Which Abatement Policies Are Best Away from Optimality?

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Which Abatement Policies Are Best Away from Optimality?

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Abstract

We consider the case of a non-infinitely-altruistic social planner who does not know the true climate and economic parameters of the DICE model and who, because of political or social constraints, cannot act optimally. We find that the impact of parameter uncertainty on economic outcomes is much more pronounced away from optimality than along an optimal path. We also find that for this non-omniscient and politically constrained social planner the most desirable of the ‘feasible’ courses of actions depends strongly on which quantity is known with uncertainty. A gradual ramp-up is preferred to a steep (‘Stern-like’) abatement schedule when we consider (symmetric) uncertainty in the growth rate of the economy, or in the cost of abatement. This result is extremely robust to the choice of a number of non-expected-utility-maximization decisional criteria that do not make use of probabilities: minimal regret, maximax and maximin all give the same recommendation. Ambiguity aversion does not change these results. However, when uncertainty in the damage function is considered, a steeper abatement schedule becomes a strong contender, and is preferred by some decisional criteria. This suggests that researching this aspect of climate modelling would have the greatest policy relevance. Finally, we note that a gradual (‘Nordhaus-like’) ramp-up of the abatement efforts is always preferred to a slower (‘business-as-usual’) schedule of abatement even in the case of much stronger future economic growth or much milder climate damage than the central estimates of the DICE model.

Keywords: Decisional Criteria, DSGE models, Integrated Assessment Models, Optimal Climate Policies
JEL Codes: D58, D81, Q28, Q54, Q58

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1 Statement of the Problem

The DICE model has become one of the best-known and most widely quoted Integrated Assessment Models (IAMs). It has been created with a policy focus in mind: given a set of equations describing the evolution of the economy and of the climate system, a chosen utility function and utility discount rate, it prescribes the optimal course of investment and abatement. As is well known, the DICE model suggests that a social planner should engage in a gradual ramp-up of the abatement efforts. This should be contrasted with the recommendations from Integrated Assessment Models such as the one presented in the Stern review, which calls for a much steeper ramp-up of abatement initiatives.

It is widely recognized that the origin of the sharp differences in policy recommendations based on cost-benefit analysis can be traced to the different choices for the preference parameters. In the choice of these all-important preference parameters one can follow a market-based, a normative or an ‘introspective’ approach. The market-based approach, which tends to suggest a higher discount rate than the Stern choice, has well-known shortcomings, especially given the extremely long time horizons of the problem. (See, in this respect, the discussion in Gollier.) However, one does not have to embrace a market-based approach to find the Stern-like choice of the implicit social discount rate questionable: thought experiments such as the ‘wrinkle experiment’ in Nordhaus raise serious doubts about the reasonableness of applying effectively zero discounting to future benefits and damages.

Since there seems to be little hope of resolving what are ultimately different philosophical views in the near future, we by-pass the question of the ‘correct’ discount rate, and we try to answer the following questions: if one leaves the DICE utility discount rate and the distributional features of the problem unchanged, how robust is the key DICE recommendation of a gradual ramp-up? Are there plausible circumstances under which, despite DICE-model-like preference parameters, one should still prefer a steep (‘Stern-like’) abatement schedule? Does one need Stern’s almost infinite altruism to justify a fast ramp-up schedule?

In this work we answer these questions from a rarely explored perspective. We consider in fact the

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1 A description of earlier versions of the model, and the changes to the model and its parameters, can be found in Nordhaus and Sztorc (2013). The structure of the model and its relationship to similar Integrated Assessment Models can be found in Nordhaus and Moffat (2017) and Nordhaus (2007b). The links between the DICE model and DSGE models are discussed in detail in Hassler, Krusell, and Smith (2016).


3 See Chapter 1 in particular.

4 See page 158 and passim.

5 The value of the ‘utility impatience’ parameter, $\beta$, of $0.0010$ chosen by Stern does not even allow for a very modest reduction of the degree of concern for the welfare of infinitely distant generations: it is only meant to cater for the probability that humanity may be wiped out by an asteroid-like event.
case of a (non-almost-infinitely altruistic) planner whose actions are politically and socially constrained and who must choose the best feasible course of action in the presence of parameter uncertainty. These policy constraints have deep economic implications, and, as Gollier (2020) has recently argued, can explain a number of aspects of mitigation policies, such as the so-called carbon-pricing puzzle (the much higher growth rate for the social cost of carbon than the interest rate in the economy, as the Hotelling’s rule (Hotelling (1931), and, more recently, Chakravorty, Moreaux, and Tidball (2008)) would imply).

As we discuss in Sections 3 and 6, parameter uncertainty is large—indeed, it is not even clear that one is dealing with a situation of risk (with known probability distribution for the parameter values or the severity of the outcomes) rather than uncertainty. This being the case, well-established decisional techniques, such as maximization of expected discounted utility, become difficult to use, and decisional criteria that require less rich information (such as the maximin, minimal regret, maximax or the smooth-ambiguity-analysis criteria) come to the fore. Our strategy is to examine the congruence of the recommendations from these very different decisional criteria in order to assess the robustness of different policy choices under parameter uncertainty.

Parameter uncertainty is more closely linked to non-optimality than usually appreciated. This is because, while in the neighbourhood of the optimal solution the sensitivity of the optimal policies to the model input parameters is indeed modest, we show in Section 3 that along non-optimal paths the model parameter sensitivity increases greatly, and deviations from optimality magnify the effects of parameter uncertainty. As the goal of our work is to provide robust decisional guidance to the social planner, we must take into account how policy constraints (via the deviations from optimality they impose) compound the problems caused by parameter uncertainty.

Given this setting, our main findings are as follows. Depending on the parameter, uncertainty has a very different effect on which of the feasible courses of action is deemed preferable. As far as uncertainty in the cost of the abatement efforts or in the growth of the economy is concerned, the schedule that we call Gradual is preferred to both the Aggressive and the Slow schedules by all the decisional criteria we have examined. Taking ambiguity aversion into account does not change these conclusions. We can actually considerably strengthen this statement: even if we stress the cost and

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6 Constrained policies are also considered in Nordhaus (2007b); however, these still explore the optimal policies consistent with the achievement of exogenous targets, such as a maximum acceptable increase in the temperature anomaly by a certain date. As discussed in Section 2, Gollier (2020) also considers policy actions away from optimality, but from the perspective of a ‘selfish bureaucrat’ is the spirit of Niskanen (1960).

7 A possible explanation of the carbon pricing puzzle is based on the existence of political constraints related to the social acceptability of climate policies... Gollier (2020), page 5, emphasis added.

8 Parameter uncertainty in IAM models is often dealt with by calculating the optimal paths associated with parameter sets drawn from a posited parameter distribution. Needless to say, this falls well short of reaching a decision on the basis of a procedure that feeds the uncertainty into a convex utility function.

9 Here we use the term ‘risk’ to denote collectively what Knight (1957, 2006) describes as ‘a priori probability’ and ‘statistical probability’, and we use the term ‘uncertainty’ to denote what he calls ‘estimates’. See p. 225.

10 In this respect, see, eg, the discussion in Heal and Millner (2013) and Weitzman (2009). For a critique of Weitzman’s position, see Nordhaus (2008).

11 We have mainly focussed in our study on the uncertainty in those model parameters (the growth rate of the economy, the ‘damage exponent’ and the abatement cost) that have been recognized as having the greatest effect on the optimal policy (see in this respect the discussion in Nordhaus and Sztörz (2013)).

