). As a result, determining the optimal quantity of empty containers to store in container yards has become a critical concern for liner companies. In practice, a solution to this problem has long been available. In a container yard, liner companies regularly overstock empty containers, which means that liner companies will store excess empty containers to fully meet demand. Although overstocking is a simple and straightforward process, it results
in increased expenditures due to excess storage. As a result, an increasing number of liner companies are partnering with container rental companies (or other liner companies, the same below). Their partnership spawns another operation strategy known as the combination of storage and renting (Wang et al. 2008; Yu and Li 2020; Braga et al. 2020). Under this strategy, liner companies can rent empty containers for temporary use in addition to storing and transporting their own empty containers (Yun, Lee, and Choi 2011b; Lin 2020), and rental companies deliver empty containers to demand nodes in line with liner company specifications.

Clearly, there are several advantages of this strategy, including assisting liner companies in reducing the stock level of empty containers, compressing empty container administrative and transportation costs, and enhancing liner companies’ ability to deal with fluctuations in empty container demand. However, it may also result in hefty rental bills. This compels liner companies to consider the following two operational decisions carefully and simultaneously: 1) How can the empty container stock level in the container yard be determined according to the fluctuation of demand? 2) How can the number of empty containers be determined to be leased? In this paper, we address this combined optimization problem of self-owned empty container storage and leased container utilization (ECIP).

ECIP is not straightforward and easy. There are two challenges inherent in the decision-making process. First, if the overstocking strategy is to be abandoned, a more precise estimate of empty container storage is necessary. However, random fluctuations in empty container demand have increased in recent years, which could be attributed to the increased frequency of noncyclical adjustment of container shipping networks, the increasing prevalence of shippers returning empty containers inter-regionally, and the rapid growth of intercontinental rail container transportation. It has proven challenging to quantify the stochastic nature of empty container demand using a definite probability distribution function.
In such circumstances, when the demand probability distribution function is virtually unknown, determining the appropriate amount of empty container storage is a difficult challenge. Second, the cost of transporting for self-owned containers and leased containers may vary greatly by area. For instance, an inland freight station may be closer to the rental company’s container yard than to the liner company’s container yard, resulting in a reduced operational cost when renting containers. This creates a mutually reinforcing and interdependent link between self-owned container volume and leased container volume. Due to the complexity of this connection, optimizing the empty container inventory becomes a difficult task. To the best of our knowledge, previous research has not been able to provide optimal answers to the abovementioned two challenges.

Therefore, in this paper, we propose a model for solving ECIP considering stochastic empty container demand based on inventory control theory to fill the abovementioned research gap. In the model, we allow liner companies to use leased empty containers at any time and fully consider the difference in transportation costs of different types of containers. Three major contributions are made by this paper.

(1) A novel model for container inventory management is developed. We loosen the requirement that the probability distribution function of empty container demand should be known in this model and do not employ the traditional overstock inventory optimization principle, allowing liner companies to utilize leased containers more frequently to reduce operational costs.

(2) A new resource allocation approach for empty containers is proposed, dubbed the largest-debt-first policy. On the one hand, this policy is straightforward and simple to apply for liner companies; on the other hand, it has the potential to cut liner companies’ operational expenses while still optimizing their use of self-owned and leased containers.
Numerical experiments demonstrate that our proposed method can accomplish collaborative optimization of self-owned container inventory control and leased container utilization when a variety of practical factors are taken into account.

The remainder of this paper is organized as follows. Section 2 reviews the literature related to IETS. Our ECIP is then described in two phases in Section 3. We begin by defining the operation mode of the inland empty container storage and transshipment system. Then, we describe the assumptions and notations used in ECIP. Section 4 develops a stochastic decision model for ECIP. Although this model is difficult to solve directly, it can be transformed equivalently by implementing the largest-debt-first policy. After the transformation, the new model can be solved using a heuristic algorithm. We present this algorithm in detail in Section 5. In Section 6, numerical experiments are conducted to evaluate the effectiveness of the model and algorithm. Finally, Section 7 summarizes the paper and makes several recommendations for further research.

2. Literature review

In essence, ECIP can be viewed as a problem combining inventory management with container transportation. According to existing research, this topic is also known as the empty container allocation problem or the empty container transportation problem. Representative studies can be found in Jula, Chassiakos, and Ioannou (2006), Shintani et al. (2007), Deidda et al. (2008) and Song and Dong (2015). These studies fall into two major categories: deterministic and stochastic.

In the context of deterministic studies, Olivo, Di Francesco, and Zuddas (2013) proposed a method to realize the reasonable allocation of self-owned empty containers between ports and inland freight stations. The authors predicated on the idea that demand for empty containers is predictable in advance. They not only address different modes of
transportation in the paper but also include aspects such as long- and short-term container leasing, as well as challenges associated with inland transit route design. Wang and Jing (2020) is a comparable study recently published to that of Olivo, Di Francesco, and Zuddas (2013). The distinction between the two is that Wang and Jing (2020) further took container transhipments into account. They assumed that empty containers could be delivered to hubs first and then dispersed to inland freight stations, which significantly reduces transportation costs by taking advantage of the economies of scale of centralized transportation. Similarly, Brouer, Pisinger, and Spoorendonk (2011), Meng, Wang, and Liu (2012), Vojdani, Lootz, and Rösner (2013), Bell et al. (2013), Ambrosino and Sciomachen (2014), and Yu, Fransoo, and Lee (2018) performed in-depth studies on ECIP in a port’s hinterland from a variety of viewpoints.

