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Friction and slip measured at the bed of an Antarctic ice stream

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Abstract:

The slip of glaciers over the underlying bed is the dominant mechanism governing the migration of ice from land into the oceans, contributing to sea-level rise. Yet glacier slip remains poorly understood or constrained by observations. Here we observe both frictional shear-stress and slip at the bed of an ice stream, using 100,000 repetitive stick-slip icequakes from Rutford Ice Stream, Antarctica. Basal shear-stresses and slip-rates vary from $10^4$ to $10^7 Pa$ and 0.2 to 1.5 m day$^{-1}$, respectively. Friction and slip vary temporally over the order of hours and spatially over 10s of meters, caused by corresponding variations in ice-bed interface material and effective-normal-stress. Our findings also suggest that the bed is substantially more complex than currently assumed in ice stream models and that basal effective-normal-stresses may be significantly higher than previously thought. The observations also provide previously unresolved constraint of the basal boundary conditions of ice dynamics models. This is critical for constraining the primary contribution of ice mass loss in Antarctica, and hence the endeavor to reduce uncertainty in sea-level rise projections.
Main Text:

Glacier slip is the primary mechanism governing the migration of ice from land into the oceans, providing a major contribution to sea-level rise\(^1,2\). Friction at the bed of a glacier fundamentally limits the speed at which the ice can slip. This friction is controlled by a number of factors, including bed material, the presence of debris in basal ice, and hydrological systems that modulate effective-normal-stresses. However, basal friction and slip remain poorly understood or constrained by observations\(^1,3,5\). Such observational constraint of friction and slip is critical for the verification of ice-bed boundary condition assumptions in ice dynamics models, which are required to reduce uncertainty in corresponding sea-level rise projections\(^4,6,7\).

Previous contributions to address this critical observational void come from laboratory-based experiments\(^8–13\) geophysical studies\(^14–23\), and borehole measurements\(^24,25\). However, to date there have been challenges with such approaches. Laboratory experiments provide insight into fundamental physical properties of the bed material (till)\(^10\) and ice-bed interface interactions\(^12\) but are limited by scale and the diversity of natural glacier beds. Geophysical studies have measured the in-situ bed strength, but with sparse spatial and temporal resolution\(^14\). Borehole measurements of slip are not only sparse, but have not been accompanied by measurements of shear-stress, making quantitative interpretations difficult. The ice streams and outlet glaciers that contribute the majority of ice flux into the oceans likely have active, spatially- and temporally-varying hydrological systems\(^26,27\), perturbing basal friction and slip over short time- and length-scales. An observational void therefore remains.

Here we address this observational void by using icequakes to provide the first spatially-mapped, in-situ observations of both frictional drag and slip-rate at the bed of an ice stream. These icequakes are generated by the sudden release of strain at or near the ice-bed interface. The dataset analyzed comprises 100,000 icequakes\(^28\) from Rutford Ice Stream (RIS), Antarctica (see Fig. 1). The icequakes originate approximately at the center of the ice stream, where the dominant source of drag is postulated to originate from the bed rather than from the shear margins. These icequakes nucleate in clusters that are highly repetitive (see Extended Data Fig. 1), with near-constant inter-event times of the order of 100s of seconds and icequakes clusters active for hours to days\(^28\). These icequakes are inferred to be at the bed from: their hypocentral depths; the consistent flow- and bed- parallel orientation of their double-couple focal mechanism slip-vectors; and full-waveform modelling a typical RIS icequake source\(^20,28–30\). The tight spatial clustering and repetitive nature motivate our use of a rate-and-state friction law in combination with icequake observations to investigate the glacier sliding process. This rate-and-state friction law\(^31\) also enables the calculation of other basal parameters including bed shear moduli and insight into the modulation of glaciological effective-normal-pressures.
Fig. 1. Seismic network and icequake data at Rutford Ice Stream, Antarctica. (a) Location of Rutford Ice Stream (RIS) relative to the Antarctic continent. Topography is from Bedmap2. (b) Map of network with respect to RIS shear-margins. (c) Map of the experiment and icequake data at Rutford Ice Stream, from November 2018 to February 2019. Red scatter points show icequake locations. All icequakes are approximately at ice stream bed. Green inverted triangles show geophone locations. Bed topography data are from the literature. Pink dashed line indicates a bed-character boundary from the literature.