12 The precise definition of the Gradual, Aggressive and Slow schedules is given in Section 4.
growth parameters by as much as ±70% the Gradual schedule still dominates the other two schedules, and therefore no reasonable decisional criterion could alter this ordering. This is somewhat surprising, because one of the recurrent explanations of the DICE gradual ramp-up of abatement efforts is that, if we are going to be rich in the future, it is more efficient to delay investment. So, greater economic growth should plausibly point to an even slower abatement schedule. This consideration is however tempered by the fact that higher economic growth is accompanied by higher emissions. Therefore, for a fixed investment, the future sacrifice on richer generations will indeed be smaller, but the size of the climate problem to bring under control will be greater exactly because of the higher economic growth. The two effects play against each other, partially cancelling each other out.

The dominance, or at least robustness, of the Gradual solution, however, no longer holds when we stress over a wide, but reasonable, range of values the exponent of the difficult-to-estimate damage function. Now the Aggressive schedule becomes a serious contender, and whether the Gradual or the Aggressive schedule is chosen depends on the precise choice of decisional criterion, and on the relatively fine details of the competing non-optimal schedules (described in Section 4). One obvious practical conclusion of this finding (echoed in Nordhaus (2021)) is that devoting research efforts to a more accurate estimation of the damage function would have far greater policy relevance than improving our understanding of any other aspects of the DICE model.

Finally, for no parameter choice and no decisional criterion do we find a significantly slower abatement schedules preferable to the Gradual one. Economically, this means that the saving on the abatement costs do not compensate for the higher damages that occur even if the most favourable growth of damage scenarios materialize.

Finally, we acknowledge that using feasible, but non-optimal, abatement schedules introduces an element of arbitrariness. We therefore discuss in detail in Section 4 how we have mapped the outputs of the abatement schedules used in our analysis to the outcomes of the well-known Representative Concentration Pathways used by the IPCC in its 2014 Assessment Report.

2 Literature contribution

The analytical approach taken in this article builds on two important strands of the economic literature. First, we look at actions away from optimality, and, in particular, study deviations from optimal control. Second, we evaluate these potential courses of action when the decision-maker is confronted with uncertainty about the structural parameters: we do so by analysing the impact of model ambiguity.

Policy responses produced by optimal control analyses, of which DICE is a prime example, are the output of dynamic economic studies that emerged in the first half of the twentieth century. The decarbonization rate and the abatement costs are in the DICE model a deterministic function of time.

14We follow and further the cost-benefit analyses of climate change in the line of Nordhaus (2008), Stern (2007), Hope (2006) and Hope (2013) and related Integrated Assessment Models. We are aware of objections such as those in Pindyck (2013), who have criticized these cost-benefit analyses for their unrealistic assumptions, but we consider the addressing of these concerns beyond the scope of this work.

15In the seminal contribution by Ramsey (1928), the consumer’s lifetime utility was shown in the late 1920s to depend on the time path of consumption; the goal of dynamic optimization was then to find the time path of consumption that maximizes the consumer’s lifetime utility. Almost at the same time, Hotelling (1931) showed that dynamic optimization
connections between the DICE model and DSGE models is discussed in detail in Hassler, Krusell, and Smith (2016). Our approach, while firmly situated in this strand of research, differs from most of the literature because, for the reasons explained in the Introduction, we consider feasible action plans to be equivalently, if not more, interesting than optimal schedules. As far as we are aware, very few papers have considered the effect of sub-optimality in the DICE model; among these, Tol (2020) looks at departures from optimality from the perspective of the Niskanen (1960) model by introducing what he calls ‘selfish bureaucrats’ (ie, budget-maximizing bureaucrats). In our approach we are agnostic as to the origin of the socio-economic constraints faced by policy-makers. What matters from our perspective is that the candidate abatement schedules should be feasible. Feasibility is, of course, subjective. It is for this reason that we justify our schedules on the basis of their proximity to the well-established Representative Concentration Pathways used by the IPCC (Pachauri and Meyer (2014)) (see in this respect the discussion in Section 4).

Once the practically possible courses of action are specified, the policy makers who use climate models to inform their choices still have to face a major hurdle: the radical uncertainty of the physical features of the model. Given this background of very imprecise knowledge of the physical and economic models, parameter uncertainty has been widely recognized to be a serious problem with Integrated Assessment Models in general, and with the DICE model in particular. Typically, this uncertainty has been dealt with by large-scale sensitivity analysis and Monte Carlo Simulations (Yumashev (2020), for instance, quotes 150 uncertain inputs to the PAGE-ICE model by Hope (2006)), but each Monte-Carlo is usually deterministic in time. As a results of these analyses, the policy maker is therefore faced with a range of possible outcomes, and with an array of decisional tools (such as minimal regret, maximin, maximax, smooth-ambiguity aversion, etc) but with little guidance as to how this information can be used to reach decisions. We bring parameter uncertainty to the heart of our analysis by considering decisional criteria that do not require knowledge of the probability distribution of the parameters, and by focussing on the decisional robustness that can be obtained by bringing together the results from different criteria. The unknown physical earth reactions under settings never

yields the time path of extraction of an exhaustible resource that maximizes the value of a resource project, such as a mine. After World War Two, the tools of dynamic programming and optimal control theory were developed simultaneously by applied mathematicians across the iron curtain (Bellman (1957), Bellman and Dreyfus (1962), Pontryagin, Boltysanski, Gamkrelidze, and Mishchenko (1962)) and helped determine solutions to these economic models. Not only the DICE model, but, more generally, all the Integrated Assessment Models developed in its furrow, have borrowed extensively from Ramsey and Hotelling, and their optimal resolutions make extensive use of the Bellman and Pontryagin toolboxes.

Tol (2020) also considers the effect of policy heterogeneity.

17 Just to quote a few examples, the climate literature has identified several potential tipping elements in the climate system, including the irreversible melting of the Greenland ice sheet. Joughin, Smith, and Medley (2014) argue that marine west Antarctic ice sheet collapse is already under way, while Lenton, Held, Kriegler, Hall, Lucht, Rahmstorf, and Schellnhuber (2008) evaluate that the Greenland ice sheet melting could lead to a global sea level rise of up to one meter per century. As Cai, Lenton, and Lontzek (2016) discuss, there is a risk of multiple interacting tipping points such as the weakening or shutdown of the Atlantic thermohaline circulation (Anthoff and Tol (2016) study the competing effects of cooling due to a slowdown of the thermohaline currents on the one hand and the warming due to greenhouse gases on the other), or the thawing of the Siberian permafrost (see Biskaborn, Smith, and Noetzli (2010) for a recent global perspective). In addition, the human reaction to climate change is a potentially important source of uncertainty in itself, that could play out over long horizons as detailed by Lemoine and Traeger (2014), Jensen and Traeger (2014), Nordhaus (2016), Hambel, Kraft, and Schwartz (2021), or Cai, Judd, and Lontzek (2015) and Cai, Judd, Lenton, Lontzek, and Narita (2015).

18 There are similarities here with the approach taken by van der Ploeg and Rezai (2018), who explore the robustness of climate policies by letting the coefficient of relative ambiguity aversion vary from zero, corresponding to expected
experienced – or recorded – in the historical past, and the risks of tipping phenomena, then justify the wide range of parameter shocks we have taken into consideration. As recognized in the literature (see, eg, Nordhaus and Moffat (2017)), the damage function, the productivity paths and the magnitude of the abatement costs concentrate the highest risks over the course of the next several decades, and we therefore focus our analysis on these quantities.

