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Analysis of power law non-linearity in solitonic solutions using extended hyperbolic function method

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Abstract

This paper retrieves the optical solitons to Biswas-Arshed equation (BAE) which is deliberated in the lack of self-phase modulation by applying the new extended hyperbolic function method (EHFM). Novel constructed solutions have the shape of bright, singular, periodic singular, and dark solitons. The achieved solutions contain key applications in engineering and physics. These solutions define the wave performance of the governing models, actually. The outcomes disclose that our scheme is very active and reliable. The acquired results are illustrated by 3-D and 2-D graphs to understand the real phenomena for such sort of non-linear models.

Keywords: New EHFM; Power law non-linearity; Optical solitons.

1 Introduction

Solitons have much importance in various areas of science and nature such as waves, geology, wave propagation, population ecology, fluid dynamics, computer science, biology, plasmas, heat, birefringent fibers, mechanics and optics. There are many mathematical models that efficiently demonstrate the properties of soliton transmission [1-36]. The key object for the presence of solitons in the optical fibers is to keep balance among GVD and nonlinearity. In special conditions, there may a circumstances arise which produce small nonlinearity and low GVD. Recently, Biswas and Arshed [37-42] estimated a precise creative clue to grip the circumstances where non-linearity and GVD are small which is prented in the model called BAE. This governing model can act as a feasible model to study the properties of solitons in crystals, PCF, and optical fibers. This article studies the BAE with power law via new EHFM [43-45] that lead to construct singular, bright, dark-bright and periodic-singular optical soliton solutions.
The layouts of this paper are as. The governing model is described in section 2. Analysis of the proposed method new EHFM is presented in section 3. In section 4 the new EHFM is applied and section 5 consists the findings and discussions. Conclusions of this paper are posted in section 6.

2 Biswas-Arshed Equation

The BAE with full non-linearity [37-42] is given by

\[ \imath u_t + a_1 u_{xx} + a_2 u_{xt} + \imath (b_1 u_{xxx} + b_2 u_{xxt}) = \imath [\phi(|u|^{2n})_x u + \sigma(|u|^{2n})_x u + \theta |u|^{2n} u], \quad (1) \]

\( u(x, t) \) and \( n \) are complex-valued function and full nonlinearity parameter respectively. On the left of the (1), \( a_1 \) and \( a_2 \) indicate the temporal evolution, coefficient of GVD and coefficient of STD respectively. Next, \( b_1 \) and \( b_2 \) are coefficients of 3OD and STD. \( \theta \) and \( \sigma \) represent the effect of self-steepening and non-linear dispersion in the absence of SPM. This effect of dispersion and nonlinearity provides required balance for solitons existence.

Assume that

\[ u(x, t) = P(\eta)e^{i\Psi(x, t)}, \quad (2) \]

where \( P(\eta) \) indicate the amplitude and

\[ \eta = x - ct, \quad \Psi(x, t) = -kx + wt + \epsilon. \quad (3) \]

where \( c \) and \( \Psi(x, t) \) are the velocity and the phase component. While \( k, \ w \) are frequency and wave number, accordingly, while \( \epsilon \) is phase constant of solitons.

Inserting (2), (3) in (1) yields:

The real part gives

\[ -(w + a_1 k^2 + b_1 k^3 - a_2 wk - b_2 wk^2)P + (a_1 + 3b_1 k - a_2 c - 2b_2 c k - b_2 w)P'' = k(\theta + \phi)P^{2n+1}, \quad (4) \]

while the imaginary part gives

\[ (-3b_1 k^2 + b_2 c k^2 + 2b_2 wk - 2a_1 k - c + 2a_2 c k + a_2 w)P' + (-b_2 c + b_1)P'' = [\theta + 2n\sigma + (2n + 1)\phi]P^{2n+1}P'. \quad (5) \]

Integrating (5) and taking the integration constant to be zero, we obtain

\[ (-3b_1 k^2 + b_2 c k^2 + 2b_2 wk - 2a_1 k - c + 2a_2 c k + a_2 w)P + (-b_2 c + b_1)P'' = [\theta + 2n\sigma + (2n + 1)\phi]P^{2n}P'. \quad (6) \]