Results

Observed ice-bed friction and slip-rate

The icequake source properties and inter-event times are used in combination with a rate-and-state friction law to calculate: fault effective-normal-stress ($\sigma$); total frictional shear-stress, or drag per unit area ($\tau$); shear-modulus ($G_{\text{bed}}$); slip ($d$); and slip-rate ($v_{\text{slip}}$) at the bed of RIS. Fig. 2 shows these results for a representative subset of icequake clusters. Fault effective-normal-stress, shear-stress and shear-modulus (Fig. 2a-c) vary by orders of magnitude between clusters, even after accounting for uncertainty. However, these parameters are all confined within expected physical limits. Effective-normal-stresses remain below the maximum ice overburden pressure, which is the upper possible limit for the average effective-normal-stress over the entire fault. The observed shear-stress ranges from $\sim 10^4$ to $10^7$ Pa. If the icequake cluster locations, or sticky-spots, contribute more drag than the surrounding bed, then sticky-spot shear-stresses could theoretically have a much higher limit than the average bed shear-stress. Although bed shear moduli vary significantly between clusters, the majority of the clusters’ shear moduli agree with one of the only previous seismically-derived in-situ measurements, 70 MPa, from Whillans Ice Stream. Additionally, measurements do not exceed the shear-modulus of ice.
Slip-rates show smaller variations in amplitude, from ~0.2 to 1.5 m day\(^{-1}\), but have higher associated uncertainties due to their dependence on both shear-modulus and fault-area. While a number of clusters exhibit time-averaged slip-rates approximately equal to the steady-state surface velocity of RIS (dashed line, Fig. 2f\(^{34}\)), other clusters have significantly lower slip-rates.

**Fig. 2. All icequake-derived basal sliding parameters through time.** Data are a subset of icequake clusters over the period of 5\(^{th}\) to 15\(^{th}\) January 2019. Each colored line represents an individual icequake cluster. Uncertainties are shown by shaded regions. (a) Effective-normal-stress. Red dashed-dotted line indicates the maximum possible ice overburden pressure. (b) Total frictional basal shear-stress. (c) Bed shear-modulus. Previous estimates from literature are indicated by the dashed lines\(^{14,15}\). (d) Slip associated with individual icequakes. (e) Inter-event time between icequakes in a cluster. (f) Equivalent daily slip-rate calculated from the slip and inter-event times in (d) and (e). All data is smoothed by applying a 100-event moving-average window. All uncertainties are estimated using calculus-derived uncertainty propagation methods. Sensitivity analysis of the rate-and-state model is shown in Extended Data Fig. 5.

Fig. 3a-c show the variation in effective-normal-stress, shear-stress and slip-rate for the entire experiment duration. Histograms of the stress and slip-rate distributions are shown in Fig. 3d-f. The normal and shear-stress histograms show a bimodal distribution, with more than two thirds of the icequakes having effective-normal-stresses lower than \(\sim 5 \times 10^5\) Pa and shear-stresses
lower than $2 \times 10^5$ Pa. Conversely, the slip-rates exhibit a unimodal distribution, tailing off below $0.2 \, m \, day^{-1}$ and above $1.5 \, m \, day^{-1}$.

**Fig. 3. Basal effective-normal-stress, shear-stress and slip-rate for the entire experiment.**
Colored lines represent individual icequake clusters. (a) Effective-normal-stress on the fault. (b) Shear-stress through time. (c) Slip-rate through time. (d) to (f) Histograms of the respective time-series data in (a) to (c). Uncertainties in (a) to (c) are given by the shaded regions. Other labels as in Fig. 2. Uncertainties are as defined in Fig. 2.

Individual icequake clusters switch on and off, being active for the order of hours to days (see Fig. 2). Within single clusters, bed friction and slip are modulated by signals with dominant periods of ~6 to 12 hours (see Fig. 2). However, although this alludes to tidal modulation of basal friction, and indeed surface velocities are known to be modulated by tidal frequencies$^{34,35}$, we cannot decipher a clear relationship between tidal signals propagated 40 km upstream from RIS’s grounding line and our signals$^{28}$. We therefore do not discuss any link with tidal signals further.

The spatial distribution of average basal shear-stress, slip-rate and fault radius for each cluster over a 7 x 6 km region are shown in Fig. 4. Shear-stresses are largest at the clusters farthest upstream, approximately where the bed properties are inferred to transition from unconsolidated to consolidated till$^{33}$ (pink dashed-line, Fig. 1) and where the bed has shorter wavelength
topography than upstream that likely inhibits ice flow. Average slip-rate is spatially consistent
across all clusters. This is expected, as our study site is located near the center of the ice stream,
with no spatial variation in surface slip-rate. Fault radius, defining the area of an icequake
cluster sticky-spot is also measured (see Fig. 4c). Fault radii indicate that individual seismically
active sticky-spots have areas < 2800 m$^2$. Only a small number of sticky-spots are active at any
instant. This suggests that regions of sufficiently high basal friction to generate seismicity are
confined to the minority of the bed at a given point in time, yet invoke significant basal drag.
Aseismic regions between icequake clusters likely also contribute to the basal drag, presumably
providing the dominant source of aseismic drag upstream of the unconsolidated-consolidated
sediment boundary (pink dashed line, Fig. 1).

Fig. 4. Spatial variability in average basal shear-stress, slip-rate and fault radius for the
clusters. (a) Average shear-stress for the clusters. (b) Average slip-rate for the clusters. (c)
Average fault radius for the clusters. Residual topography data is from ground-penetrating
radar. Size of scatter points indicates fault radius. Green inverted triangles indicate the
locations of the network of receivers used in this study.