Given this pervasive uncertainty, ambiguity reflects the fact that decision-makers ultimately need to reach a conclusion while faced with multiple models, or vastly different plausible parameter values. The smooth ambiguity model originally proposed and axiomatized by Klibanoff, Marianacci, and Mukerji (2005) addresses the situation where uncertainty lies within models and is distinct from risk. In the climate science space Millner, Dietz, and Heal (2013) and Lemoine and Traeger (2016) use smooth ambiguity to analyse the robustness of their conclusions, while Olson, Sriver, Goes, Urban, Matthews, Haran, and Keller (2012) study within model ambiguity on their climate sensitivity parameter. Barnett, Brock, and Hansen (2020) formulate a social decision planner problem that includes concerns about the potential misspecification of alternative models and ambiguity over how much weight to assign to each of these models. In our non-stochastic setting, we apply non-probabilistic decision rules to rank potential actions in the face of ambiguity, and, as a by-product, we explore the robustness of the policy recommendations. We are not the first to consider decisional criteria not based on the maximization of expected utility in the context of climate change (see, for instance, the work by Heal and Millner (2013)). To our knowledge, however, only the early work by Woodward and Bishop (1997) took as direct input the outcomes from the DICE model. They did so by modelling alternative climate scenarios, one associated with Nordhaus’ base case, and the other with a ‘catastrophic’ one. As we have explained in the Introduction, our focus is very different.

3 The Importance of Being Optimal

We show in this section that, if the social planner always acts optimally, the net welfare changes relatively little even if one introduces substantial stochasticity to the deterministic behaviour postulated in DICE. This has led to the contention that reasonable parameter uncertainty will have little effect on society’s welfare. This conclusion must be strongly qualified. The near-invariance of total welfare in the neighbourhood of an optimal consumption plan – ie, if the planner can act optimally – is to be expected, as the total utility that is maximized is by construction an extremum of the function(al) of the control variables. This can be seen more precisely as follows.
Consider the total utility, \( U(\{c(t)\}) \) as a function of the consumption choices, \( c(t) \), over the discrete time steps over which the DICE optimization is carried out\(^{22}\). Let \( x_k \) be one of the parameters on which the optimal consumption path depends, and let \( \tilde{x}_k \) denote its optimal value. Close to the optimal consumption programme, \( \tilde{c}(t; \tilde{x}_k) \), one can write for the change in welfare associated with a change in the parameter, \( x_k \),

\[
U(\{c(t; \tilde{x}_k + dx_k)\}) = U(\{\tilde{c}(t_j; \tilde{x}_k)\}) + \sum_j \frac{\partial U}{\partial c(t_i)} \frac{\partial c(t_j)}{\partial x_k} dx_k
\]  

(1)

The effect of a small but finite change in the \( k \)th parameter on the optimal consumption (the term \( \frac{\partial c(t_j; \tilde{x}_k)}{\partial x_k} \)) may well be large. However, if the consumption path is the optimal one, each term \( \frac{\partial U}{\partial c(t_j)} \) will vanish to first order, and the resulting changes in total welfare will therefore be muted. This, however, is no longer the case if the consumption path is not the optimal one, as in this case the parameter sensitivity multiplies terms \( \frac{\partial U}{\partial c(t_j)} \) that need not be small. If the abatement programme followed is not the optimal one, parameter uncertainty can therefore have a significant effect on the resulting welfare. If this is the case, asking which choice criteria the planner should employ becomes very important.

To illustrate this point with maximum clarity without having to rely on arguably subjective choices of what an Aggressive, Gradual or Slow abatement programme would look like (see in this respect the discussion in Section 4), we follow a slightly different procedure from the one employed in the rest of the paper, and we focus on one of the most important (and most-difficult-to-estimate) parameters of the model, namely, the damage exponent. This parameter links the damage fraction, \( damfrac \), to the temperature anomaly via the following equation\(^{23}\).

\[
damfrac = a_2 \times T^{a_3}
\]

(2)

Specifically, we proceed as follows. We consider four distinct situations: in the first three, the social planner consistently and optimally acts on the basis of the correct knowledge that the damage exponent, \( a_3 \), is either 1.34, or 2, or 2.98, respectively. (The DICE default value is \( a_3 = 2 \).) In the fourth case, we assume that the true exponent is 2.98 but the social planner, either out of ignorance or because of political constraints, acts on the basis of the optimal path obtained for \( a_3 = 1.34 \).

Figs 1 and 2 convey a striking message about the economic variables, consumption and economic output, directly linked to the maximand (the total discounted utility): as the top three lines of both figures show, as long as one behaves in a consistently optimal way, there is very little change in economic welfare even when the damage exponent varies over as large a range as 1.34 to 2.98. However, this ceases to be the case when one moves away from optimality. As the lower curves in the same Figs 1 and

\(^{22}\)In the DICE set-up the abatement fractions, \( \mu(t) \), and the savings rates, \( sav(t) \), are the control variables. However, for a given schedule of savings rate, there is a one-to-one mapping between abatement fraction and consumption. In this section, we therefore cast our discussion in terms of consumption. We note that savings rates are always much more constant than the abatement fractions, and therefore we ignore them in the discussion. Adding the savings component only makes the notation a bit heavier, but does not change any of the conclusions.

\(^{23}\)The output available for non-abatement investment and consumption, \( y \) is given by \( y = y_{gross} (1 - damfrac - abatefrac) \), with \( abatefrac \) equal to the fraction of output devoted to abatement efforts.

\(^{24}\)To facilitate cross-referencing with the DICE model, we have used the same symbols for the variables used in this paper as the variable names in the GAMS code of the DICE model made public by Prof Nordhaus.
show, there are now significant differences in economic outcomes between the first three consumption patterns on the one hand, and the fourth on the other – the one obtained by assuming that the $a_3$ parameter is not consistent with the optimization path. What is happening is that for all the consistently optimal solutions relating to the different values of the exponent $a_3$ the physical, non-economic variables, such as, eg, the temperature anomaly, do change significantly, but they do so in such a way as to produce small overall changes in the variables linked to economic welfare. (We have displayed consumption and net output, but the same considerations apply, for instance, to capital or investment.)

![Consumption and Net Output Graphs](image1.png)

Figure 1: The consumption and net output ($y = y_{gross}(1 - damfrac - abatefrac)$) obtained from savings and abatement decisions consistently based on the assumption that the true damage exponent, $a_3$ is 2, 1.34, or 2.98 (curves labelled ‘2’, ‘1.34’ and ‘2.98’), and based on the assumption that the correct exponent is 1.34 when it actually is 2.98 (curve labelled ‘1.34/2.98’). Time in years from 2015 on the $x$ axis.

The results presented in these figures therefore show with great clarity that exploring the loss in welfare coming from acting away from optimality in the presence of parameter uncertainty is key to determining how to make prudent choices using the DICE model. Of course, there is only one way to be optimal, and infinitely many ways of being sub-optimal. We therefore turn in Section 4 to the extremely important topic how the sub-optimal paths can be chosen in a reasonable and robust manner.

4 Choice of the Three Abatement Schedules

To make our analysis amenable to quantitative analysis, we consider three possible abatement programmes, which we dub Aggressive, Gradual and Slow. Since these schedules are not optimal, there is a degree of arbitrariness associated with them. We have, however, tried as much as possible to anchor our schedules around well-established ‘pathways’, namely the Representative Concentration Pathways in the 2014 Assessment Report of the IPCC. The DICE model and the various IPCC pathways do not share the same underlying physical models, and a precise match is therefore not possible. As we
Figure 2: The ‘physical’ quantity damage fraction obtained from savings and abatement decisions consistently based on the assumption that the true damage exponent, $a_3$ is 2, 1.34, or 2.98 (curves labelled ‘2’, ‘1.34’ and ‘2.98’), and based on the assumption that the correct exponent is 1.34 when it actually is 2.98 (curve labelled ‘1.34/2.98’). Time in years from 2015 on the $x$ axis.

show below, however, there is a close match between the Aggressive, Gradual and Slow schedules on the one hand, and the RCP 4.5, RCP 6.0 and RCP 8.5 concentration pathways on the other.

