An Energy Efficient Control Strategy for Electric Vehicle Driven by In-Wheel-Motors Based on Discrete Adaptive Sliding Mode Control

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An Energy Efficient Control Strategy for Electric Vehicle Driven by In-Wheel-Motors Based on Discrete Adaptive Sliding Mode Control

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Abstract: This paper presents an energy efficient control strategy for electric vehicle (EV) driven by in-wheel-motors (IWMs) based on discrete adaptive sliding mode control (DASMC). The nonlinear vehicle model, tire model and the IWM model are established at first to represent the operation mechanism of the whole system. Based on the modeling, two virtual control variables are used to represent the longitudinal and yaw control efforts to coordinate the vehicle motion control. Then DASMC method is applied to calculate the required total driving torque and yaw moment, which can improve the tracking performance as well as the system robustness. According to the vehicle nonlinear model, the additional yaw moment can be expressed as a function of longitudinal and lateral tire forces. For further control scheme development, a tire force estimator using unscented Kalman filter is designed to estimate real-time tire forces. On these bases, energy efficient torque allocation method is developed to distribute the total driving torque and differential torque to each IWM, considering the motor energy consumption, the tire slip energy consumption and the brake energy recovery. Simulation results of the proposed control strategy using co-platform of Matlab/Simulink and CarSim® demonstrate that it can accomplish the vehicle motion control in a coordinated and economic way.

Keywords: Electric vehicle • Energy optimization • Motion control • Discrete adaptive sliding mode control

1 Introduction

Electric vehicles (EV) have been considered as a substitution of the traditional vehicle with internal combustion engine for the advantages of clean energy source and emission[1-3]. EV driven by in-wheel-motors (IWMs) has been considered as a promising architecture for its noticeable advantages comparing with other kinds of EV[4,5]. Firstly, the elimination of the transmission mechanism can save the producing cost and make more space for drivers and passengers. Secondly, the application of motorized wheels can improve the motor-drive operation efficiency [6-8].

Besides the merits mentioned above, another significant advantage of EV driven by IWMs is that comprehensive performance can be elevated because the independent driving approach makes it possible to accomplish integrated optimization and control, then obtain more flexible responses under different driving condition [9,10]. Some prior researches have been done in this area. Y. Yang[11] proposed a current distribution control for dual directly driven wheel motors for EVs. They considered the status difference between two IWMs caused by fabricating qualities and different aging rates and so on, and then developed an internal controller serves to distribute the current, instead of torque, to the driving wheels, thereby enhancing the robustness and stability of the system. M Demirici [12] proposed a control method for IWM drive EV based on the direct yaw-moment control, giving the optimized wheel force distribution as well as the coordination control of the hydraulic braking and the motor torque, which improves the stability of the four-wheel-drive electric vehicle effectively. D. Wu [13] presented a layered vehicle dynamic control system, which is composed of an adaptive optimal control allocation method using neural networks for optimal distribution of tire forces and the sliding mode yaw moment observer for robust control of yaw dynamics to solve the stability control problem. Chang Hu [14] utilized the front-wheel differential drive-assist
steering to achieve the path-following control for independently actuated electric autonomous ground vehicles. The differential torque between the left and right wheels can be utilized to actuate the front wheels as the sole steering power when the regular steering system fails, avoiding the dangerous consequences.

Moreover, torque distribution, namely distributing the IWM torque properly to accomplish the given targets such as high dynamic demand, driving stability demand or good energy efficiency is also an important issue. A lot of reports on this subject and relevant studies have been published. Y. Li[15] presented an ideal force distribution control method for the EV based on the friction circle of tire force, making the front and rear wheels reach the adhesion limits at the same time in different conditions. Another optimal torque distribution for EV driven by IWMs was introduced by X. Zhang[16]. The linear quadratic regulator via a weighted least square method was used to calculate the required longitudinal force of each wheel. Then the IWM torques were obtained by a tire slip ratio controller because they considered the longitudinal force of each wheel was decided by the tire slip ratio. To increase the cruising range of EVs equipped with front and rear in-wheel-motors, an optimal torque distribution algorithm for longitudinal motion by considering the transfer of weight between front and rear axles and motor losses was proposed by Y. Wang[17]. The EV was modeled as a linear-time-invariant system with generalized frequency variables and then the output power of IWM was modeled as a convex function of the distribution ratio, providing an added value for front-rear-independent-drive EVs. Y. Li [18] proposed a coordinated control algorithm of vehicle dynamics.