Discussion
Frictional shear-stress and slip-rate
The most important, immediate finding of this work is the ability to observe in-situ frictional
shear-stress and slip-rate, the two critical parameters for constraining the basal drag boundary
conditions of ice dynamics models. Our approach could be applied to any glacier that generates
icequakes. Most fast-moving glaciers likely generate such icequakes, with the majority of
glaciers on which seismometers have been deployed exhibiting at least some basal
seismicity. Seismic tremor associated with sliding can also occur, thought to initiate
at the boundary between the conditionally-stable and unstable regimes of the rate-and-state
friction model. Indeed, the premise of this study was inspired by such observations. However,
due to the inability to extract both corner frequency and inter-event time information
from tremor, it cannot be used to measure shear-stress and slip using our approach.
Our confidence in the frictional shear-stress and slip-rate measurements is founded partially on the uncertainty amplitudes, but also fundamentally on the agreement between the observed basal slip-rates and GNSS-derived surface displacement. This agreement validates assumptions of slip-dominant rather than deformation-dominant flow at RIS and the use of a rate-and-state model and assumptions of the icequake source properties. The small discrepancy between the surface and basal slip-rates is primarily due to uncertainty, except for a minority of particularly sticky-spots. These sticky-spots exhibit particularly strong frictional drag that significantly inhibits local ice flow, albeit for short durations of the order of hours to days.

Observed basal shear-stresses are of the order of $10^4$ to $10^7$ Pa, acting at sticky-spots with diameters of the order of 10 to 60 m (see Fig. 4). Basal shear-stresses of the order $10^5$ Pa are typical values used in ice dynamics models and laboratory experiments for RIS’s surface slip-rate of $\sim 400 \text{ m/yr}$. Basal shear-stresses of $10^6$ to $10^7$ Pa might initially appear inconsistently high compared to models and experiments. However, these high friction sticky-spots are spatially small compared to the total bed area. Our results therefore imply that certain icequake clusters accommodate a considerable proportion of the total basal drag.

**Icequake generation mechanisms**

We propose that the icequakes are generated by at least one of two mechanisms, or sliding regimes. The presence of two sliding regimes is motivated by the physical system and the bimodal distributions observed in Fig. 3d,e. The regimes (see Fig. 5) are: regime I, rock-on-rock friction between ice-entrained clasts and bedrock at the fault interface; and regime II, where clasts plough through till, with failure accommodated by a till-on-till fault interface. Clasts are pieces of rock partially entrained into the ice (see Fig. 5). The presence of such clasts is discussed in the literature. The motivations for these clast-based icequake models are that they can explain the rate-weakening friction required to generate icequakes, that clasts are required to generate icequakes in laboratory environments, and that such icequakes likely originate within one seismic wavelength of the ice-bed interface. We suggest that the highest effective-normal-stress icequakes exhibit regime I sliding, since this regime allows for the average effective-normal-stress over the entire fault-area to be concentrated over much smaller clast-bedrock contact areas. Similarly, we postulate that the lower effective-normal-stress icequakes are associated with regime II sliding, although we cannot rule out that all icequakes are generated via regime I.

**Effective normal-stress vs. effective fluid pressure**

Our results imply significant temporal variation in basal effective-normal-stress. Such increases and decreases in effective-normal-stress are inferred to be caused by corresponding decreases and increases in basal water pressure. However, while the icequake-derived effective-normal-stresses, $\bar{\sigma}$, averaged over the entire fault are equivalent to the average glaciological effective pressure, $P_{\text{eff}} = P_{\text{ice overburden}} - P_{\text{water}}$, within that same fault-area, asperities and bed heterogeneity on length-scales shorter than the fault diameter could significantly perturb local effective-normal-pressures. Although all our measured effective-normal-stresses remain below the ice overburden pressure, current hydrological models cannot reconcile glaciological effective pressures greater than $\sim 0.5 \text{ MPa}$ for expected till porosities. Sparse observations of
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effective-normal-pressures at RIS from borehole measurements find \( P_{\text{eff}} \approx 0.2 \, MPa^{49} \), although
till acoustic impedance measurements at RIS suggest that dewatering is possible\(^{23} \). Dewatered till
would imply \( P_{\text{eff}} = P_{\text{Ice overburden}} \). Our highest observed effective-normal-stresses therefore
suggest either: that our understanding of bed characteristics and associated physical models may
have to be revisited, at the very least for RIS; or that the rate-and-state model does not
adequately describe icequake physics.

We suggest three possible explanations for resolving the discrepancy between observed and
theoretical maximum effective-normal-stresses. Firstly, prominent bedrock outcrops could
significantly inhibit ice flow, allowing stoss-side effective-normal-stresses of the order of at least
1 \( MPa \) to develop\(^{52} \). A second explanation again lies with bedrock outcrops, whereby
impermeable bedrock might inhibit the transport of fluids, facilitating dewatered regions. A third
explanation is that till porosities are far lower than conceived in current models of bed properties,
possibly supported by till impedance measurements\(^{23} \). These suggestions are not exhaustive. We
only suggest here that our results motivate new models to explain such observations.