Before describing the scenarios in detail, we must clarify one important point. In our setting the planner commits at time 0 to a schedule that then remains unchanged over time. This is clearly a very strong, and arguably unrealistic, assumption, because, presumably, the planner will try to change her course of action as new information is revealed. We make the assumption for the sake of tractability, and we simply note that the irrevocable nature of the policy is not as central a feature as it would be in an optimization setting, because, when the planner is socially or politically constrained, it is not obvious how she may be able to change her abatement schedule as new information becomes available. We therefore use a single-choice policy as an approximation to a more complex state-dependent abatement policy.

The three abatement paths were created using the following function:

$$\mu_{subj}(t) = \mu_0 + \theta [1 - \exp(-\kappa t)]$$

with $\theta = \mu_T - \mu_0$, $\mu_0 = 0.05$, and $\kappa = 0.0185, 0.0120, 0.0065$, $\mu_T = 1.25, 1.15, 1.00$ for the Aggressive, Gradual and Slow schedules, respectively. The chosen values for the parameter $\kappa$ correspond to decarbonization half-lives (ie, times over which the carbon intensity of the economy will halve) of 38, 58 and 106 years for the Aggressive, Gradual and Slow schedules, respectively. These half lives are of

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26The role of the abatement function $\mu(t)$ is best understood by looking at the equation that links industrial emissions, $e_{ind}(t)$, to the gross output, $y_{gross}(t)$: $e_{ind}(t) \propto y_{gross}(t) \times (1 - \mu(t))$, with the proportionality constant given by the carbon intensity.
the same order of magnitude as those quoted the study by Seshadri (2016) of the peak in CO2 concentrations, in which he sets a range for the half lives between 14 and 84 years. To put these numbers in a more precise empirical context, one has to distinguish between energy carbon intensity and GDP carbon intensity. The first refers to decarbonization due to the replacement of fuels at high carbon content with fuels at lower carbon content. This decarbonization rate has been roughly constant over the last two centuries (Gruebler and Nakicenovic, 1996) and estimates for this quantity are around 0.3-0.4% per annum (for instance Gruebler (2004) quotes in 2004 a decarbonization rate of 0.35% per annum). In addition, there is the energy requirement per unit of GDP produced, which has fallen more rapidly, at a rate of approximately 0.9% per annum over the last two centuries. Combining the two rates of decarbonization would give a half life of approximately 60 years (with an overall effective rate of 1.2% per annum). A more recent PWC report quotes decarbonization rates of 1.5% per annum for the period 2000-2019, corresponding to a half-life of approximately 50 years. Naively extrapolating recent trends is of course problematic, because on the one hand the urgency of reducing carbon emissions has greatly increased in the last decade and may well continue to increase; on the other hand, the recent decarbonization rate reflects the picking of the ‘low-hanging fruits’, and deeper decarbonization (eg, associated with the production of concrete or steel) may progress more slowly. Taking these considerations into account, the chosen half-lives for the Aggressive, Gradual and Slow schedules of approximately 40, 60 and 100 years appear reasonable. We compare in Figs the Aggressive, Gradual and Slow abatement schedules, ie, the control variable, $\mu$, referred to in the DICE documentation as the emission reduction rate.

As for the savings rates, we note that in all the DICE-model optimizations they always display far smaller variations than the abatement fractions. We have therefore assigned constant savings rates of 0.30, 0.25 and 0.20 for the Aggressive, Gradual and Slow schedules, respectively, that very closely mirrors the optimal savings patterns obtained by the DICE model with different preference parameters. We note that the average world savings rates ranged between 22% and 27% for the period 1975 to 2020 (source: World Bank (2021), and therefore the values chosen appear very reasonable.

As a next step, we show how our scenarios map to the Representative Concentration Pathways that have acquired benchmark status in the climate-change literature. The Representative Concentration Pathways (RPCs), prepared by the four modelling teams/models (NIES/AIM, IIASA/MESSAGE, PNNL/MiniCAM, and PBL/IMAGE), have been designed to serve as input for climate and atmospheric chemistry modelling for the development of scenarios for the IPCC’s Fifth Assessment Report. They include a stringent mitigation scenario, RCP2.6, two intermediate scenarios, RCP4.5 and RCP6.0, and one scenario with very high emissions, RCP8.5. Each scenario is designated by a suffix X.X as in RPCX.X, which indicates the forcing in $W/m^2$.

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27Available at https://www.pwc.co.uk/services/sustainability-climate-change/insights/net-zero-economy-index.html.
30See Riahi, Gruebler, and Nakicenovic (2007).
31‘Forcing’ is the balance of energy in and energy out for the Earth system, ie, the difference between solar irradiance absorbed by the Earth, and the energy radiated (not reflected) back into space via black-body radiation. Reflection is accounted for, via albedo, as an effective reduced irradiance.
Figure 3: The Aggressive, Gradual and Slow abatement schedules for $\mu$ compared.
Due to the different modelling assumptions underlying the DICE model and the RCPs, a perfect match between the output quantities produced by the DICE model and by the RCPs is not to be expected. However, as shown in what follows, there is a strong correspondence between our scenarios and the RCP pathways. The most relevant RCPs for our purposes are RCP 2.6, RCP 4.5, RCP 6.0 and RCP 8.5, which can be roughly characterized as follows:

1. RCP 2.6: emissions peak by 2030, and then decline around zero by 2080; CO2 concentrations peak in mid-century at around 440 ppm and then start to decline slowly.
2. RCP 4.5: emissions peak around 2040 at a level about 50% higher than the 2000 level, then decline.
3. RCP 6.0: emissions peak around 2080, and then fall sharply.
4. RCP 8.5: emissions continue to rise throughout the 21st century.

To establish a correspondence between the RCP pathways and our scenarios, we have first looked at the total CO2 atmospheric concentration (in parts per million) along the four RPCs and along our scenarios. As we show in the second panel of Fig [1], there is a very close correspondence in CO2 concentration out to the year 2100 between the Gradual scenario and the RCP6.0 pathway; between the Aggressive scenario and a mixture of the RCP4.5 and RCP 6.0 pathways (with a 40 % weight on the RCP4.5 concentration); and between the Slow scenario and a mixture of the RCP6.0 and RCP8.5 pathways (with a 80% weight on the RCP6.0 concentration). The match follows the expected pattern, with the concentrations associated with the Aggressive, Gradual and Slow schedules matching concentrations associated with higher and higher forcings.

Since concentration is only one aspect of the RCPs narrative, we have also looked at the temperature anomaly and CO2 emissions by the year 2100, as predicted by the DICE model, and by the various RCPs, and at the location of the peak of the CO2 emission function. We found the following close matches, broadly in line with the results above. For the Gradual schedule the best match is invariably the RCP 6.0 scenario, whose emission of 51 Gt CO2/yr matches closely the DICE-predicted emissions of 47 Gt CO2/yr. The likely range for the RCP 6.0 likely temperature anomaly by the end of the century is K 1.6 to K 3.3, to be compared with the DICE temperature anomaly of K 3.3. In the DICE Gradual schedule the CO2 emissions peak in 2075, and in 2080 along the RCP 6.0 pathway.

For the Slow schedule the best match is obtained with the RCP 8.5 pathway: the average RCP 8.5 temperature anomaly by 2100 is K 3.7 (likely range 2.6 to 4.8), and the corresponding DICE quantity is K 4.00. The average RCP 8.5 CO2 emissions by 2100 is 105 Gt CO2/yr, to be compared with a DICE emissions of 100 Gt CO2/yr. In the DICE Slow schedule emissions keep on rising throughout the 21st century, as they do in the RCP 8.5 pathway.