Meanwhile, other scholars do deterministic research on ECIP in conjunction with other aspects of shipping management (e.g., fleet deployment, shipping network design). For example, motivated by an industrial project, Monemi and Gelareh (2017) proposed an integrated modelling approach for the converging challenges of network design, fleet deployment, and empty repositioning in liner shipping. Jeong et al. (2018) designed a shipping line network considering empty container management. The authors believed that the assumption that container transportation demand is aggregated at ports is unrealistic in the context of bilateral trade routes between countries. Therefore, an empty container management strategy for container supply chains for bilateral trade was proposed. Based on accelerated particle swarm optimization and heuristic rules, a hybrid solution algorithm was designed. Very recently, Dong et al. (2020) addressed a composite problem of ECIP and marine fleet deployment in the context of roll-on/roll-off shipping. The authors created large-scale linear programming by converting ECIP to numerous inventory management constraints and solved it with Xpress. Since most of the abovementioned studies assume that
the transportation demand for empty containers is known and constant, the approaches provided are inapplicable to the ECIP addressed in this paper.

In recent decades, a growing number of scholars have focused on the unpredictability of empty container demand and attempted to address stochastic ECIP using a variety of theories or methodologies. For example, Song (2007) investigated a shuttle service system with unpredictable demand and limited repositioning capacity. Chou et al. (2010) considered the uncertainties of empty container demand, transportation costs, operational costs, and other factors when evaluating empty container relocation between several ports. They highlighted that the changing patterns of these factors are difficult to grasp and characterize. As a result, the authors applied fuzzy decision theory to conduct the uncertainty analysis of the problem, and a two-stage fuzzy optimization framework was developed. Specifically, the first stage involved developing a fuzzy decision model to assess the optimal empty container stock level of a port. In the second stage, the authors employed a classical transportation model to determine the ideal transportation scheme for self-owned empty containers between ports based on the stock level determined in the first stage. Very recently, Lu, Lee, and Lee (2020) investigated simultaneous pricing and ECIP decisions considering stochastic demand in two-depot shipping services. They solved the problem by developing a large-scale dynamic programming model. The authors analyzed several complicated elements in the model, including price, empty container storage, empty container leasing, and empty container transportation, and thoroughly addressed the model's solvability. Most of the literature makes use of a mathematical programming methodology. However, mathematical programming models are inevitably sophisticated and computationally intensive, making them difficult to execute in practice because their underlying logic is concealed from the liner company (Du and Hall 1997).
To remedy this shortcoming, a school of researchers has attempted to investigate stochastic ECIP using inventory control theory. This theory has the advantage of assisting liner companies in developing a straightforward policy for empty container management. Typically, such a policy includes an upper and lower bound for the empty container inventory. When an empty container yard’s inventory falls below the lower bound, empty containers should be shipped in. Conversely, when the inventory exceeds the upper restriction, empty containers must be transported out. As a result, this policy is typically referred to as an X-threshold policy. Addressing instance, Yun, Lee, and Choi (2011a) discussed the decision-making problem of a two-threshold policy in the hinterland of ports. Although the authors considered the time-varying characteristics of transportation demand, they did not present a quantitative optimization decision-making method but employed simulation for analysis. Li et al. (2004) proposed a two-threshold policy for the issue of single-port empty container shipping. Song and Carter (2008) developed a three-phase threshold policy for empty container repositioning. Zhang, Ng, and Cheng (2014) investigated the repositioning problem of empty containers in a stochastic and dynamic environment characterized by lost sales to determine an optimal two-threshold policy. From a mathematical standpoint, their approach is novel in that it was the first to incorporate stochasticity, dynamics, and lost sales into an optimal control problem. The authors validated the existence of the optimal policy and created a model of dynamic stochastic programming to find it.

The preceding studies conducted extensive studies on ECIP under stochastic demand from a variety of perspectives, yielding several significant conclusions. Most of these studies make the assumption that all demands must be met. If self-owned containers are not accessible, rental companies can always provide infinite empty containers to fulfill demand (e.g., Cheung and Chen (1998), Li et al. (2004), Li et al. (2007) and Song and Zhang (2010), Didenkulova (2020)). Therefore, the proposed approaches can be considered to be a
combination of storage and renting strategies. They do, however, share one assumption: they all presume the availability of the probability distribution function of empty container demand. As previously indicated, this assumption no longer meets the actual requirements for empty container management. Therefore, it is critical to deviate from this assumption and propose a new method for optimizing empty container management policies.

3. Problem description

To adequately explain the ECIP addressed in this paper, a more complete explanation of the input IETS operation mode is required (see Figure 2). A liner company is the decision maker of the system, and it can use both self-owned containers and leased containers to meet shippers’ demand. Among them, leased containers are provided by rental companies, while self-owned containers have two sources. One is steady supply, i.e., empty containers are carried into the container yard on a regular basis and in a predefined number by the liner company. The second source is containers that the consignee returns after usage. Because the return time and quantity of these empty containers are difficult to predict accurately, this supply source is referred to as random supply.

![Figure 2](image-url) The process of container export and import
When empty containers are required for an export operation (as illustrated in Figure 2(a)), the shipper must first make an application to the liner company for the reservation of shipping slots. The liner company then decides where to obtain empty containers based on available stock and related costs. If the liner company uses self-owned containers, then it is responsible for transporting the empty containers from the container yard to the inland freight station, as well as for the transportation costs. When leased containers are used, the liner company rents empty containers from the rental company, which carries them to the inland freight station. The procedure of container importation is depicted in Figure 2(b). After unloading from a ship, laden containers are delivered to the consignee’s designated location. The consignee then unloads the containers and delivers the empty containers to the liner company’s appointed destination. Finally, the liner company is responsible for the storage (if the container is self-owned) or return (if the container is leased) of empty containers and pays the associated costs. To facilitate the modelling, we introduce the following assumptions for the abovementioned operation mode (see Figure 3).

(1) Only inland freight stations are used to load containers. In other words, we consider inland freight stations to be demand nodes for empty containers in this paper. We assume that the demand for empty containers at each inland freight station is distributed independently and uniformly.

(2) Only the container yard has the capacity to keep empty containers. The container yard’s operational cost is a strictly monotonic function of the total number of empty containers stored in the yard.