If effective-normal-stresses are related to \( P_{\text{eff}} \) at sticky-spots, then they provide an observational
foundation for calibrating basal-fluid-pressures assumed in: laboratory-experiments\(^{8} \); ice
dynamics models\(^{46} \); and glacier basal hydrology and tidal forcing\(^{34,35} \).

Enhanced knowledge of Rutford Ice Stream bed conditions

The considerable variation in bed properties observed at RIS are presented as an example of the
enhanced knowledge of the bed properties that our approach provides. Firstly, for the
unconsolidated till (label 1, Fig. 5) and much of the consolidated till (label 2, Fig. 5), the
effective-normal-stresses are too low to generate the unstable stick-slip conditions required for
icequake nucleation. Within consolidated till regions (label 2, Fig. 5), there are small zones that
become seismically active if the effective-normal-stress is sufficiently high (label 3, Fig. 5). These sticky-spots turn on and off, modulated by changes in effective-normal-stress, bed strength
and bed material.

Frictional shear-stress at an individual sticky-spot can vary temporally by up to an order of
magnitude and spatially by several orders of magnitude. This variation occurs over the order of
hours and 100s meters. This implies that there are both till and bedrock outcrops, combined with
an active hydrological system or permeable bed capable of such variable spatial and temporal
variations in basal fluid pressure. The regions exhibiting the highest shear-stresses are at the
upstream edges of local topographic highs near the unconsolidated-consolidated bed boundary
(label 6, Fig. 5). This is likely because resistive stresses of these materially stronger, topographic
highs can accommodate more basal drag. Some regions of consolidated till might contain
pockets of melt water (label 5, Fig. 5). However, we cannot observe such a phenomenon
seismically as these patches would have an approximately zero shear-modulus.
Fig. 5. Schematic diagram summarizing the findings of this study in relation to basal friction and slip with bed characteristics. Bed properties are labelled in the legend. Numbered points are referred to in the text. Note that features not to scale, but arranged approximately according to spatial trends in Fig. 4. Regime I and regime II are shown schematically, with regime I being clast-on-rock icequake slip behavior and regime II being till-on-till slip behavior.

Wider implications

Our results show that much of the basal drag at an ice stream can be accommodated within small zones of significantly higher-than-average friction. This could have a profound impact on how sliding is formulated in ice dynamics models. However, although friction varies significantly, average basal slip-rates remain predominantly stable at RIS. This is encouraging for current modelling efforts since if the temporally- and spatially-averaged slip-rates are approximately constant, then perhaps such models are not required to be sensitive to small-scale, rapid variations in bed friction. Our observations quantify the highly variable bed properties over a sufficient duration required to test such a hypothesis.

Another important question to address is how our approach could be implemented at ice sheet scale. One could deploy temporary seismic arrays on important ice streams and outlet glaciers for short durations. A number of targeted deployments would allow verification of the link between surface- and basal- velocity at ice sheet scales.
A further question that this study raises is could a rate-and-state friction model used for the icequake sliding analysis also be used as a mathematical basis for informing sliding laws used in ice dynamics models more generally. Such a model was recently proposed to describe surging glacier behavior\textsuperscript{53} and has been validated at laboratory scale\textsuperscript{12,13}. The rate-and-state model meets the conditional stability requirement, not allowing runaway acceleration of a glacier. A more comprehensive comparison to sliding laws for deformable beds, showing agreement for surface velocities $> 100$ m/yr, is provided in the supplementary text.

Finally, these icequakes observations can aid the understanding of earthquake mechanics more generally. Even the smallest magnitude icequakes ($M_w \approx -1.5$) have high signal-to-noise-ratios, and so could elucidate any lower limits on the fundamental size of earthquake nucleation for given fault properties\textsuperscript{54–56}. Additionally, icequakes in this study have stress-drops that vary with magnitude, contrary to magnitude-invariant stress-drops observed for larger earthquakes\textsuperscript{57}.

Our findings show that icequakes can provide the critical observations required to constrain the highly variable friction at the bed of an Antarctic ice stream. Applying such observational constraint to ice dynamics models would reduce uncertainty in corresponding sea-level rise projections.

References


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Methods

The icequake dataset

This study uses 100,000 icequakes at the bed of Rutford Ice Stream (RIS), Antarctica. An example of such an icequake arrival can be found in Extended Data Fig. 1. These data were collected over the period of November 2018 to February 2019. The icequakes are detected using QuakeMigrate and relocated using NonLinLoc. A full description of the detection, location and clustering analysis of this seismicity can be found in. The hypocentral depths, orientation of focal mechanisms, and full waveform modelling provide us with confidence that these icequakes are associated with sliding within one seismic wavelength (~10 m) of the bed.

Observable parameters from stick-slip icequakes

Earthquake source models can be used to calculate the size of the earthquake, its duration, the fault radius and the shear-stress-drop associated with the release of seismic energy. These observable parameters are required for any analysis of frictional behavior at the bed of glaciers using icequakes. The methods we use to obtain these parameters from the icequake signals are described below.