To get the solutions, we use following substitution

\[ P = W^{1/2n} \quad (7) \]
Eqs. (5) and (6), this gives yields
\[ (a_1 + 3b_1k - a_2c - b_2w - 2b_2ck)(1 - 2n)(W')^2 + 2nWW' = 0 \]
\[ -4n^2 (w + a_1k^2 + b_1k^3 - a_2wk - b_2wk^2)W^2 - 4n^2k(\theta + \phi)W^3 = 0 \] (8)

\[ (2n + 1)(b_1 - b_2c)[(1 - 2n)(W')^2 + 2nWW'] - 4n^2[2n\sigma + \theta + (2n + 1)\phi]W^3 \]
\[ + (-2a_1k - 3b_1k^2 + b_2ck^2 + 2b_2wk + 2a_2ck - c + a_2w)4n^2(2n + 1)(1 - 2n)(W')^2 \]
\[ + 2nWW'W^2 = 0. \] (9)

As \( W(\xi) \) satisfies (8) and (9), the constraint conditions are as follows
\[ \frac{(a_1 - a_2c + 3b_1k - 2b_2ck - b_2w)}{(2n + 1)(b_1 - b_2c)} = \frac{(\theta + \phi)k}{2n\sigma + \theta + (2n + 1)\phi} \]
\[ = \frac{-(w + a_1k^2 + b_1k^3 - a_2wk - b_2wk^2)}{(2n + 1)(b_2ck^2 - 3b_1k^2 + 2b_2wk - 2a_1k + 2a_2ck - c + a_2w)}. \] (10)

Now (8) is analyzed using extended hyperbolic function method.
Setting \( A = b_2ck^2 - 3b_1k^2 + 2b_2wk - 2a_1k + 2a_2ck - c + a_2w \)
and \( B = w + a_1k^2 - a_2wk + b_1k^3 - b_2wk^2 \), gives
\[ A[(1 - 2n)(W')^2 + 2nWW'] - 4n^2BW^2 - 4n^2k(\theta + \phi)W^3 = 0. \] (11)

### 3 New EHFM

Two phases of the new EHFM are

**Form 1:** Let (11) has the solution as
\[ W(\eta) = \sum_{i=0}^{N} F_i \Phi^i(\eta), \] (12)

where \( F_i \) are constants and \( \Phi(\eta) \) satisfies the auxiliary ODE as
\[ \frac{d\Phi}{d\eta} = \Phi \sqrt{\tau + \mu \Phi^2}, \tau, \mu \in R. \] (13)

To find the number \( N \), we use balancing rule on (11). Substituting (12) in (11) along with (13) produces a system of algebraic equations for \( F_i (0 \leq i \leq N) \). A set of solutions is acquired by solving this system, that accepts (13) as

**Set 1:** If \( \tau > 0 \) and \( \mu > 0 \),
\[ \Phi(\eta) = -\sqrt{\frac{\tau}{\mu}} \csc h(\sqrt{\tau}(\eta + \eta_0)). \] (14)

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Set 2: If $\tau < 0$ and $\mu > 0$,

$$\Phi(\eta) = \sqrt{-\frac{\tau}{\mu}} \sec \left(\sqrt{-\tau}(\eta + \eta_0)\right).$$

(15)

Set 3: If $\tau > 0$ and $\mu < 0$,

$$\Phi(\eta) = \sqrt{\frac{\tau}{-\mu}} \sech \left(\sqrt{\tau}(\eta + \eta_0)\right).$$

(16)

Set 4: If $\tau < 0$ and $\mu > 0$,

$$\Phi(\eta) = \sqrt{-\frac{\tau}{\mu}} \csc \left(\sqrt{-\tau}(\eta + \eta_0)\right).$$

(17)

Set 5: If $\tau > 0$ and $\mu = 0$,

$$\Phi(\eta) = \exp \left(\sqrt{\tau}(\eta + \eta_0)\right).$$

(18)

Set 6: If $\tau < 0$ and $\mu = 0$,

$$\Phi(\eta) = \cos \left(\sqrt{-\tau}(\eta + \eta_0)\right) + i\sin \left(\sqrt{-\tau}(\eta + \eta_0)\right).$$

(19)

Set 7: If $\tau = 0$ and $\mu > 0$,

$$\Phi(\eta) = \pm \frac{1}{\sqrt[4]{\mu}(\eta + \eta_0)}. $$

(20)

Set 8: If $\tau = 0$ and $\mu < 0$,

$$\Phi(\eta) = \pm \frac{i}{\sqrt{-\mu}(\eta + \eta_0)}. $$

(21)

Form 2: Adopting the same pattern as above, assume (12) satisfies the auxiliary ODE as follows

$$\frac{d\Phi}{d\eta} = \tau + \mu \Phi^2, \ \tau, \mu \in R. $$

(22)

Substituting (12) into (11) with (22) yields value of $N$, provides a set equations. Assume (22) has the solutions as