(3) The whole demand for empty containers must be met. For each inland freight station, the liner company must determine the proportion (called the ‘utilization rate’) of self-owned empty containers to the total supply of empty containers. The utilization rate is also the basis for determining the number of leased containers.
(4) Although we do not assume knowledge of the probability distribution function of empty container demand, we do assume knowledge of its mathematical expectation and variance based on the demand samples.

(5) The liner company is responsible for transporting self-owned empty containers from the container yard to the freight station. Accordingly, the liner company is liable for the transportation costs connected with this. In addition, if self-owned empty containers are utilized, the consignee is responsible for returning the emptied containers to the container yard, including all the related transportation costs.

(6) Shippers can obtain leased empty containers from the freight station, and consignees must return the leased empty container to the freight station. The utilization of leased empty containers incurs no transportation costs; nonetheless, leasing fees are incurred, which are paid by the liner company.

(7) We presume that the probability distribution information for the random supply is known. For steady supply, we assume that the liner company can use this supply once in a time interval to replenish the empty containers in the container yard. Both the random and steady supply determine the periodically empty container stock of the container yard together.

(8) Rental container resources are unlimited, and rental container transit time is not considered.

Based on these settings and assumptions, the ECIP is described as follows. Under the premise that the empty container demand should be fully satisfied, the decision maker must optimize the following issues throughout a decision period to minimize the expected operational cost: the quantity of self-owned empty containers from the steady supply source and the utilization rate of self-owned empty containers at each inland freight station. The random supply probability information, the empty container leasing fee, and the freight rate
of empty container transportation are known, but the probability distribution function of random demand for empty containers is unknown.

![Diagram showing the flow of empty containers (Figure 3)]

**Figure 3** Suppliers and consumers of empty containers in the input IETS

For the convenience of the reader, the notations frequently used in this paper are listed in Table 1.

**Table 1 Parameters and Variables**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Number of inland freight stations</td>
</tr>
<tr>
<td>$T$</td>
<td>Length of observed period (number of weeks), $t \in {1, 2, \ldots, T}$</td>
</tr>
<tr>
<td>$i$</td>
<td>No. of inland freight stations, $i = 1, 2, \ldots, N$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Adjustment coefficient for the operational cost at the container yard</td>
</tr>
<tr>
<td>$X_i(t)$</td>
<td>Empty container demand at inland freight station $i$, the stochastic variable about $t \in {1, 2, \ldots, T}$</td>
</tr>
<tr>
<td>$Y(t)$</td>
<td>Random supply, the stochastic variable about $t \in {1, 2, \ldots, T}$</td>
</tr>
<tr>
<td>$C^s(\cdot)$</td>
<td>Function for calculating the storage operational cost at the container yard (RMB/TEU)</td>
</tr>
<tr>
<td>$C_i^t$</td>
<td>Transportation cost per TEU from container yard to inland freight station $i$ (RMB/TEU)</td>
</tr>
<tr>
<td>$C_i^r$</td>
<td>Rental fee per TEU at inland freight station $i$ (RMB/TEU)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>The amount of steady supply obtained at the container yard in each interval</td>
</tr>
<tr>
<td>$\beta = (\beta_i)$</td>
<td>The utilization rate of self-owned empty containers to the total supply at inland freight station $i$</td>
</tr>
</tbody>
</table>
The quantity of self-owned containers provided by the liner company at inland freight station $i$ in interval $t$, $t \in \{1, 2, \ldots, T\}$

The quantity of leased containers at inland freight station $i$ in interval $t$, $t \in \{1, 2, \ldots, T\}$

4. Model development

4.1. Proposal of the model

In this section, a nonlinear stochastic programming empty container restoration model (ECRM) is constructed with the objective of minimizing the sum of the expected storage operational cost of self-owned containers, the expected transportation cost of self-owned containers and the expected rental cost of leased containers. The model can realize the synchronous optimization of the steady supply quantity ($S$), the utilization rate ($\beta$), and the self-owned containers supplied to each inland freight station in each interval ($Q^o(t)$). The proposed ECRM is as follows.

$$
\begin{align*}
\min_{(S, \beta, Q^o(t))} \quad & a_1 = C_h(S + E[Y]) + \rho D[Y] + \sum_{i=1}^{N} \limsup_{T \to \infty} \left( \frac{\sum_{t=1}^{T} Q^o_i(t)}{T} C_i^o + \frac{\sum_{t=1}^{T} Q'_i(t)}{T} C'_i \right) \\
\text{s.t.} \quad & \liminf_{T \to \infty} \sum_{t=1}^{T} Q^o_i(t) \geq \beta_i \\
& a.s., \forall i = 1, \ldots, N \quad (2) \\
& Q^o_i(t) + Q'_i(t) = X_i(t) \quad \forall i = 1, \ldots, N, t = 1, \ldots, T \quad (3) \\
& \sum_{i=1}^{N} Q^o_i(t) \leq S + E[Y] \quad \forall t = 1, \ldots, T \quad (4) \\
& S \geq 0, Q^o_i(t) \geq 0, Q'_i(t) \geq 0 \quad \forall i = 1 \ldots N, t = 1, \ldots, T \quad (5) \\
& 0 \leq \beta_i \leq 1 \quad \forall i = 1, \ldots, N \quad (6)
\end{align*}
$$
Equation (1) is the objective function of the proposed ECRM. The first item is the expected value of the storage operational cost of the container yard, which is determined jointly by the expected values of steady supply $S$ and random supply $Y$. The second item represents the cost incurred by the liner company to cope with the fluctuation of random supply. In reality, the number of empty containers returned by the consignee is random. This characteristic requires the liner company to take special measures to handle the randomness (for example, reserve some containers and temporary rental containers for such situations). These measures increase the operational cost at the container yard. The resulting influence is characterized by $\rho$. The third item represents the expected sum of the transportation cost of self-owned containers and the rental cost of leased containers. Notably, since it is impossible to determine whether the limit exists at this time, we use the supremum in the function. The left part of inequality (2) represents the ratio of self-owned containers supplied by the liner company to the total empty container supply at inland freight station $i$. This ratio is almost surely not less than $\beta$. Equation (3) indicates that for any time interval $t \in \{1, 2, \ldots, T\}$, the empty container demand of inland freight station $i$ must be met. Equation (4) indicates that the total quantity of self-owned containers supplied by the liner company to all inland freight stations in any time interval shall not exceed the sum of steady supply and random supply obtained in that time interval.