Seismic moment

The seismic moment, $M_0$, of an earthquake describes the energy released and is defined as

$$M_0 = \frac{4 \pi \rho v_i^2 r \Omega_0}{A_{rad,i} C_{free-surface}}$$

where $\rho$ is the density of the medium at the earthquake source, $v_i$ is the velocity of the seismic phase $i$ (P or S), $r$ is the hypocentre-receiver distance, $\Omega_0$ is the long-period spectral amplitude, $A_{rad,i}$ is the amplitude of radiation of seismic phase $i$ for the particular source-receiver azimuth and take-off angle, and $C_{free-surface}$ is the free surface correction term, which depends upon the angle of inclination of the seismic phase arrival at the surface. For this study, we assume typical ice values of $\rho = 917$ kg m$^{-3}$, $v_{p,ice} = 3841$ m s$^{-1}$, $v_{s,ice} = 1970$ m s$^{-1}$. $A_{rad,i}$ is calculated as described in, based on the assumption that all the icequakes in this study are double-couple (DC) sources with strikes aligned with the ice flow direction. This assumption is based upon previous observations at Rutford Ice Stream. $\Omega_0$ is calculated by fitting a Brune source model to the noise-removed spectrum of the icequake.

Corner frequency

The spectrum of an earthquake contains more information than just the long-period spectral amplitude. If one assumes that an earthquake’s spectrum can be described by a Brune model then one can also measure the corner frequency, $f_c$, of the earthquake. However, an earthquake’s spectrum is also particularly sensitive to seismic attenuation.
Seismic attenuation, often described by the quality factor, $Q$, reduces the amplitude of an earthquake spectrum non-linearly across all frequencies. If path attenuation is poorly constrained then it can lead to detrimental uncertainty in the measured corner frequency, as evidenced by the trade-off between $Q$ and $f_c$ in the Brune model,

$$\Omega(f) = \frac{\Omega_0 e^{-\pi f / Q}}{1 + (f / f_c)^2} \tag{2},$$

where $\Omega(f)$ is the amplitude of the spectrum for a certain frequency $f$ and $t$ is the travel-time.

To obtain an accurate measurement of corner frequency, we therefore use a linearized spectral ratios method to constrain $Q$. This spectral ratios method isolates the path effects from the source effects. An example of the linearized Brune model fit and the observed spectrum for an example icequake is shown in Extended Data Fig. 1c. We obtain estimates of $Q$ from this method of the order of 200 to 800 (see Extended Data Fig. 2b), which are in agreement with other measurements for Antarctic ice. This then allows Equation 1 to be fit to the earthquake spectrum with only $\Omega_0$ and $f_c$ as variables. We find that the icequake corner frequencies at RIS fall approximately within the range of 40-100 Hz (see Extended Data Fig. 2c).

**Fault radius and stress-drop**

One can estimate the fault radius, $r$, and stress-drop, $\Delta\tau$, of an earthquake from the corner frequency.

The relationship between corner frequency and fault radius, $R$, is given by,

$$f_c = k_1 \frac{\beta}{R} \tag{3},$$

where $f_c$ is the spherically-averaged corner frequency for the earthquake, $\beta$ is the shear-wave speed near the source and $k_1$ is a constant relating the spherically-averaged corner frequencies for a specific fault model for the seismic phase $i$. Here, we use the fault model of, which gives $k_p = 0.38$ and $k_s = 0.26$ for a rupture speed of 0.98 $\beta$. We let $\beta$ equal the shear velocity of ice (1970 m s$^{-1}$). For clast-on-bedrock slip (Regime I, Fig. 5, main text), this is valid as rupture will propagate through the bedrock and the ice that the clasts are embedded within, with us only observing the rupture propagation through the ice. For till-on-till slip (Regime II, Fig. 5, main text), our assumption of $\beta$ is likely an overestimate, resulting in an overestimate of fault radius. As the seismic properties of the till are unknown, we are limited in assigning a lower value of $\beta$ for any Regime II events. We assume a symmetric circular fault for this analysis. We therefore calculate average corner frequencies for each event based on the corner frequencies observed at all receivers. The potential effects of the symmetric circular fault assumption are shown in.

The uniform stress-drop of an earthquake can then be found using the fault radius and the relationship given by.
\[ \Delta \tau = \frac{7}{16} \frac{M_0}{R^3} \quad (4), \]

We now have all the observable parameters required to constrain a friction model at an icequake source.

### Using a rate- and state- friction law for deriving frictional shear-stress and slip from icequakes

#### Calculating shear-stress

Earthquakes are typically generated as the result of stick-slip frictional instabilities at a fault interface \(^{31}\). We hypothesize that icequakes associated with sliding at the bed of a glacier can be described by a similar model. For our investigation, we assume that the fault-interface is at or near (< 1 wavelength) the ice-bed interface. Schematic diagrams describing the model are given in Extended Data Fig. 3. Within this framework, we can apply the following rate- and state- friction law given by \(^{31}\),

\[ \tau = \left[ \mu_0 + a \ln \left( \frac{v}{v_0} \right) + b \ln \left( \frac{v_0 \theta}{L} \right) \right] \bar{\sigma} \quad (5), \]