Fig.1 Energy efficient control strategy for EV driven by IWMs based on DASMC
performance and energy consumption for an EV driven by four IWMs and steered by two steer-by-wire systems. In their study, multi controllers were designed for each subsystem at first and then a rule-based coordinated control scheme is developed according to vehicle driving states to accomplish whole control target.

According to the abovementioned studies, an energy efficient control strategy for electric vehicle driven by in-wheel-motors is designed in this paper. The overall control scheme is displayed in Fig. 1. Different from previous research studies, we are aiming to obtain a good comprehensive vehicle performance in both dynamic motion control and energy efficiency. The overall control scheme consists of three parts. The first part is the vehicle motion controller using discrete sliding mode control (DSMC) algorithm to calculate the longitudinal and yaw control efforts, guaranteeing good dynamics performance and stability. The second part is the lateral tire force estimation. Rather than linear tire force calculation, an unscented Kalman filter (UKF) is applied considering the tire nonlinear characteristics because the yaw moment is closely related to the tire lateral force, so precise estimation can contribute to further torque allocation. In the third part, the longitudinal and yaw control efforts will be distributed to each IWM considering the energy efficiency. More specifically, the optimal allocator is designed considering IWM energy consumption, tire slip energy consumption and the brake energy recovery.

The rest of this paper is organized as follows. The vehicle nonlinear model and IWM model are built in Section. 2, followed by the details about proposed control for IWM driven EV based on optimal torque allocation are presented in Section. 3. Then the co-simulation results based on Matlab/Simulink and CarSim® are presented in Section .4, together with the related analyses. Finally, the conclusion will be summarized in Section. 5.

2 Modeling

The vehicle nonlinear dynamic model, tire model and IWM model are constructed for further analysis and subsequent control strategy design.

2.1 Vehicle dynamic model

In this study, only longitudinal, lateral and yaw dynamics are concerned. A schematic diagram of the vehicle three degree-of-freedom nonlinear model [19] with four independently driven IWMs is shown in Fig. 2.

Fig. 2 Schematic diagram of vehicle nonlinear model

The longitudinal, lateral and yaw motions of this model can be expressed as

\[
\begin{align*}
\dot{v}_x &= v_y - F_x - F_y + \frac{1}{m} \sum F_i \\
\dot{v}_y &= -v_y + \frac{1}{m} \sum F_i \\
\dot{\gamma} &= \frac{1}{I_z} M_z
\end{align*}
\]  

(1)

where \(v_x, v_y\) and \(\gamma\) are the longitudinal velocity, lateral velocity, and yaw rate of the vehicle, respectively; \(m\) is the vehicle total mass; \(I_z\) is the vehicle yaw inertia; \(\Sigma F_i\) is the total longitudinal tire force; \(\Sigma F_i\) is the total lateral tire force; \(M_z\) is the total yaw moment provided by four wheels; \(F_w\) and \(F_f\) are the aerodynamic resistance and rolling resistance, respectively, which can be expressed as

\[
F_x = \frac{1}{2} C_d \rho A v^2 ; \quad F_f = f m g .
\]  

(2)

(3)

where \(C_d\) is the aerodynamic resistance coefficient; \(\rho\) is the air density; \(A\) is the windward area; \(f\) is the rolling resistance coefficient; \(g\) is the gravity acceleration.

According to Fig. 2, \(\Sigma F_x, \Sigma F_y\) and \(M_z\) can be written as

\[
\begin{align*}
\Sigma F_i &= (F_{i,fl} + F_{i,fr}) \cos \delta_i - (F_{i,fl} + F_{i,fr}) \sin \delta_i + F_{i,rl} + F_{i,rr} \\
\Sigma F_i &= (F_{i,fl} + F_{i,fr}) \cos \delta_i + (F_{i,fl} + F_{i,fr}) \sin \delta_i + F_{i,rl} + F_{i,rr} \\
M_z &= c((F_{i,fl} - F_{i,fr}) \cos \delta_i + (F_{i,fl} - F_{i,fr}) \sin \delta_i) \delta_i - b(F_{i,fl} + F_{i,fr}) \\
M_z &= a((F_{i,fl} + F_{i,fr}) \cos \delta_i + (F_{i,fl} + F_{i,fr}) \sin \delta_i) \delta_i - b(F_{i,fl} + F_{i,fr}) \\
\end{align*}
\]

where \(\delta_i\) is the steering wheel angle; \(F_{i,\cdot}\) is the longitudinal tire force of the \(i\)th wheel \((i = fl, fr, rl, rr)\); \(F_{i,\cdot}\) represents the lateral tire force of the \(i\)th wheel; \(a\) and \(b\) is the distances from front and rear axle to center of gravity, respectively; \(c\) is the half of the track width. The longitudinal tire force can be calculated by the wheel rotational dynamics equation, which is

\[
\dot{\omega}_i = \frac{1}{T_i} T_r F_{i,\cdot} (i = fl, fr, rl, rr) .
\]  