The conventional way to handle stochastic variables in a stochastic programming model is to replace them with the mathematical expression of the expected value. For example, equation (3) can be converted to $\mathbf{E}[Q^o_i] + \mathbf{E}[Q^r_i] = \mathbf{E}[X_i] \quad \forall i = 1, \ldots, N$. However, this method is not applicable in the proposed ECRM. On the one hand, the stochastic decision variable $Q^o_i(t)$ is a function dependent on $S$ and $X_i(t)$, and the mathematical expression of $X_i$ is unknown at this time. As a result, obtaining the mathematical expression for $\mathbf{E}[Q^o_i]$ is
theoretically impossible. On the other hand, even if we obtain the expression of $E[Q^o_i]$, it is unreasonable to write $E[X_i]$ at the right end of the equation because IETS requires $Q^o(t) + Q'_i(t)$ to be equal to $X_i(t)$, not $E[X_i]$. Thus, if $X_i(t)$ is replaced with $E[X_i]$, a gap will exist between the actual empty container transportation volume and the real demand. This is different from the original intention of our research and the requirements of IETS operation. Therefore, how to describe and characterize $Q^o_i(t)$ becomes key to solving ECRM. We may circumvent this issue by learning from the research on X-threshold policy settings.

The common practice in research on X-threshold policy settings is to introduce a policy into the model to generate $Q^o_i(t)$. In established research, optimization models incorporating samples of random variables as decision variables are not uncommon. In such cases, it is standard practice in the academic community to incorporate a generation policy for $Q^o_i(t)$ into the model to convert the formation process of $Q^o_i(t)$ into a stochastic process connected to $X_i(t)$ and thereby simplify the model. In general, such a generation policy should meet the following two criteria: 1) convenience, i.e., the strategy should make it simple for decision makers to generate random variable samples; in other words, the logic for generating random variable samples should be simple and straightforward to apply in practice; 2) feasibility, i.e., the random variable samples generated by the strategy should satisfy the model's constraints. To create $Q^o_i(t)$ in this paper, we employ the largest-debt-first policy. This strategy enables us to convert the ECRM model to a more manageable stochastic programming model.
4.2. Largest-debt-first policy

The largest-debt-first policy allocates self-owned containers based on the gap between past demand for empty containers and historical replenishment. Specifically, we let the vector 
\[ \Delta(t) = (\Delta_i(t)), i = 1, \ldots, N \]
represent the cumulative shortage of self-owned containers at all inland freight stations in time interval \( t \), that is, the cumulative ‘debt’. Therein, \( \Delta_i(t) \) represents the shortage of self-owned containers of inland freight station \( i \) at time interval \( t \), and the calculation method is shown in formula (7). \( \Delta_i(t) \) equals the accumulation of the difference between the number of required self-owned empty containers (\( \beta_i E[X_i] \)) and the actual cumulative number of supplied containers (\( Q^o(s) \)) before time interval \( t \).

\[
\Delta_i(t) = \sum_{s=t}^{t-1} \left[ \beta_i E[X_i] - Q^o(s) \right] \quad \forall i = 1, \ldots, N \tag{7}
\]

Notably, when all \( \beta_i \) values are known, equation (7) depicts only the theoretical value of the cumulative shortage of inland freight station \( i \) from interval 1 to \( t-1 \). We assume that once a shortage occurs in an inland freight station, we will use leased containers to compensate for the shortage; therefore, there is no actual shortage at any inland freight station in any interval. According to the accumulated debt of each inland freight station in interval \( t \) (\( \Delta_i(t) \)), we determine the priority of empty container allocation of each inland freight station in descending order. An inland freight station with larger accumulated debt will have priority to obtain self-owned containers from the liner company. Specifically, we assume that the allocation order at time interval \( t \) (\( P(t) \)) is \([1]<[2]<\cdots<[N]\), where \([i]\) represents the supply order of inland freight station \( i \) at time interval \( t \). If \([1]<[2]\), the requirement of inland freight station 1 should be satisfied prior to that of inland freight station 2. Based on the above symbols, the method for allocating empty containers can be written as equation (8).
\[
\sum_{k=1}^{n} Q_{[k]}^n(t) = \min \left\{ S + E[Y], \sum_{k=1}^{n} X_{[k]}^n(t) \right\} \quad \forall n = 1, \ldots, N
\]  

(8)

According to Equation (8), empty containers available at the container yard are allocated to the inland freight station with the largest accumulated debt \( \Delta_i(t) \) at present. After the demands are satisfied, if there are still self-owned containers left, they will be allocated to the inland freight station with the second largest accumulated debt, and so on. In interval \( t \), if the liner company allocates all self-owned empty containers according to the above principles and the empty container demands of the inland freight stations are still not met, additional empty containers are leased. The implementation process can be summarized as follows.

**Step 1**: Set the time interval as \( t = 1 \), and sort all \( N \) inland freight stations in descending order according to the container demand at \( t = 1 \). Thus, we can obtain \( P(t) \).

**Step 2**: Based on \( P(t) \), the liner company determines the quantity of self-owned containers supplied to each inland freight station according to Equation (8), and the insufficient part is supplemented by leased containers.

**Step 3**: Set \( t = t + 1 \), update the accumulated debt values \( \Delta_i(t) \) according to Equation (7) and the available quantity of self-owned containers.

**Step 4**: Update \( P(t) \) based on the accumulated debt values of all the inland freight stations.

**Step 5**: Return to **Step 2** until \( t = T \).

4.3. Model transformation

According to theorem 1 in Zhong et al. (2018), for a given \( \beta = \{ \beta_i \} \), the ECRM can be equivalently transformed into model ECRM-1.