where \( \tau \) is the total frictional shear-stress, \( \mu_0 \) is the steady-state friction coefficient at \( v = v_0 \), \( v \) is the slip velocity, \( v_0 \) is a reference velocity, defined in this case to be the background slip-rate, \( a \) and \( b \) are material properties, \( L \) is the characteristic slip distance over which the system returns to steady-state and renew surface contacts, and \( \theta \) is the state variable. The variation of the state variable, \( \theta \), through time can be defined by the aging or the slip laws \(^{69}\), given by,

\[ \frac{\partial \theta}{\partial t} = 1 - \frac{v \theta}{L} \quad \text{(aging law)} \quad (6), \]

\[ \frac{\partial \theta}{\partial t} = -\frac{v \theta}{L} \ln \left( \frac{v \theta}{L} \right) \quad \text{(slip law)} \quad (7). \]

The state variable, \( \theta \), represents the characteristic contact lifetime of a fault. In order to apply the rate- and state- model to the stick-slip icequake system in a mathematically tractable way, we assume that the state variable of the system is constant over the duration of an icequake cycle, i.e. \( \frac{\partial \theta}{\partial t} = 0 \) through all time during a cycle. For a destructive frictional failure process, \( \theta \) likely changes with time during earthquake nucleation and as the fault heals. However, for the icequake generation mechanisms proposed in this study (see Fig. 5, main text), damage at the fault interface that affects the frictional properties is likely less significant than at traditional earthquake fault interfaces. This lack of damage is evidenced to some extent by the highly repetitive nature of the icequakes \(^{20,28}\). We assume that at least part of the icequake patch is near steady-state, or approximately at steady-state if it slips sufficiently fast. A caveat to this is that some of the icequake patch could have remained below the steady-state sliding limit, which we do not explore this here. Overall, we deem the approximation of \( \frac{\partial \theta}{\partial t} = 0 \) between individual icequakes as acceptable in this case. This assumption can be used to find the state variable, \( \theta \), as an expression of \( v, L \) from either the aging law (Equation 6) or the slip law (Equation 7), which both yield,
\[ \theta = \frac{L}{v} \]  
(8).

Equation 5 can then be reduced to a rate-dependent friction law, given by,

\[ \tau = \left[ \mu_0 + (a - b) \ln \left( \frac{v}{v_0} \right) \right] \bar{\sigma} \]  
(9).

The coefficient of friction in Equation 9 can then be thought of as \( \mu = \mu_0 + \Delta \mu \), where \( \mu_0 \) is the static friction component, and the dynamic friction component, \( \Delta \mu \), is given by,

\[ \Delta \mu = (a - b) \ln \left( \frac{v}{v_0} \right) \]  
(10),

which when multiplied by the effective-normal-stress, \( \bar{\sigma} \), can be assumed as equal to the earthquake stress-drop (see Equation 13).

One can then parametrize Equation 9 in such a way so that it can be solved for individual icequakes. We take \( \mu_0 = 0.4 \), \( a = 5 \times 10^{-3} \) and \( b = 15 \times 10^{-3} \) from 19. We approximate the ratio of the instantaneous sliding velocity to the reference velocity, \( \frac{v}{v_0} \), as,

\[ \frac{v}{v_0} = \frac{\left( \frac{d}{T} \right)}{\left( \frac{d}{t_{inter-event}} \right)} \]  
(11),

where \( d \) is the slip associated with an event (unknown), \( T \) is the slip duration, which we approximate to be equal to the inverse of the icequake corner frequency, \( f_c \), \( t_{inter-event} \) is the time between two consecutive icequakes. The correspondence of these parameters to the stick-slip cycle is shown in Extended Data Fig. 3b. With this parametrization, the velocity ratio then becomes,

\[ \frac{v}{v_0} = f_c \cdot t_{inter-event} \]  
(12).

Assuming that the friction at the interface is velocity-weakening and therefore unstable, one can then assume that the dynamic part of Equation 9 is equal to the stress-drop measured during an icequake, \( \Delta \tau \). One should note that this assumption implies that all the dynamic stress-release during slip is accommodated seismically (see red shaded region of Extended Data Fig. 3b). However, there is also frictional shear-stress present that cannot be measured directly using stress-drop measurements. We also assume a seismic radiation efficiency of 1, which is obviously an approximation, with the actual seismic radiation efficiency unknown. Although the radiation efficiency will in reality be <1, due to thermal heating and the generation of additional surface area during abrasion, fracture tip energy, and other phenomena such as off-fault cracking are likely insignificant in comparison to standard earthquakes 72, so we deem our first-order approximation as reasonable in this case. For tectonic earthquakes, the seismic radiation efficiency typically might be of the order of 0.1 (for example, see 73). If the icequake seismic radiation efficiencies were similarly low, then this would be approximately equivalent to reducing the magnitude of \( M_0 \) by a factor of 10. Sensitivity analysis in the supplementary text suggests that such a reduction in \( M_0 \) would reduce the shear-stress, \( \tau \), by an order of magnitude, but the slip velocity, \( v_{slip} \), would only be reduced by a factor of 3. Assuming velocity-weakening friction and a radiation efficiency of one results in the definition of the effective-normal-stress at the fault interface, given by,
\[ \sigma = \frac{\Delta \tau}{(a - b) \ln \left( \frac{V}{V_0} \right)} \]  (13).