Set 1: If $\tau \mu > 0$,

$$\Phi(\eta) = sn(\tau) \sqrt{\frac{\tau}{\mu}} \tan(\sqrt{\tau \mu}(\eta + \eta_0)). $$

(23)
Set 2: If $\tau \mu > 0$,

$$\Phi(\eta) = -sn(\tau) \sqrt{\frac{\tau}{\mu}} \cot(\sqrt{\mu}(\eta + \eta_0)).$$  \hspace{1cm} (24)

Set 3: If $\tau \mu < 0$,

$$\Phi(\eta) = sn(\tau) \sqrt{\frac{\tau}{-\mu}} \tanh(\sqrt{-\mu}(\eta + \eta_0)).$$  \hspace{1cm} (25)

Set 4: If $\tau \mu < 0$,

$$\Phi(\eta) = sn(\tau) \sqrt{\frac{\tau}{-\mu}} \coth(\sqrt{-\mu}(\eta + \eta_0)).$$  \hspace{1cm} (26)

Set 5: If $\tau = 0$ and $\mu > 0$,

$$\Phi(\eta) = -\frac{1}{\mu(\eta + \eta_0)}.$$  \hspace{1cm} (27)

Set 6: If $\tau \in R$ and $\mu = 0$,

$$\Phi(\eta) = \tau(\eta + \eta_0).$$  \hspace{1cm} (28)

Note: $sn$ is well-known sign function.

4 Application of the new EHFM

Form 1: In this section, we utilize the above said method to solve the BAE with power law non-linearity. Using balance principal in (11), yields $N = 2$, so (12) converts to

$$W(\eta) = F_0 + F_1 \Phi(\eta) + F_2(\Phi(\eta))^2,$$  \hspace{1cm} (29)

where $F_0$, $F_1$ and $F_2$ are constants. Inserting (29) in (11) and attains a set of equations in $F_0$, $F_1$, $F_2$, $\tau$ and $\mu$ is obtained. On working the set of equations, we achieve

$$F_0 = 0, \quad F_1 = 0,$$

$$F_2 = F_2, \quad \tau = -\frac{Bn^2}{2An - 2n - A},$$

$$\mu = -\frac{k n^2(\theta + \phi)F_2}{2An - 3n - A},$$  \hspace{1cm} (30)
Set 1: If $\tau > 0$ and $\mu > 0$,

$$u_1(x, t) = \left[ F_2\left( -\sqrt{\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \csc\left( \sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0) \right) \right) \right]^{1/2} \times (e^{i\Psi(x, t)}). \quad (31)$$

Set 2: If $\tau < 0$ and $\mu > 0$,

$$u_2(x, t) = \left[ F_2\left( \sqrt{\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \sec\left( \sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0) \right) \right) \right]^{1/2} \times (e^{i\Psi(x, t)}). \quad (32)$$

Set 3: If $\tau > 0$ and $\mu < 0$,

$$u_3(x, t) = \left[ F_2\left( \sqrt{\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \sech\left( \sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0) \right) \right) \right]^{1/2} \times (e^{i\Psi(x, t)}). \quad (33)$$

Set 4: If $\tau < 0$ and $\mu > 0$,

$$u_4(x, t) = \left[ F_2\left( \sqrt{\frac{B(-3n + A(-1 + 2n))}{k(-2n + A(-1 + 2n))(\theta + \phi)F_2}} \csc\left( \sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0) \right) \right) \right]^{1/2} \times (e^{i\Psi(x, t)}). \quad (34)$$

Set 5: If $\tau > 0$ and $\mu = 0$,

$$u_5(x, t) = \left[ F_2\left( \exp\left( \sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0) \right) \right) \right]^{1/2} \times (e^{i\Psi(x, t)}). \quad (35)$$

Set 6: If $\tau < 0$ and $\mu = 0$,

$$u_6(x, t) = \left[ F_2\left( \cos\left( \sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0) \right) + i\sin\left( \sqrt{\frac{Bn^2}{2An - 2n - A}}(\eta + \eta_0) \right) \right) \right]^{1/2} \times (e^{i\Psi(x, t)}). \quad (36)$$
Where $\Psi(x,t) = -kx + wt + \epsilon, \quad \eta = x - ct$.