**ECRM-1**: 

\[
\text{ECRM-1:}
\]
Based on the analysis of ECRM-1, we find that when \( \boldsymbol{\beta} \) is known, under the influence of the largest-debt-first policy, the steady supply \( S \) at the container yard in each interval satisfies Constraints (10)-(11) if and only if Constraints (4)-(5) are satisfied. More importantly, according to the proof process of the theorem, when observed period \( T \) approaches infinity, the quantities of self-owned containers and leased containers obtained by each inland freight station in each interval should satisfy Equations (12) and (13). In other words, by using the largest-debt-first policy, the quantities of self-owned containers and leased containers obtained by each inland freight station in each interval converge to \( \beta_i \) times and \( 1 - \beta_i \) times the expected value of the actual demand at inland freight station \( i \) in the long run. This conclusion is interesting; it means that we can eliminate variables \( Q_i^o(t) \) and \( Q_i^l(t) \) from the model. Based on this idea, we substitute Constraints (12) and (13) into objective function (9); then, the ECRM can be further simplified to ECRM-2.

ECRM-2:
\[
\min_{(\beta, \rho)} \alpha_1 = C_h^b \left( S + E[Y] \right) + \sum_{i=1}^{N} \left[ \beta C_i^f + (1 - \beta) C_i^c \right] E[X_i] + \rho D[Y]
\]  
\[ (14) \]

s.t. (10), (11)

Due to the presence of constraint (10), ECRM-2 is still difficult to solve directly. However, when \( \beta \) is known, objective function (14) increases in a strict monotonic manner with respect to \( S \). As a result, if \( \beta \) is known, the minimum \( S \) satisfying constraints (10) and (11) is the optimal solution to ECRM-2. Thus, we can obtain \( S \) with high efficiency through binary search and simulation. Based on this feature, we design an effective heuristic algorithm for ECRM-2 in Section 5.

5. Algorithm design

In Section 4, we obtain a concise ECRM-2 based on model transformation. However, due to constraint (10), ECRM-2 cannot yet be solved directly at this time. As a result, we devise a differential evolution (DE) algorithm to solve the model. DE is a population-based adaptive global optimization algorithm that is characterized by a simple structure, easy implementation and fast convergence (Price 2013; Fleetwood 2004). In this paper, we use DE to solve ECRM-2 to obtain a relatively satisfactory solution as quickly as possible.

In essence, the differential evolution algorithm can be regarded as an improvement of the genetic algorithm. Therefore, we must also design targeted coding methods. In the algorithm design section, we take \( \beta \) as the coding object because a binary search-based algorithm can be used to calculate the value of \( S \) for any given \( \beta \) (Zhong et al. 2018). Based on the preceding analysis, we can further regard ECRM-2 as a mathematical programming problem with a single decision variable \( \beta \). ECRM-2 can be solved by performing a differential operation on \( \beta \).
5.1. Determination of empty container stock level based on binary search

Based on the binary search, the key operation for determining the optimal empty container stock $S + E[Y]$ is to judge whether $S + E[Y]$ satisfies constraint (9). The calculation method is as follows.

**Step 1:** According to equation (15), calculate the upper and lower bounds of the optimal empty container stock $S + E[Y]$. Therein, $M$ is a very large positive value.

$$
\left[ \sum_{i=1}^{N} \beta_i E[X_i], \sum_{i=1}^{N} \beta_i E[X_i] + M \right]
$$

**Step 2:** Adopt binary search. Take the midpoint of the upper and lower bounds obtained in the previous step as the stock level required for the simulation operation process and perform the following steps:

**Step 2.1:** Randomly generate the empty container demand of every inland freight station in all $T$ intervals. Based on the largest-debt-first policy, the priority of each interval $t$ $(P(t))$ is generated, the self-owned empty containers available are allocated to each inland freight station in each interval according to the priority list $P(t)$, and the inland freight stations that do not have their demands fully satisfied are allocated an appropriate quantity of leased containers.

**Step 2.2:** Randomly select a priority list from $P(t)$, $t = 1,2,\ldots,T$ with equal probability. Based on the selected priority list, calculate the utilization rate of self-owned containers obtained by each inland freight station $i$ ($\tilde{\beta}_i$).

**Step 3:** If for all $\tilde{\beta}_i$, $i \in N$ are no less than $\beta_i$, the algorithm terminates; otherwise, set the value of the midpoint as the upper bound in the next iteration process, and return to **Step 2**.
5.2. Coding design and fitness calculation

In this section, we use the real number coding method to characterize $\beta$, as shown in Figure 4. The coding of each individual is composed of several genes, and the length of the genes is equal to the number of inland freight stations. For example, if the number of inland freight stations is 20, then the length of the individual is 20. The value of each gene indicates the utilization rate of self-owned containers at inland freight station $i$ (where $i$ is the location value of the gene number), and the range of the value is $[0, 0.99]$. For example, the first gene in an individual represents the utilization rate of allocated self-owned empty containers at the container yard to inland freight station No. 1. The fitness calculation includes two steps. First, according to $\beta$ (determined by the code), the algorithm in Section 5.1 is used to obtain the optimal value of $S$. Then, $S$ and $\beta$ are substituted into equation (14) to obtain the fitness value.

![Figure 4 Individual coding](image)

5.3. Differential operator

The basic idea of the difference operator is as follows. First, in a randomly generated initial population, two individuals are arbitrarily selected (A and B), and their vector difference is multiplied by a decimal in the range of $[0, 1]$ (we call this decimal the ‘differential mutation factor’) and added to another individual vector (C) to generate a new individual (D). Second, the crossover operation is performed on individuals D and A to generate child individual (E). Third, the fitness values of individuals E and A are compared. If the fitness value of
individual E is better than that of individual A, individual E replaces individual A in the next generation; otherwise, individual A is retained. Through continuous iteration, the search results approach the optimal solution. The operation flow of the differential operator is shown in Figure 5 (Price, Storn, and Lampinen 2006).