Once we know the effective-normal-stress, \( \sigma \), we can find the overall shear-stress on the fault, \( \tau \), from Equation 9.

We emphasize that the effective-normal-stress, \( \sigma \), is the normal stress on the fault, which is not necessarily equivalent to a traditionally defined glaciological effective pressure, \( P_{\text{eff}} = P_{\text{ice}} - P_{\text{water}} \). The fault effective-normal-stress, \( \sigma \), is the effective-normal-stress that acts over the fault-area, \( A_{\text{fault}} \), derived from the earthquake corner frequency (Equation 3). The actual normal stress acting through clasts in contact with the underlying contact surface might increase the normal stress acting through these clasts (see sliding regime I Fig. 5). However, fault-average normal stress, \( \sigma \), must be equal to the average glaciological effective pressure, \( P_{\text{eff}} \), over the same area of the bed.

**Calculating slip**

The second glaciologically important parameter to measure at the bed is the slip, and hence the basal slip-rate. To calculate slip, we assume that while an individual icequake cluster is active, all (or at least the vast majority of) slip is accommodated seismically. This is likely the case for RIS, as evidenced by the close agreement between surface slip-rate and seismically measured basal slip-rates (see Fig. 2f). Calculating the basal slip, \( d \), from an icequake is challenging because one first has to determine a method of estimating the bed shear-modulus, \( G_{\text{bed}} \), since the slip is given by,

\[ d = \frac{M_0}{G_{\text{bed}} A} \]  (14),

where \( M_0 \) is the seismic moment released by an earthquake and \( A \) is the area of the fault.

The bed shear-modulus, \( G_{\text{bed}} \), is calculated by assuming a further behavior of the rate-and state-friction law. This behavior is that an earthquake can only nucleate if it is in the unstable regime. In this study, we assume that the temporally-averaged driving shear-stress at the fault varies over longer time-scales than the icequake inter-event time, with the shear-stress at which the fault fails governed by the effective-normal-stress acting on the fault, \( \sigma \). The approximately constant inter-event time between individual consecutive icequake pairs (see Fig. 2e) within a single cluster validates this assumption. \(^{31} \) define the effective-normal-stress at which a fault becomes unstable is defined as the critical normal stress, \( \sigma_c \), with velocity-weakening behavior prevailing above this stress. \( \sigma_c \) is given by \(^{31} \),

\[ \sigma_c = \frac{k L}{b - a} \]  (15),

where \( k \) is the spring constant of the system (see Extended Data Fig. 3a), which is given by,

\[ k = \frac{G^*}{R} \]  (16),

where \( G^* \) is the effective shear-modulus of the bimaterial interface \(^{19} \) and \( R \) here is the radius of the fault, which can be found from the icequake corner frequency, if assuming a
symmetric, circular fault \textsuperscript{66,67}. However, this equation still has two unknowns: $G^*$, the effective shear-modulus that we require to calculate the slip; and $L$, the critical slip distance, otherwise referred to as the state evolution distance. For the purposes of this study, we approximate $L$ to remain constant, but allow $G^*$ to vary with effective-normal-stress, which from granular material theory \textsuperscript{74} is assumed to take the generic empirical form,

$$G^* = A \bar{\sigma}^n + C \quad (17),$$

where $A$, $n$ and $C$ are constants to invert for. We use a least squares approach to minimize the function,

$$f(\bar{\sigma}_c, R, a, b, A, n, C, L) = \ln\left(\frac{(A \bar{\sigma}_c^n + C) L}{b - a}\right) - \ln(R \bar{\sigma}_c) \quad (18),$$

where $\bar{\sigma}_c$ and $R$ vary for each icequake, and $A$, $n$, $C$ and $L$ are varied to minimize the function. $\bar{\sigma}_c$ is taken to be the effective-normal-stress for the first 100 icequakes when a cluster becomes active, as calculated using Equation 13. These parameters are found to be $A = 22,000$, $n = 0.78$, $C = 8,200$ $Pa$ and $L = 7.7 \times 10^{-5}$ $m$, with the result of the minimization shown in Extended Data Fig. 4. Now $L$ can be substituted into Equation 15 to find the bimaterial shear-modulus, $G^*$. The shear-modulus of the bed, $G_{\text{bed}}$ can then be found using the Poisson ratios of ice ($1/3$) and till (0.49), which gives $G^* \approx 3.5$ $G_{\text{bed}}$ \textsuperscript{19}. Granular material theory, or at least the relationship of Equation 17, is thought to still hold for clast-over-bedrock sliding since the shear-modulus will still be related to some exponent, $n$, of $\bar{\sigma}$, even if that exponent were $\sim$0.