(5)
where $T_i$ and $\omega_i$ are the driving torque and the angular speed of the $i$th wheel; $I_w$ and $R_w$ are the wheel moment of inertia and tire rolling radius, respectively. $T_i$ is the $i$th IWM output torque.

### 2.2 Tire model

The tire forces can be expressed by a Dugoff tire model [20] as follows

$$F_i = \mu F_s k_i \frac{\lambda_i}{1 + \lambda_i} f(L_i)$$

$$F_{yi} = \mu F_s k_i \tan \alpha_i \frac{\lambda_i}{1 + \lambda_i} f(L_i)$$

$$f(L_i) = \begin{cases} (2 - L_i) L_i & L_i \leq 1 \\ 1 & L_i > 1 \end{cases}$$

$$L_i = \frac{(1 - \lambda_i)(1 - \cos \alpha_i) \sqrt{k_i^2 \lambda_i^2 + k_i^2 \lambda_i^2 \cdot \tan^2 \alpha_i}}{2}$$

where $k_i$ is the longitudinal stiffness; $k_i$ is the lateral stiffness; $\mu$ is the road adhesion coefficient; $\alpha_i$ is the sideslip angle; $\lambda_i$ is the longitudinal slip rate; $F_{yi}$ is the vertical load of the tire. More specifically, $\alpha_i$, $\lambda_i$ and $F_{yi}$ can be written as

$$\alpha_p = \alpha_p + \delta \frac{a_y + v_y}{v_x}$$

$$\alpha_i = \alpha_i - \frac{v_x - b \gamma}{v_x}$$

$$\lambda_i = \frac{a_0 R_w - v_y}{v_x}$$

$$F_{qi} = \left( \frac{a_0}{c} \right)^2 \frac{a_0}{c} \frac{b}{l} \frac{1}{2} \frac{a_0}{c} \frac{h}{l}$$

$$F_{qi} = \left( \frac{a_0}{c} \right)^2 \frac{a_0}{c} \frac{b}{l} \frac{1}{2} \frac{a_0}{c} \frac{h}{l}$$

$$F_{qi} = \left( \frac{a_0}{c} \right)^2 \frac{a_0}{c} \frac{b}{l} \frac{1}{2} \frac{a_0}{c} \frac{h}{l}$$

$$F_{qi} = \left( \frac{a_0}{c} \right)^2 \frac{a_0}{c} \frac{b}{l} \frac{1}{2} \frac{a_0}{c} \frac{h}{l}$$

2.3 IWM model

Four permanent magnet synchronous motors (PMSMs) are applied as the driving motor[21]. Based on power invariant requirement, the $d-q$ equivalent circuit of a PMSM is displayed as Fig.3.

![Fig.3 D-q equivalent circuit of a PMSM](image)

The voltages of equivalent circuit are derived as

$$\begin{align*}
    u_d &= R_p i_d - \omega_L L_d i_q \\
    u_q &= R_p i_q + \omega_L (L_d i_d + \varphi_d)
\end{align*}$$

(13)

The electromagnetic torque equation can be expressed as

$$T_e = p_n (\omega_L i_d + (L_d - L_q) i_d i_q)$$

(14)

where $L_d = L_q$ and the $d$-axis current can be controlled to be zero. Subsequently,

$$T_e = p_n \varphi_d i_q$$

(15)

Then the input power to a PMSM can be calculated by the following equation:

$$P_{in} = u_d i_d + u_q i_q$$

$$= (R_p i_d - \omega_L L_d i_q) i_d + (R_p i_q + \omega_L (L_d i_d + \varphi_d)) i_q$$

$$= R_p (i_d^2 + i_q^2) + \omega_L^2 (L_d^2 i_d^2 + L_q^2 i_q^2) / R_p + \omega_L \varphi_d i_q$$

(16)

### 3 Control design for IWM driven EV

The control method design is proposed in this section, including vehicle motion controller design, lateral tire force estimation and the optimal energy torque allocation.