**Form 2:**
Using balance principal on (11), attains $N = 2$, so (12) gives

$$W(\eta) = F_0 + F_1\Phi(\eta) + F_2(\Phi(\eta))^2,$$

where $F_0, F_1$ and $F_2$ are constants. Inserting (37) in (11) and comparing the coefficients of each polynomial of $\Phi(\eta)$ to zero, we retrieve a set of equations in $F_0, F_1, F_2, \tau$ and $\mu$. Working on the set of equations, we acquire

$$F_0 = \frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta + \phi A(1-2n)+2n\phi)},$$

$$F_1 = 0, \quad F_2 = \frac{\mu^2(A(1-2n)+3n)}{kn^2(\phi + \theta)},$$

$$\tau = -\frac{Bn^2}{(2n + A(1-2n))\mu}, \quad \mu = \mu.$$  

(38)

**Set 1:** If $\tau \mu > 0$,

$$u_9(x,t) = \left[ \frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta + \phi A(1-2n)+2n\phi)} \right]^{1/2n} \times (e^{i\Psi(x,t)}).$$

(39)

**Set 2:** If $\tau \mu > 0$,

$$u_{10}(x,t) = \left[ \frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta + \phi A(1-2n)+2n\phi)} \right]^{1/2n} \cot(\sqrt{-\frac{Bn^2}{A+2n-2An}(\eta + \eta_0)}) \times (e^{i\Psi(x,t)}).$$

(40)

**Set 3:** If $\tau \mu < 0$,

$$u_{11}(x,t) = \left[ \frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta + \phi A(1-2n)+2n\phi)} \right]^{1/2n} \tanh(\sqrt{-\frac{Bn^2}{A+2n-2An}(\eta + \eta_0)}) \times (e^{i\Psi(x,t)}).$$

(41)
Set 4: If $\tau \mu < 0$,

$$u_{12}(x,t) = \left[ \frac{(A(1-2n)+3n)B}{k(\theta A(1-2n)+2n\theta + \phi (1-2n)+2n\phi)} \right] \times \left( e^{\psi(x,t)} \right).$$

where $\chi = \text{sgn}\left(-\frac{Bn^2}{(2n+A(1-2n))\mu}\right)$, $\Psi(x,t) = -kx + wt + \epsilon$, $\eta = x - ct$.

5 Results and Discussions

we illustrated the solutions of BAE in the form of optical solitons using new EHFM. These solutions have applications in telecommunication to transfer information as solitons have the proficiency to travel long spaces without distortion and without altering their shapes. In this work, we added selected graphical representations of some solutions of BAE to dodge overcrowded the manuscript. Figs. 1-6 represent 2-D and 3-D plots of some optical solitons of (1). Figures 1,6 demonstrate the solutions obtained given in (31) and (42) accordingly, which are singular soliton waves. Whereas figures 2,4 demonstrate the solutions (32) and (39) which are periodic singular soliton waves. Solution (33) given by Figure 3 represents bright soliton and solution (41) provide dark soliton and is shown by Figure 5.

Figure 1: (a) 3D plot of (31) with $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$ $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $c = 2.5$, $\epsilon = -4$.(b) 2D representation of (31) using $t = 1$. 
Figure 2: (c) 3D plot of (32) with $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$ $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = 4$, $c = 2.5$. (d) 2D representation of (32) with $t = 1$.

Figure 3: (e) 3D plot of (33) for $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$ $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = -4$, $c = 2.5$. (f) 2D representation of (33) for $t = 1$. 


Figure 4: (g) 3D graph of (41) for \( F_2 = 1.7, \ A = -2.7, \ B = 11.4, \ n = 1 \ k = -2, \ \theta = -4, \ \phi = 0.8, \ \omega = 3, \ \epsilon = -4, \ c = 2.5. \) (h) 2D representation of (41) for \( t = 1. \)

Figure 5: (i) 3D graph of (43) for \( F_2 = 1.7, \ A = -2.7, \ B = 11.4, \ n = 1 \ k = -2, \ \theta = -4, \ \phi = 0.8, \ \omega = 3, \ \epsilon = -4, \ c = 2.5. \) (j) 2D representation of (43) for \( t = 1. \)
Figure 6: (k) 3D graph of (44) with $F_2 = 1.7$, $A = -2.7$, $B = 11.4$, $n = 1$ $k = -2$, $\theta = -4$, $\phi = 0.8$, $\omega = 3$, $\epsilon = -4$, $c = 2.5$. (l) 2D representation of (44) for $t = 1$.

6 Conclusion

In this paper, we have successfully employed the EHFM on BAE and constructed the bright, periodic, dark, singular and combo solitons. These constructed results have much importance in many areas of non-linear sciences such as physics, birefringent fibers, applied mathematics, optical fibers, engineering, pulse propagation and many more. The outcomes of this study are motivating and improve our understanding for the optical soliton solutions. By these outcomes, we can recognize that the present method is appropriate, useful and skilled for attaining the exact solutions of such kind of problems. The acquired results are fresh, correct and not reported earlier in literature.

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