Finally, the Aggressive schedule is best matched by the RCP 4.5 (and occasionally by the RCP 2.6): the CO2 emissions by 2100 of RCP 2.6 and RCP 4.5 are 0 and 17 Gt CO2/yr, and the DICE

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32 The concentrations for the RCPs are CO2 equivalent gases, aggregating gases under the Kyoto Protocols. The CO2 concentration for the DICE model is what is referred to in the documentation as ‘concentration in the upper strata’.

33 The technical meaning of the term ‘likely’ in the IPCC reports is ‘with likelihood higher than 66 percent’.
(a) The CO2 concentration in parts per million associated with the *Aggressive* schedule and with a mixture of the RCP4.5 and RCP6 pathways (with weight of 0.4 on RCP4.5).

(b) The CO2 concentration in parts per million associated with the *Slow* schedule and with a mixture of the RCP6 and RCP8.5 pathways (with weight of 0.8 on RCP6).

Figure 4: Comparison of the CO2 concentrations (in parts per million) associated with our fast and slow abatement schedules and with various Representative Concentration Pathways (or combinations thereof).
emissions by the same date are 0. As for the temperature anomalies these are \( K \) 2.3 by 2100 for the DICE model and in the likely range 1.3 to 2.9 for the RCP 4.5 pathway, with an average value of 2.1. In the DICE exogenous Aggressive schedule the CO2 emissions peak in 20 years from now, and in 10 years and 20 years along the RCP 2.6 and RCP 6.0 pathways, respectively. We can therefore conclude that our scenarios can be mapped with good accuracy to the well-established and well-studied RCPs used in the Assessment report by the IPCC.

5 The Investigation Methodology

Having chosen these sets of abatement schedules, our investigation proceeds as follows.

We assume that several key parameters (see the discussion in Section 6) that characterize the DICE economy and the climate system can assume one of five values: the DICE default one, or two ‘up’ and ‘down’ values, symmetrically positioned around the DICE value, and chosen to represent moderate and severe up and down shocks to the parameter in question. We can now put ourselves in the shoes of a social planner who is uncertain about the true value of the parameters, and has to choose between one of the three feasible courses of action. How is she going to make her choice?

For each of the parameters, given its possible five values we calculated the total discounted utility corresponding to the three abatement schedules. More precisely, we denote by \( U_{ij} \) the total discounted utility associated with abatement and savings schedule, \( i \), and uncertain parameter, \( j \):

\[
U_{ij} = K_1 \left[ \sum_{k=1,n} L(t_k) u \left( \frac{c_{ij} \times 1000}{L(t_k)} (t_k) \right) r r(t_k) \right] + K_2
\]

where \( c_{ij}(t_k) \) is the value at time \( t_k \) of consumption per person under the abatement and savings schedule \( i \), for the parameter value \( j \), \( r r(t_k) \) is the factor used to discount utility from time \( t_k \) to today, \( L(t_k) \) is the population at time \( t_k \), and \( K_1 \) and \( K_2 \) are economically irrelevant multiplicative and additive scaling constants added for consistency with the DICE code.\(^{34}\) As for the power utility function, \( u \), it is given by

\[
u(x) = \frac{x^{1-elasmu}}{1 - elasmu} - 1
\]

and \( elasmu \) is the elasticity of marginal utility (set in the DICE code at 1.45).

By following this procedure, for each set we produce a \( 3 \times 5 \) matrix, with states of the world (parameter values) as columns, and courses of actions as rows. See Tab 1 for an example. To be clear: the entry under the columns, say, +25\% for parameter \( a_3 \) and aligned with the row labelled, say, Aggressive gives the sum of the discounted per-period utilities obtained by running the DICE code with the Aggressive abatement schedule and the parameter value shocked by +25\%.

Next, given this tableau, we employ three decisional criteria that do not require knowledge of probabilities attaching to the five parameter values – namely the maximax, the maximin and the minimal regret criteria. Recall that the maximax criterion maximizes the best outcome. The maximin \( ^{Wald} \)

\(^{34}\)The values for \( K_1 \) and \( K_2 \) are 0.03025 and \( -10,993.704 \), respectively.
(1949)) has a Rawlsian (Rawls (1972)) flavour to it, in that it chooses the course of action that maximizes the worst outcome in all the possible states of the world. Finally, the minimal regret criterion (Savage (1954) ) minimizes the opportunity cost of not having chosen the best course of action for each state of the world. More precisely, if $A$ designates the set of actions, $a(s)$ a specific action under state $s$, $j(a, s)$ its associated utility, $S$ the set of states and $s$ a specific state, maximin, maximax and minimal regret criteria are defined as follows:

**maximin:** $\max_{a \in A} \left[ \min_{s \in S} j(a, s) \right]$

**maximax:** $\max_{a \in A} \left[ \max_{s \in S} j(a, s) \right]$

**minimal regret:** $\min_{a \in A} \left[ \max_{s \in S} (\max_{a' \in A} j(a', s) - j(a, s)) \right]$

While the maximax and maximin criteria are often criticized for looking exclusively at ‘what can go right’ or ‘what can go wrong’, respectively, the minimal regret criterion is more appealing, and constitutes a more interesting benchmark against which to pit the expected utility approach that we describe below.

If we make the stronger assumption that the social planner can assign probabilities to the discrete set of five possible parameter values, we also explore the policy recommendations offered by the Expected Utility Maximization and by the Smooth Ambiguity Aversion (Klibanoff, Marinacci and Mukerji, 2009) decisional criteria. We make use of the Smooth Ambiguity Aversion criterion in order to capture the planner’s aversion to ambiguity. This can be important because studies carried out using non-separable (Zinn-Epstein-like) utility functions (although under settings simpler than the DICE model) have found that there is significant value in the early resolution of uncertainty, and that this recommends a steeper optimal abatement programme. See in this respect Ackerman, Stanton, and Bueno (2013).

More precisely, if $\{q_i\}$ are the probabilities attaching to the $n$ parameter values, $\{x_i\}$, and $\{U_i(x_i)\}$ the associated total discounted utility, a generic Smooth Ambiguity Welfare, $SAW$, is defined as

$$SQW = \phi^{-1}\left[ \sum_i q_i \phi(U_i(x_i)) \right]$$

for some concave aggregator function $\phi$. For our analysis, we chose as aggregator function

$$\phi(x) = \frac{x^{1-\chi} - 1}{1 - \chi}$$

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35 Several other non-probabilistic criteria have been proposed, such as the $\alpha$-maximin criterion, defined by $\max_{a \in A} [\alpha \min_{s \in S} a(s) + (1 - \alpha) \max_{s \in S} a(s)]$ by Arrow and Hurwicz (2010). However, they often depend on an arbitrary ‘mixing’ parameter $\alpha$. A detailed analysis over a range of possible values for $\alpha$ would make the present work, already very rich in permutations, too heavy.

36 Actually, we simply impose the milder requirement that the social planner only knows the first two moments of the distribution of the parameter values. Given this level of knowledge, a Gaussian distribution is the corresponding maximum-entropy distribution. The planner therefore uses the discrete probabilities $q_i$ associated with a Gaussian density with these two moments. For the optimization over five values the discrete probabilities were chosen to be $q_{\pm40\%} = 0.13$, $q_{\pm25\%} = 0.20$ and $q_0 = 0.34$. 