![Flowchart of the differential operator](image)

**Figure 5** Flowchart of the differential operator

Taking Figure 6 as an example, we further illustrate the individual generation process based on the differential evolution operation (except the crossover and selection operations). First, the values of each gene in parents A and B are subtracted to obtain differential individual 1, which is multiplied by the mutation factor to obtain differential individual 2. Then, differential individual 2 is added to parent C to obtain new individual A after the differential operation. Notably, during the operation shown in Figure 5, the gene value of the last gene point in new individual A is 1.38, which exceeded the upper limit. In this case, we randomly generated an effective value within the limits to replace the original gene value.
Then, the new individual A and the parent individual A are subjected to the traditional crossover operation, and part of the genes of the new individual A are randomly replaced with genes at the corresponding gene points of parent A according to the crossover probability (as shown in Figure 7).

**Figure 6** Generation method of a new individual

**Figure 7** The crossover operation

6. Numerical experiments

To verify the effectiveness of the abovementioned models and algorithms, this section considers the stochastic container demands of 20 inland freight stations in the hinterland and analyses the changes in related indicators, such as the expected total cost of the liner company and the empty container stock level at the container yard, under the following four configuration schemes.
Scheme 1: The liner company uses the largest-debt-first policy to meet the empty container demands at each inland freight station. Therein, $S$ and $\beta$ are both decision variables.

Scheme 2: The liner company uses the traditional overstock method to meet the empty container demands of each inland freight station. Under this scheme, $S$ is set as the safe stock level that can completely avoid a shortage of empty containers, and the calculation method is shown in equation (16); where $\beta$ is the only decision variable and $u_i$ and $\sigma_i$ are the mean and variance of the demand samples for empty containers at inland freight station $i$.

$$S=\sum_{i=1}^{N}(u_i + 3\sigma_i)\beta_i - E[Y]$$

(16)

Scheme 3: The liner company uses only its own containers to meet the container demands of each inland freight station. That is, $\beta_i = 0.99$, $\forall i = 1,2,\cdots,N$. In this case, $S$ is the only decision variable.

Scheme 4: The liner company uses only leased containers to meet the container demands of each inland freight station. That is, $\beta_i = 0$, $\forall i = 1,2,\cdots,N$. In this case, $S$ is the only decision variable.

In Section 6.1, we introduce the setting of the experimental data and the demand information of each inland freight station. Section 6.2 examines the convergence of our suggested algorithm before determining its effectiveness. Then, Section 6.3 analyses the influence of the change in the length of the observed period on the expected total cost of the liner company under the four configuration schemes. Moreover, we compare the optimized value of $\beta_i$ for each inland freight station $i$ with the real value acquired from the simulation. Finally, Section 6.4 analyses the influence of a change in the variance of the demands of each inland freight station on the expected total cost of the liner company and the empty container stock level at the container yard under the four configuration schemes.
6.1. Experimental data setting and demand information of inland freight stations

(1) Storage operational costs

The empty container storage operational cost function is related to the total quantity of empty containers stored at the container yard. We set the mathematical expression of the cost function as follows, where \( \alpha = 20 \), \( b = 150 \), and \( \lambda = 1.05 \).

\[
C_b(S + E[Y]) = \begin{cases} 
\alpha(S + E[Y]), & 0 < (S + E[Y]) \leq 500 \\
\frac{(S + E[Y] - 500)^\lambda}{b} + \alpha(S + E[Y]), & S + E[Y] > 500
\end{cases}
\]

(2) Empty container demand and transportation and rental costs of every inland freight station

Each inland freight station has a varied location, as well as a different time for cargo owners to use empty containers. Therefore, the unit empty container rental fees paid by the liner company vary depending on different inland freight stations. We assume that the container demand of each inland freight station is independent and follows a normal distribution. The average demand of each inland freight station in each interval, the unit empty container rental cost of the liner company in each inland freight station and the unit empty container transportation costs from the container yard to each inland freight station are shown in the following table.

<table>
<thead>
<tr>
<th>No. of inland freight station</th>
<th>Average demand (TEU/interval)</th>
<th>Rental rate (RMB/TEU)</th>
<th>Transportation rate (RMB/TEU)</th>
<th>No. of inland freight station</th>
<th>Average demand (TEU/interval)</th>
<th>Rental rate (RMB/TEU)</th>
<th>Transportation rate (RMB/TEU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48</td>
<td>500</td>
<td>381</td>
<td>11</td>
<td>27</td>
<td>480</td>
<td>165</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>340</td>
<td>150</td>
<td>12</td>
<td>44</td>
<td>380</td>
<td>541</td>
</tr>
<tr>
<td>3</td>
<td>14</td>
<td>360</td>
<td>113</td>
<td>13</td>
<td>30</td>
<td>450</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>300</td>
<td>102</td>
<td>14</td>
<td>58</td>
<td>340</td>
<td>225</td>
</tr>
<tr>
<td>5</td>
<td>59</td>
<td>480</td>
<td>260</td>
<td>15</td>
<td>40</td>
<td>420</td>
<td>468</td>
</tr>
<tr>
<td>6</td>
<td>51</td>
<td>460</td>
<td>265</td>
<td>16</td>
<td>44</td>
<td>460</td>
<td>273</td>
</tr>
<tr>
<td>7</td>
<td>45</td>
<td>380</td>
<td>282</td>
<td>17</td>
<td>49</td>
<td>360</td>
<td>450</td>
</tr>
<tr>
<td>8</td>
<td>34</td>
<td>400</td>
<td>272</td>
<td>18</td>
<td>25</td>
<td>440</td>
<td>201</td>
</tr>
<tr>
<td>9</td>
<td>12</td>
<td>460</td>
<td>159</td>
<td>19</td>
<td>31</td>
<td>400</td>
<td>241</td>
</tr>
<tr>
<td>10</td>
<td>34</td>
<td>420</td>
<td>247</td>
<td>20</td>
<td>56</td>
<td>440</td>
<td>506</td>
</tr>
</tbody>
</table>
6.2. Analysis of the convergence of the algorithm

In this section, the differential evolution algorithm is used to solve the ECRM under scheme 1. Figure 8 depicts the expected total cost as a function of the number of iterations. When the number of iterations reaches approximately 350, the value converges, demonstrating the algorithm’s effectiveness.