#### 3.1 Vehicle motion controller design

Road interferences and the parameter uncertainties are not considered in the modeling in Section 2. In order to preserve system stability as well as maintain good system robustness, the ADSMC is applied for vehicle motion control design[22].

Assuming $\sin \delta \approx \delta$, $\cos \delta \approx 1$, because of the small magnitude of front wheel steering angle, and substituting Eq.2, Eq.3, Eq.4 and Eq.5 into Eq.5, then Eq.1 can be
derived as
\[
\begin{align*}
\dot{v}_s &= v_s \gamma + \frac{1}{mR_v^s} u_t - F_v - F_f - d_1, \\
\dot{v}_r &= -v_r \gamma + \frac{1}{m} (F_{sg} + F_{g} + F_{sr} + F_{mr}) \\
\dot{\gamma} &= a(F_{sg} + F_{g}) - b(F_v + F_f) + \frac{c}{I_R} u_2 + d_2
\end{align*}
\]
where \( u_t = T_\beta + T_\rho + T_v + T_r \) and \( u_2 = -T_\beta + T_\rho - T_v + T_r \) are the virtual control inputs to this nonlinear system; \( d_1 \) and \( d_2 \) are considered as modeling errors, which can be expressed as
\[
\begin{align*}
d_1 &= -\frac{J}{mR_v^s} (\dot{\phi}_t + \dot{\phi}_x + \dot{\phi}_z + \dot{\phi}_g) \\
d_2 &= \frac{I}{mR_v^s} (\dot{\phi}_t - \dot{\phi}_x - \dot{\phi}_z - \dot{\phi}_g)
\end{align*}
\]
\( \gamma \) is an error that is the system asymptotic stability surface, which means any starting state will eventually reach the switching surface \( S_j = 0 \), \( j = 1, 2 \). The reaching condition Eq. 23 can be achieved.

The reference longitudinal velocity and yaw rate are defined as \( v_s^r \) and \( \gamma^r \). Errors between the reference values and the actual values are given as \( \epsilon_i = v_s^r - v_i \) and \( \epsilon_r = \gamma^r - \gamma \). To realize \( \epsilon_i \rightarrow 0 \) and \( \epsilon_r \rightarrow 0 \), the sliding faces are defined as \( S_j(k) = \rho_x e_r(k) \) and \( S_j(k) = \rho_\gamma e_r(k) \), where \( \rho_x \) and \( \rho_\gamma \) are positive defined.

The Lyapunov function candidates are chosen as
\[
\begin{align*}
V_1(k) &= \frac{1}{2} S_j^2(k) \\
V_2(k) &= \frac{1}{2} S_j^2(k)
\end{align*}
\]
and then
\[
\Delta V_1 = S_j^2(k + 1) - S_j^2(k) \\
\Delta V_2 = S_j^2(k + 1) - S_j^2(k)
\]
According to the Lyapunov theorem of asymptotic stability, \( S_j = 0 \), \( j = 1, 2 \) is the system asymptotic stability surface, which means any starting state will eventually reach the switching surface \( S_j = 0 \), \( j = 1, 2 \). The reaching condition as
\[
S_j^2(k + 1) < S_j^2(k), (j = 1, 2)
\]
When sampling time \( t_s \) is small enough, the existing and reaching condition can be derived as
\[
\frac{[(S_j(k + 1) - S_j(k))\text{sgn}(S_j(k))] < 0}{t_s}
\]
where \( \epsilon_i > 0, \epsilon_r > 0, 1 - q_j t_s > 0 \).

For Eq. 25, it can be derived
\[
[(S_j(k + 1) - S_j(k))\text{sgn}(S_j(k))] = \epsilon_i t_s \text{sgn}(S_j(k))\text{sgn}(S_j(k)) (j = 1, 2)
\]
\( = (2 - q_j t_s) |S_j(k)| - \epsilon_i t_s |S_j(k)| > 0 \)

Meanwhile, when sampling time \( t_s \) is small enough, \( 2 - q_j t_s \leq 0 \), then
\[
[(S_j(k + 1) + S_j(k))\text{sgn}(S_j(k))] = [(2 - q_j t_s) |S_j(k)| - \epsilon_i t_s |S_j(k)|] (j = 1, 2)
\]
\( = (2 - q_j t_s) |S_j(k)| - \epsilon_i t_s |S_j(k)| > 0 \)