This leaves open the question of the choice of the value for the parameter \( \chi \). In this respect there is little agreement in the literature, with two limiting cases providing limited guidance: one the one hand we know that, if we define by \( A_{\phi} \) the absolute ambiguity aversion,

\[
A_{\phi} \equiv -\frac{\phi''}{\phi'} \tag{8}
\]
as \( A_{\phi} \) tends to infinity, smooth ambiguity aversion produces the same decisional outcome as the max-min criterion (see Gollier (2013), Chapter 6); at the opposite end, ambiguity aversion vanishes if the \( \chi \) coefficient is lower than the risk aversion coefficient. We therefore take guidance from the empirical and review work carried out in Altug, Cakmakli, Collard, Mukerji, and Ozsoylev (2010) and Gallant, Jahan-Prevar, and Liu (2015), and consider values of 9 (in line with Ju and Miao (2012)) and 19 (in line with Jahan-Parvar and Liu (2014)).

### 6 Justification of the Parameter Shocks

As explained in the introductory section, we account for parameter uncertainty by calculating the discounted utility corresponding to the three abatement schedules discussed in Section (4) after shocking in turn each of the following parameters symmetrically around the default DICE value. The parameters that we shocked and the justification for the chosen shocks are detailed below. In all cases we applied both a ‘mild’ symmetric shock (very approximately one standard deviation) and a more severe (still symmetric) one (approximately one-and-half-to-two standard deviations).\(^{37}\)

1. **Temperature damage exponent,** \( a_3 \). In the DICE model the parameter \( a_3 \) enters the damage function as \( \text{damfrac} = a_2 \times T^{a_3} \) and has a default value of 2. In the PAGE Integrated Assessment Model, the damage parameter is sampled via Monte Carlo simulations from a triangular distribution with support 1 to 3. This gives a standard deviation for the distribution of values for \( a_3 \) of 0.41. In their study of the effect of a higher damage exponent on the optimal abatement effort, Dietz, Hope, Stern and Zenghelis (2007) raise the exponent to a value of 3. Akerman, Stanton and Bueno (2010) argue that since ‘there is virtually no empirical evidence on the likely damages from large temperature changes, estimates of the shape of the damage function remain highly uncertain.’ Nordhaus and Moffat (2017) undertake a thorough meta-analysis of damage data, and conclude that an exponential with no linear term gives the best functional fit to the damage data (which are concentrated in the 2-to-4 C region – see their Figure 1). They find an optimal exponent in the region of 2 (as in the DICE specification). They also consider the possibility of sharp threshold in the damage function in the vicinity of 2 C: they do so by arguing that the existence of a ‘kink’ in the damage function above some threshold level would manifest itself as a high exponent (high convexity), and they find no evidence for this.\(^{38}\) However, a damage function with a kink need not be characterized by the same exponent, and may even by described by two almost-linear segments linked by a very-high-convexity juncture. Again, there is virtually no evidence of what the damage could be for very high values of the temperature anomaly. In

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\(^{37}\)For ease of cross-reference with the DICE code we use in this section the symbols and variable names used in the DICE code.

\(^{38}\)...An important example that has been used in policy discussions is an approach which assumes a sharp threshold at a temperature increase of 2 C; implicitly this implies a very sharp kink in the curve near that threshold...’, page 16
the light of this, the ‘up’ and ‘down’ values we have chosen of 2.57 and 1.56 appear well in the range of plausible values for \( a_3 \), and values of 2.98 and 1.32 would correspond to more extreme realizations.

2. **Temperature damage proportionality constant, \( a_2 \).** To our knowledge, much less attention has been given to the confidence on the proportionality constant, \( a_2 \), in the damage function as \( \text{damfrac} = a_2 \times T^{a_3} \), than to the exponent \( a_3 \). In his discussion of the parameters of the DICE 2007 model, Nordhaus (2007) quotes in Appendix Table VII-1 a standard deviation of 0.0013 (in units of fraction of global output), for a mean value of 0.00236. The mean value for the 2016 version of DICE is 0.00236. The change in value simply due to DICE parameter updating from 2007 to 2107 is therefore close to 20%. In the present study we therefore apply a variability of 25% for the medium shocks and of 40% for the severe shocks.

3. **Rate of growth of total factor productivity, \( ga_0 \).** The growth in factor productivity plays an important role in the DICE model, as it governs the rate at which the total factor productivity (TFP) will grow over time.\(^{39}\) Growth in the TFP is, in turn, the key engine in pro capita gross output via a Cobb-Douglas production function. At a qualitative level, high growth makes future generation richer, and, because of declining marginal utility, the same sacrifice to abate climate change less painful. The same high growth, however, exacerbates the climate damage via higher emissions. The net outcome therefore comes from balancing these two competing factors. The relatively slow ramp-up advocated by the optimal DICE path is normally explained (see, eg, Nordhaus (2007b) and Nordhaus and Moffat (2017)) in terms of the first effect prevailing over the second.

There is a wealth of theoretical studies about the adjustment to the social discount rate (of which the rate of growth in the economy is a key component) in the case of an unknown trend of economic growth (see, eg, Gollier (2013)), but a dearth of empirical estimates of what the trend volatility should be. Not surprisingly, there are correspondingly large differences in estimates of long-term trends in Total Production Factor (both for developed and emerging economies). Gordon (2016), for instance, in a well-known if controversial study argues that, at least in the case of the US economy, the growth experienced in the second half of the twentieth century was due to a non-repeatable combination of circumstances, and that similar growth in output are not to be expected. Grauwe (2019) and Picketty (2014) argue along similar lines. In a recent study, Kim and Loayza (2019) conduct simulation studies of the rate of growth (by region) of the Total Production Factor (TFP), and in most cases they find that TFP growth follows a convex path that increases at a decreasing rate, reaches a maximum, and then decreases or stabilizes.\(^{40}\) Their study covers much shorter horizons than the DICE horizon, yet find large variations (from -0.6% to 2.55% across regions and simulations) in the TFP growth rate. Christensen, Gillingham, and Nordhaus (2018) present projections of economic growth to the end of the century, and find a median value of 2.1% for the growth rate, with a higher-than-usual annual standard deviation of 1.1%. We also note that the standard deviation in Nordhaus (2007b) for \( ga_0 \) is 0.0040 (for a

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\(^{39}\)”...By far the most important uncertain variable for climatic outcomes is the growth in total factor productivity. This is the main driver of economic growth in the long run, and output trends tend to dominate emissions and therefore climate change...” Nordhaus (2007b), page 109.

\(^{40}\)emphasis added
mean value of 0.0092 per annum. In the light of this, we find that, again, ‘mild’ and ‘severe’ percentage shocks of ±25% and ±40% are reasonable for the task at hand.

4. Exponent of the cost function, expcost2. In the DICE model the abatement costs have the form

\[ abatecost(t) = y_{gross}(t) \times cost1(t) \times \mu(t)^{expcost2} \]  

with \( cost1(t) = pback(t - 1) \times \sigma(t)/expcost2/1000 \). The discussion of the mitigation cost function, \( abatecost(t) \), in Nordhaus (2007b) is mainly focussed on the ‘backstop technology, ie, on the yet-to-be-fully-implemented technology (such as nuclear fusion) that will replace, albeit at a high cost, all carbon-emitting sources of energy. Based on our findings on the sensitivity of the optimal schedule on the exponent of the damage function, and since the abatement cost grow exponentially in the abatement fraction, \( \mu \), we focus the attention on the damage exponent, expcost2.

In the documentation of the 2007 version the DICE model Nordhaus (2007a), the results of a log-linear regression are reported where the quantity \( cost1(t) \times \mu(t)^{expcost2} \) is regressed against abatement costs estimates. The value reported for expcost2 is 2.05 with a standard error of 0.344. In the 2013 and 2106 versions of the DICE model, the same quantity was given the value of 2.8 and 2.6. In the absence of more precise information, we therefore propose the standard shocks used elsewhere in the paper of 25% and 40% as reasonable values for one- and two-standard-deviation moves in the parameter expcost2.