![Figure 8: The convergence characteristics of the proposed algorithm](image)

6.3. Sensitivity analysis of length of the observed period

In this section, we conduct a sensitivity analysis on the length of the observed period. In the model, we require the length of $T$ to approach infinity. In the experiment, however, this is neither possible nor necessary. We report the changes in the liner company’s expected total cost under the four different schemes considering different situations of $T = 60, 90, 120, 150$ and 180, as shown in Table 3 and Figure 9.

<table>
<thead>
<tr>
<th>$T$ (days)</th>
<th>Scheme 1 (10 thousand RMB)</th>
<th>Scheme 2 (10 thousand RMB)</th>
<th>Scheme 3 (10 thousand RMB)</th>
<th>Scheme 4 (10 thousand RMB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>25.12</td>
<td>31.74</td>
<td>32.13</td>
<td>31.91</td>
</tr>
<tr>
<td>90</td>
<td>24.97</td>
<td>31.53</td>
<td>31.81</td>
<td>31.91</td>
</tr>
<tr>
<td>120</td>
<td>24.87</td>
<td>31.30</td>
<td>31.60</td>
<td>31.91</td>
</tr>
<tr>
<td>150</td>
<td>24.79</td>
<td>31.14</td>
<td>31.43</td>
<td>31.91</td>
</tr>
<tr>
<td>180</td>
<td>24.73</td>
<td>31.07</td>
<td>31.29</td>
<td>31.91</td>
</tr>
</tbody>
</table>
As shown in Figure 9, with the increase in the value of $T$, the order of expected operational costs under the four schemes changes. Specifically, when $T$ is 60, the ranking of the expected total costs under the four schemes in ascending order is scheme 1 < scheme 2 < scheme 4 < scheme 3. When $T$ is 90, the order becomes scheme 1 < scheme 2 < scheme 3 < scheme 4. This ranking does not change in subsequent calculations. In addition, in all the previous calculations, the expected total cost of scheme 1 is always remarkably lower than that of the other schemes. In addition, as $T$ increases, the expected total cost of scheme 1 shows a gradual downward trend. These results are consistent with our expectations. The empty container allocation scheme based on the largest-debt-first policy can realize empty container inventory management in a reasonable manner. Moreover, as the length of the observed period increases, the effect of the proposed policy improves because this policy makes full use of the offset effect of demand fluctuation to significantly reduce the expected total cost.
Figure 9 also shows the difference (root mean square error, RMSE) between $\beta^*_i$ (the optimized value of the decision variable) and $\beta_i$ (samples actually observed in the simulation experiment) under scheme 1 with different observed periods. The results show that the RMSE decreases with increasing length of the observed period. Therefore, for scheme 1, the longer the observed period is, the closer the value of $\beta$, and the utilization rate of self-owned containers can be. The results confirm the effectiveness of the largest-debt-first policy in allocating empty containers.

Next, we consider the scenario of $T = 180$ to analyse the differences between $\beta^*_i$ and $\beta_i$ at each inland freight station $i$. The reason why we choose this scenario is that the expected total cost of this scenario is the lowest, and the differences between $\beta^*_i$ and $\beta_i$ are the smallest (as shown in Table 4). Under this scenario, the largest-debt-first policy ensures that the optimization results are very close to the simulation results (the errors are all within 0.02). In addition, according to the optimized results, different inland freight stations have different optimal values of $\beta_i$. For example, the value of $\beta_i$ for inland freight station 9 is close to 1, which means that almost all the empty container demand of inland freight station 9 can be met by self-owned containers at the container yard. In contrast, the value of $\beta_i$ for inland freight station 7 is 0, which means that it is more reasonable to use leased containers to meet this station’s demand. The results show that the liner company should optimize the empty container allocation structure of every inland freight station in combination with the actual situation.
Table 4 Values of $\beta_i$ and $\overline{\beta}_i$ for each inland freight station

<table>
<thead>
<tr>
<th>No. of inland freight station</th>
<th>$\beta_i$</th>
<th>$\overline{\beta}_i$</th>
<th>No. of inland freight station</th>
<th>$\beta_i$</th>
<th>$\overline{\beta}_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.93</td>
<td>0.93</td>
<td>11</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>2</td>
<td>0.97</td>
<td>0.96</td>
<td>12</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>3</td>
<td>0.98</td>
<td>0.99</td>
<td>13</td>
<td>0.96</td>
<td>0.97</td>
</tr>
<tr>
<td>4</td>
<td>0.93</td>
<td>0.91</td>
<td>14</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>5</td>
<td>0.92</td>
<td>0.91</td>
<td>15</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>6</td>
<td>0.93</td>
<td>0.94</td>
<td>16</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.02</td>
<td>17</td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td>8</td>
<td>0.95</td>
<td>0.95</td>
<td>18</td>
<td>0.97</td>
<td>0.99</td>
</tr>
<tr>
<td>9</td>
<td>0.98</td>
<td>0.97</td>
<td>19</td>
<td>0.96</td>
<td>0.98</td>
</tr>
<tr>
<td>10</td>
<td>0.95</td>
<td>0.95</td>
<td>20</td>
<td>0</td>
<td>0.02</td>
</tr>
</tbody>
</table>

6.4. Sensitivity analysis of variance of the demands for inland freight stations

In this section, we consider an observed period of $T = 180$ to test the influence of a change in variance in inland freight stations’ demand on the liner company’s expected total cost. For comparison, we take the ratio of the variance to the mean value of empty container demand ($\sigma_i / \mu_i$) as the parameter of the sensitivity analysis. This parameter describes the fluctuation situation of empty container demand of inland freight stations. The greater this value is, the greater the volatility of demand. Table 5 and Figure 10 show the calculation results under the four schemes.