Equation 14 can then be used to find the slip, $d$, associated with a single icequake, for the effective-normal-stress applied to the fault at that particular time. We also calculate the approximate slip-rate associated with these highly repetitive icequakes. If one assumes that all the slip when an icequake cluster is active is accommodated seismically, then one can calculate the slip-rate per day, $v_{\text{slip}}$,

$$v_{\text{slip}} = \frac{d}{t_{\text{inter-event}}} \quad (19),$$

The methods described above allow us to calculate the total shear-stress, $\tau$, and the slip, $d$, at the bed. These two parameters can provide observational constraint on ice dynamics models of ice streams.

\textbf{A note on assumptions}

A number of assumptions are made to make the derivation of basal shear-stress and slip from icequake observations and a rate-and-state friction model mathematically tractable. There are several assumptions that warrant particular emphasis. The first is the assumption that all slip at an individual sticky-spot is accommodated seismically while that cluster is active. The highly repetitive nature of the icequakes (see Extended Data Fig. 1 and \textsuperscript{28}), with approximately constant inter-event times between consecutive icequakes in a cluster, is indicative of the stability of each sticky-spot (see Fig. 2), justifying this assumption. Secondly, a Brune model \textsuperscript{63} is assumed to describe the earthquake source characteristics. While such a model is likely an approximation for the complex physics of earthquake rupture, it is a common assumption for other earthquake studies that is likely also a valid approximation for the stick-slip icequakes presented here. Thirdly, we approximate that the time-derivative of the state-variable in the rate-and-state
friction model, $\frac{\partial \sigma}{\partial t}$, equals zero during an individual icequake cycle. This approximation is valid if slip on the fault is sufficiently fast and if little damage occurs at the fault, compared to more complex earthquake faults. Obviously, this is only an approximation, as damage does likely occur at the fault, at least for the clasts-over-bedrock slip case (regime I, see Fig. 5).

Furthermore, an underestimation bias in slip may be introduced by the assumption of no fault frictional heating. Fault frictional heating would reduce the seismic radiation efficiency from our approximation of one$^{72}$. The final assumption we emphasize here is that we assume that the icequakes at the beginning of an icequake cluster nucleate at approximately the critical normal stress for nucleation, $\sigma_c$, rather than at some arbitrary value above it. The icequake slip calculations are dependent upon this assumption. This assumption would not be valid for sporadic earthquakes on complex faults, as shear-stresses could build to different values before failure for each earthquake, even with constant effective-normal-stresses, due to fault heterogeneity. Nor would it necessarily be valid if the driving shear-stress were perturbed over time-scales shorter than the inter-event time, for example by interactions with other icequake clusters. However, although icequake faults still exhibit a degree of heterogeneity due to an inhomogeneous distribution of clasts, this heterogeneity has negligible impact upon the consistency of both the inter-event times and shear-stresses between consecutive icequakes at a given sticky-spot (see Fig. 2). Furthermore, there are only a small number of active icequake clusters at any given time, which are spatially isolated from one another. The consistency in inter-event times and shear-stresses observed in our data, in agreement with similar, laboratory-generated icequakes$^{12}$, provides us with confidence in our assumption of icequakes nucleating at the critical nucleation stress, $\sigma_c$.

Additional references:


Extended Data Figures

Extended Data Fig. 1. Examples of icequake waveforms and spectra. (a) 30 minutes of continuous data for the Z component of station R3030. Approximate icequake P-phase arrival times associated with a single cluster are shown by the green lines. (b), (c), (d) Stacked waveform data on the Z-, N- and E- components for 173 events in a cluster at station R3030, located at the center of the network. Red line indicates P-phase arrival. Blue lines indicate S-phase arrivals. Grey shading represents the standard deviation of the stacked data. (e) Spectrum for one event within the cluster at station R3030. Waveform data in (a) to (d) are filtered between 10 Hz and 120 Hz.
Extended Data Fig. 2. Quality factor (Q) and corner frequency ($f_c$) distributions for the icequakes in this experiment. (a) Histogram of Q. (b) Histogram of $f_c$. Values for each icequake are averaged for all individual station observations.

Extended Data Fig. 3. Schematic Fig. describing the rate- and state- frictional model as a block-slider model. (a) Diagram of the block-slider model, showing the driving shear-stress, $\tau$, the effective-normal-stress, $\bar{\sigma}$, and the system spring constant $k$. (b) Accumulated shear-stress vs. time for a series of consecutive icequakes. (c) Shear-stress at the fault at a particular time as...
predicted by the rate-and-state model $^{31}$. (d) The stick-slip icequake cycle, with the numbers corresponding to the relevant stress states labelled in (b).

**Extended Data Fig. 4. Results of the least squares inversion of Equation 17.** Blue scatter points are the data and red scatter points show the least-squares inversion result.
Extended Data Fig. 5. Rate-and-state friction model sensitivity analysis. Plot of the sensitivity in frictional shear-stress at the bed, $\tau_{\text{bed}}$, and slip-rate at the bed, $v_{\text{slip}}$, with perturbation of the key observational parameters. The reference values used to normalize the variations are the average values of $\tau_{\text{bed}}$ and $v_{\text{slip}}$ observed at all the clusters. The magnitude of variation in each parameter are summarized in Table S1. See supplementary text for further details.
Supplementary Files

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