The list of parameters is a subset of the parameters shown in Nordhaus (2007a) to have the highest impact on the results produced by the DICE model. We have not shocked the pure rate of social time preference, because this is not a parameter about the physical world or the economy about which a planner may have uncertainty (of course, different planners may have different social time preferences, but this is a different matter). We have also not shocked the rate of growth of the population, or the rate of atmospheric retention of CO2.

Finally, we note that the parameters \( a_2 \), \( a_3 \) and expcost2 must be strictly positive, and, for a generic strictly positive parameter, \( x \), with unperturbed value \( x_0 \) we therefore applied the shocks:

\[ x_i = x_0 \exp^i \text{ for } i \in [-0.40, -0.25, 0, +0.25, +0.40] \]  

7 Results

In this section we take as inputs to the various decisional criteria under consideration the total discounted utility, \( U_{ij} \), (Equation 4) associated with a specific abatement and savings schedule \( i \) when

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\footnote{This value refers to an earlier version of the DICE model, and the expected growth per annum cannot therefore be directly compared. The standard deviation, however, should be more transportable. See the discussion on page 226 of Nordhaus (2007b).}

\footnote{We have considered the impact of endogenizing, rather than shocking, the carbon intensity (\( \sigma(t) \)) and abatement cost (\( pback(t) \)) functions, ie, to make them dependent on the cumulative level of abatement undertaken to a given time. We draw in this respect on the learning-by-doing literature as applied to problems of climate change, as in Messner (2013), Gruebler and Messner (1998), Barreto and Kypreos (1999), and van der Zwaan, Gerlach, Klassen, and Schrattenholzer (2002). For the sake of brevity the results are not reported in this study, but, qualitatively they reinforce the conclusions of our paper.}
parameter $j$ is shocked. The preference parameters are those in DICE 2016. More precisely, given an abatement schedule and a shocked value of the parameter we calculate the total discounted utility for each of our three possible courses of action (Aggressive, Gradual and Slow) and apply the decision criteria described in Section 5 to see which strategy is recommended by each of them.

The courses of action recommended by the different decisional criteria are displayed in Tab 2 for parameters $a_2$, $a_3$, $ga0$ and $expcost2$. The first observation is that, with the important exception of the damage exponent, $a_3$ – discussed in detail below –, the Gradual strategy is always preferred. It is perhaps surprising that a slow ramp-up of abatement investment (as is recommended by the Slow schedules) is not deemed optimal by the Maximin criterion (which focuses on the best possible outcome) even in the case of a much lower damage exponent or a much higher growth in the total factor of production than is assumed in the DICE model. One may have in fact expected that, if a low value for the damage exponent were to materialize, postponement of the abatement efforts would have been deemed desirable by the Maximin criterion. Similarly, with a CRRA utility function, the richer we are going to be in the future, the more it is advisable to postpone a fixed abatement sacrifice; also in this case one could therefore have expected that deferring the abatement would have been preferred by the Maximin criterion. In reality, neither prediction is correct. As for the growth in the total factor of production is concerned, it is because states of high growth are also associated with high emissions, and hence high damage. The two competing factors, declining marginal utility with higher wealth levels and greater climate damage, therefore keep the Gradual solution as the optimal one even in high-growth states. And as for the low realizations of the damage exponent, the welfare gains from saving on abatement costs is not compensated from the incurred damages, even if the damage exponent is much lower than the DICE model assumes.

When it comes to the damage parameter, $a_3$, the analysis is more complex. We first note that if schedule A has a higher utility than schedule B for all values of the shocked parameter (ie, if schedule A dominates schedule B), then no reasonable decisional criterion can prefer B to A.\footnote{To prove this for the minimum-regret criterion, for which the property is not obvious, let’s denote by $A^*$ the dominant action such that, for all actions $a$ and for all states $s$, $j(A^*(s)) \geq j(a(s))$. The minimum-regret criterion becomes in this case $\min_a [\max_s [j(A^*(s)) - j(a(s))]]$. Let’s now assume that the implied minimum regret action $a^*$ is different from $A^*$. Then, the minimum regret criterion implies that there must exist at least one state $s$ such that $j(A^*(s)) - j(a^*(s)) < j(A^*(s)) - j(A^*(s)) = 0$. This contradicts the premise on $A^*$, leading to the conclusion that $A^*$ is necessarily the minimum-regret action.} In the light of this observation, we note from Tab 1 that the Gradual schedule dominates the Slow schedule for all shocks of the parameter $a_3$. Therefore, we do not discuss the Slow schedule further in what follows. Moving to the comparison between the Gradual and Aggressive schedule, we find that, when the damage exponent is uncertain, the Gradual schedule not only fails to dominate the Aggressive schedule; it is also inferior to the Aggressive schedule according to both the Minimal Regret and the Maximin criteria. Since the Maximin criterion places the emphasis on avoiding the worst outcomes, this is not very surprising. What is more interesting is that also the Minimal Regret criterion, which minimizes the opportunity cost of not having chosen the best course of action, also gives the same recommendation.\footnote{We note that when the carbon intensity, $\sigma$, and and the cost of the backstop technology, $pback$, are endogenized as described in footnote 42, the utility associated with the Aggressive schedule is higher than for the Gradual schedule even for a mild up shock for the damage exponent, $a_3$. We also note that endogenization of the functions $\sigma$ and $pback$ always increases the relative utility of Aggressive versus Gradual schedules.}
Shocking the temperature damage exponent parameter $a_3$: Utilities

<table>
<thead>
<tr>
<th>Log-normal shock</th>
<th>-40%</th>
<th>-25%</th>
<th>0%</th>
<th>+25%</th>
<th>+40%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter value</td>
<td>1.34</td>
<td>1.56</td>
<td>2</td>
<td>2.57</td>
<td>2.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Utilities</th>
<th>Aggressive</th>
<th>Gradual</th>
<th>Slow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4396.56</td>
<td>4392.12</td>
<td>4558.33</td>
</tr>
<tr>
<td></td>
<td>4380.42</td>
<td>4516.86</td>
<td>4534.15</td>
</tr>
<tr>
<td></td>
<td>4358.05</td>
<td>4440.17</td>
<td>4492.61</td>
</tr>
<tr>
<td></td>
<td>4333.84</td>
<td>4331.58</td>
<td>4379.39</td>
</tr>
<tr>
<td></td>
<td>4315.42</td>
<td>4316.42</td>
<td>4194.23</td>
</tr>
</tbody>
</table>

Table 1: The total discounted utility, $U_{ij}$, in Equation 4, when the damage exponent, $a_3$, is shocked by the amounts in the row labelled ‘Lognormal shock’, for the Aggressive, Gradual and Slow schedules.

Shocking parameters: Criteria outcomes

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Max EU</th>
<th>MaxiMin</th>
<th>MaxiMax</th>
<th>SAA</th>
<th>Minimal Regret</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter shocked $a_2$</td>
<td>Gradual</td>
<td>Gradual</td>
<td>Gradual</td>
<td>Gradual</td>
<td>Gradual</td>
</tr>
<tr>
<td>$g_0$</td>
<td>Gradual</td>
<td>Gradual</td>
<td>Gradual</td>
<td>Gradual</td>
<td>Gradual</td>
</tr>
<tr>
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<td>Gradual</td>
<td>Gradual</td>
<td>Gradual</td>
</tr>
<tr>
<td>$a_3$</td>
<td>Gradual</td>
<td>Aggressive</td>
<td>Gradual</td>
<td>Gradual</td>
<td>Aggressive</td>
</tr>
</tbody>
</table>

Table 2: The course of action (labelled ‘Aggressive’, ‘Gradual’ or ‘Slow’) recommended by the criteria in the first row (Maximization of expected utility, labelled ‘Max EU’, MaxiMin, MaxiMax, Smooth Ambiguity Aversion, labelled ‘SAA’, and Minimal Regret).