When the variance-to-mean ratio increases, the expected total costs of schemes 2 and 3 show a significant upward trend because the liner company must prepare more empty containers to avoid a possible shortage of empty containers due to an increase in demand fluctuation. In contrast, the expected total costs of schemes 1 and 4 are almost unaffected. The results of scheme 4 are in line with our expectations, as the expected total cost of scheme 4 is related to only the expected value of empty container demand. However, for scheme 1, the results are surprising. Scheme 1, based on the largest-debt-first policy, can not only effectively reduce the safe empty container stock level but also effectively handle the adverse impact of the increase in demand fluctuation.
Table 5 Influence on the expected total cost with different values of the variance-to-mean ratio of demand

<table>
<thead>
<tr>
<th>$\sigma_i/\mu_i$</th>
<th>Scheme 1 ($\times$10000 RMB)</th>
<th>Scheme 2 ($\times$10000 RMB)</th>
<th>Scheme 3 ($\times$10000 RMB)</th>
<th>Scheme 4 ($\times$10000 RMB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24.43</td>
<td>26.20</td>
<td>29.11</td>
<td>31.91</td>
</tr>
<tr>
<td>1/5</td>
<td>24.73</td>
<td>31.07</td>
<td>31.29</td>
<td>31.91</td>
</tr>
<tr>
<td>1/4</td>
<td>24.81</td>
<td>32.75</td>
<td>32.59</td>
<td>31.91</td>
</tr>
<tr>
<td>1/3</td>
<td>24.95</td>
<td>36.48</td>
<td>35.56</td>
<td>31.92</td>
</tr>
<tr>
<td>1/2</td>
<td>25.24</td>
<td>44.86</td>
<td>45.39</td>
<td>31.93</td>
</tr>
</tbody>
</table>

Figure 10 Influence on the expected cost with different values of the variance-to-mean ratio of container demand

To explain the reason for this stability under scheme 1, we further analyse the difference in empty container stock levels between scheme 1 and scheme 2 with different variance-to-mean ratios, as shown in Figure 11. With the overstock scheme, to cope with the increasing fluctuation of empty container demand, the liner company must increase the empty container stock (even if $\theta$ has been optimized). This action directly leads to a significant increase in empty container storage operational costs in scheme 2. In scheme 1, because the largest-debt-first policy is adopted to allocate empty containers, the liner company can take advantage of the offset effect of demand fluctuation to effectively address the impact caused
by the increase in demand fluctuation. Therefore, we can ensure that the empty container stock at the container yard is always maintained at a stable level and does not increase significantly with an increase in the variance-to-mean ratio, so the expected total cost does not change significantly. In addition, as the fluctuation range of empty container demand increases, the expected cost of scheme 1 is always lower than that of the other schemes. The above results again confirm that our proposed allocation scheme is significantly better than the traditional overstock policy.

**Figure 11** Optimized empty container stock with different variance-to-mean ratios

**7. Concluding remarks**

To address the new combination of storage and rent container management modes, this paper considers the empty container allocation optimization problem for inland freight stations under the background of stochastic demands. By constructing a stochastic programming model with the minimum expected total cost of the liner company as the goal, the collaborative optimization of (1) the allocation of empty containers at container yards; (2) the transportation scheme of self-owned empty containers from container yards to every inland
freight station; and (3) the proportion of self-owned containers to the total demands in each
inland freight station can be realized. We also offer the largest-debt-first policy, which takes
into account the model’s complexity. From the standpoint of the long-term supply of empty
containers, the liner company can attain the lowest expected total cost under this policy. The
model is solved using a differential evolution technique. A differential evolution algorithm is
designed to solve the model, and numerical analysis is utilized to verify the model’s
feasibility and effectiveness. Based on the numerical experimental results, we obtain the
following conclusions.

(1) As the number of iterations of the algorithm increases, the expected value of the total
liner company cost decreases gradually. The calculation results converge after 350
iterations, demonstrating the effectiveness of the suggested approach.

(2) When compared with other schemes, the empty container allocation scheme based on
the largest-debt-first policy always obtains the lowest expected total cost as the length
of the observed period \( T \) increases. Furthermore, the longer the observed period, the
better the proposed policy’s effect will be. The policy proposed in this paper realizes
the lean inventory management of empty containers by maximizing the offset effect
produced by demand fluctuations.

(3) The best allocation pattern for different inland freight stations varies. The liner
company must take into account all the characteristics of various inland freight
stations, set the optimal allocation structure for them, and strike a balance between
self-owned and leased container supply.

(4) As the variance of the inland freight station empty container demands increases, the
liner company’s expected total cost based on the largest-debt-first policy is practically
unchanged. The findings demonstrate the advantages of our proposed policy in
handling fluctuations in empty container demand. The proposed approach may fully
utilize the demand fluctuation offset effect, significantly reduce the empty container stock levels, and effectively control the liner company’s expected total cost.

The follow-up research directions of this paper include the following three aspects. First, we consider only the storage, transportation and rental costs of empty containers. However, the composition of empty container management costs is usually complicated in practice. How to fully consider the actual operational factors and include appropriate cost elements in the model framework is a future research direction. Second, we assume that each inland freight station’s empty container demand is independent and identically distributed, ignoring the correlation of the empty container demands of inland freight stations. Therefore, the correlation of empty container demands of different inland freight stations should be considered in future research. Third, we use a differential evolution algorithm to solve the model. The algorithm can only obtain a satisfactory solution of the model. Improving the precision of the solution is also a fascinating study topic.

Acknowledgements
The funding body will be acknowledged after peer review.

References