We would also like to understand for which values of the damage parameter the Aggressive discounted utility becomes greater than the Gradual discounted utility. We therefore examine how the discounted utility behaves for more extreme values of the parameter $a_3$, and for which levels a cross-over occurs between the Aggressive and the Gradual utility. The answer is shown in Fig 5, which displays the value of the total discounted utility when we apply to the parameter a shock ranging between -70% and +70%. The cross-over between the Aggressive and the Gradual utility occurs for values of $a_3$ above approximately 3.00. (Recall that the default DICE value is $a_3 = 2$.) We also note that there are no reasonable values of the damage exponent for a cross-over between the Gradual and Slow schedules to occur.

8 Discussion and Conclusions

In this paper we have looked at the case of a non-infinitely altruistic social planner who is ignorant about the true parameters that govern the climate/economy system, and who cannot act optimally. The expected-utility-maximization criterion is therefore of limited assistance in choosing which of the available courses of action she should undertake. It is well known that, with the DICE-model preference parameters, the optimal solution calls for a gradual abatement effort. We have therefore addressed the question of whether limited information and social-political constraints can make a slow or steep abatement schedule preferable to a gradual one, even if we keep the original preference parameters of the DICE model. This is important, because the debate in the literature on the steepness of the optimal abatement schedule has mainly focussed on the ‘correct’ social discount rate.
We are aware that studying paths away from optimality introduces an element of arbitrariness: we have therefore mapped our schedules to well-studied Representative Concentration Pathways (Pachauri and Meyer (2014)), which have been extensively studied and used in the literature (see, eg, van Vuuren, Kriegler, Neil, Rihai, Carter, Edmonds, Allegatte, Kram, Mathur, and Winkler (2011)), and have gained quasi-benchmark status. Despite the differences in modelling approaches, and goals, between the DICE model and the IPCC scenario analysis in Pachauri and Meyer (2014), the mapping from our Aggressive, Gradual and Slow schedules to Representative Concentration Pathways of increasing ‘severity’ has proven very satisfactory.

The results presented in Section 7 allow us to draw the following conclusions. First and foremost, a Slow abatement schedule is a very bad policy even in the most (reasonably) benign outcomes for the uncertain parameters – in particular, even if we end up being very rich, or if the damage exponent is revealed to be much lower than currently expected. From whichever decisional angle one looks at the problem – irrespective, that is, of whether one places the emphasis on maximizing the best outcome, on minimizing the worst outcome or the opportunity cost –, ‘rolling the DICE’ and hoping that everything will work out for the best is an inferior policy.

Next, we find that, if the parameter uncertainty were limited to the rate of growth in the economy or to the cost of abatement a Gradual schedule would remain solidly preferred by all the decisional criteria we have looked at. Linear changes in the damage function (ie, uncertainty in the parameter $a_3$ in $\text{damfrac} = a_2T^{a_3}$) do not alter these results. The Smooth Ambiguity Aversion criterion of Klibanoff, Marianacci, and Mukerji (2005) (that does require knowledge of the probabilities attaching to the different parameter values) does not change the ordering. For these two sources of uncertainty, the conclusion is actually stronger: the tables with the total discounted utilities show that, when the uncertainty is limited to these parameters, the Gradual schedule dominates both the Aggressive and the Slow schedules, so that no reasonable decisional criterion could give a different answer, at least over the (wide) range of parameter uncertainty considered.
However, if the damage exponent (the parameter $a_3$ in $damfrac = a_2T^{a_3}$) were to increase significantly above the default value in the DICE model, then different decisional criteria, that place their emphasis on different aspects of welfare, can recommend a significantly faster ramp-up of the abatement efforts than a Gradual schedule would. Unfortunately, the precise behaviour of the damage function is very difficult to extrapolate to high temperature anomalies, for which we have no direct empirical information. In particular, the magnitude of feedback effects (eg, from the reduction in albedo, from the thawing of permafrost, from the increased concentration of H2O in the atmosphere at higher temperatures, etc, as discussed, eg, in Pierrehumbert\cite{2010}) is currently very imperfectly understood, and this could significantly alter the current ‘best fits’ to climate damage (see in this respect the discussion on the range of uncertainty for the parameter $a_3$ in Section\cite{5}). So, one important conclusion from our study is that devoting research efforts to resolving the current uncertainty about the damage function would have the greatest benefit when it comes to choosing among feasible courses of actions.

These results should be qualified in several directions. As Nordhaus recognizes (Nordhaus\cite{2008}), the DICE model is not built to capture the possibility of climate ‘tipping points’ or abrupt climate changes. Yes, in our analysis we have stressed many key parameters, sometimes over considerable ranges, and we have looked at the consequences of acting on erroneous knowledge of these parameters. However, our study remains firmly rooted in the DICE modelling framework. For certain outcomes, such as what the damages could be if the temperature anomaly exceeded, say, 6 C, we simply do not know whether the underlying modelling approach (let alone the parameters) still holds.

Furthermore, we have imparted what appear to be, in the light of the discussion in Section\cite{6} reasonable shocks to the key parameters. However, we have not considered the possibility of radical uncertainty, as considered, for instance, in Weitzman\cite{1998}, with very fat tails for the parameter distributions. Admittedly, we have looked at decisional criteria that do not require knowledge of the probability distributions for the model parameters. However, these criteria do depend on the ranges for the parameters, that we have implicitly truncated beyond the highest and lowest parameter values examined.

We should also say that the DICE approach (like most Integrated Assessment Models) can but awkwardly accommodate important features, such as the loss of biodiversity, of natural beauty, etc, and that it has a resolutely human-centric focus. (This focus is standard in the economics analyses, but hotly debated in the philosophical literature. For instance, for a thoughtful analysis of what role, if any, these considerations may play in reaching decisions about the environment and future generations, see Williams\cite{1995}.) One can, of course, directly add at least some of these features to the utility function, but then the answer one obtains is virtually ‘baked into’ the weight given to the non-consumption inputs: the model is then used to confirm, not to discover. The robustness results that we have presented cannot address the important, but intractable, aspect of non-consumption inputs to the utility function, or of the interests of non-human agents.

Finally, as mentioned in the introduction, a very fast ramp-up (a ‘true’ Stern optimal abatement scheduled) is easily obtained within the DICE framework by positing an elasticity of marginal utility (exponent of the power utility function) of 1 and an extremely high aversion to intergenerational inequality (with a vanishingly small utility discount rate). If these choices for the utility function are

\footnote{See, in particular, Chapter 20, “Must a concern for the environment be centred on human beings?”}
deemed normatively correct, then the very fast ramp-up of the abatement schedule is the inevitable conclusion. This is, however, a very different topic, beyond the scope of the present work.
9 Author Declarations

9.1 Author Contribution
All authors have contributed to the text, with LM focussing on the literature review in particular. RRe and RRo wrote the MatLab code. RRo prepared the figures.

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9.4 Ethics Approval and Declarations
Not applicable.

9.5 Consent to Participate
Not applicable.

9.6 Consent for Publication
Not applicable.

9.7 Availability of Data and Material
The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

9.8 Code Availability
The code used to carry out the analysis is a custom code written by the authors using the MatLab® software.

9.9 Author’s Contributions
All authors (RR, RR and LM) equally contributed to the project. All authors read and approved the manuscript.
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