Thus, the reaching condition Eq. 23 can be achieved. Substituting Eq. 20 into Eq. 25, the control laws can be derived as
\[
\begin{align*}
u_t(k) &= mR_v^s / t_s (v_s^r(k + 1) - v_s(k) + q_j t_s S_j(k) + \epsilon_i t_s \text{sgn}(S_j(k)) - t_s (v_i(k) \gamma(k) - F_v(k) - F_f(k))) \\
u_z(k) &= I_R / ct_s (\gamma^r(k + 1) - \gamma^r(k) - t_s F(k) + \epsilon_r t_s \text{sgn}(S_j(k)) + q_j t_s S_j(k))
\end{align*}
\]
where
\[
F(k) = a(F_{sg} + F_g) - b(F_v + F_f) + d_2(k)
\]

From Eq. 25,
\[
t_s < 4 / (1 + 2 q_j)
\]
Then,
\[
|p_j| = \frac{|S_j(k + 1)|}{|S_j(k)|}, p_j = 1 - q_j t_s - \epsilon_i t_s |S_j(k)|
\]

Based on the Eq. 31, there are three circumstances: 1) when \( |S_j(k)| > \epsilon_i t_s / (2 - q_j t_s) \), there is \( p_j > 1 - q_j t_s - \epsilon_i t_s (2 - q_j t_s) / \epsilon_i t_s = 1 \), then
\[
|p_j| < 1, \text{ where } |S_j(k + 1)| < |S_j(k)| \text{, which means } |S_j(k)| \text{ is decreasing; } 2) \text{ when } |S_j(k)| < \epsilon_i t_s / (2 - q_j t_s) \), there is \( p_j < 1 - q_j t_s - \epsilon_i t_s (2 - q_j t_s) / \epsilon_i t_s = 1 \), then
\[
|p_j| > 1, \text{ where } |S_j(k + 1)| > |S_j(k)| \text{, which means } |S_j(k)| \text{ is increasing; } 3) \text{ when } |S_j(k)| = \epsilon_i t_s / (2 - q_j t_s) \), there is \( p_j = 1 - q_j t_s - \epsilon_i t_s (2 - q_j t_s) / \epsilon_i t_s = -1 \), which means \( |S_j(k)| \text{ is chattering. Thus, it can be concluded that the sufficient condition of } |S_j(k)| \text{ decrease is } |S_j(k)| > \epsilon_i t_s / (2 - q_j t_s) \). To achieve this, it is required that
\[ e_j < \frac{1}{t_i} (2-t_j q_j) |S_j (k)| \]  

If we take \( e_j = |S_j (k)|/2 \) and the sampling time meets the requirement of \( t_i < 4/(1+2 q_j) \), Eq. 32 can be guaranteed. The hyperbolic tangent functions \( \tanh(S_j(k)/\sigma_j) \) is used to replace the sign switching function \( \text{sgn}(S_j(k)) \) in Eq. 28 to avoid the chattering effects in practical implementation. Here, \( \sigma_j \) is the boundary layer thicknesses.

### 3.2 Lateral tire force estimation

According to the control law designed in Eq. 28, tire forces are critical to accomplish the control target. Based on Dugoff tire model introduced in Section 2, an unscented Kalman filter is employed to estimate the real time tire forces. According to Dugoff tire model, the factors that determine the tire forces are road adhesion coefficient, tire vertical load, tire lateral stiffness, tire sideslip angle and wheel slip ratio. Among all these factors, road adhesion coefficient is the only one can not be measured or calculated directly. Here we define nominal longitudinal and lateral tire forces \( F_{x1}^0 \) and \( F_{y1}^0 \) to describe the computable part of them, which are expressed as

\[
\begin{align*}
F_{x1}^0 &= F_{x1} k_s \frac{\lambda_i}{1+\lambda_i} f(L_i) \\
F_{y1}^0 &= F_{y1} k_s \tan \alpha_i \frac{\alpha_i}{1+\alpha_i} f(L_i)
\end{align*}
\]

Then the measurement equations used for estimating are derived as

\[
\begin{align*}
\dot{a}_x &= \frac{1}{m} (\mu_p F_{x1} + \mu_p F_{y1} + \mu_i F_{x1} + \mu_i F_{y1}) \\
\dot{a}_y &= \frac{1}{m} (\mu_p F_{x1} + \mu_p F_{y1} + \mu_i F_{x1} + \mu_i F_{y1}) \\
\dot{\gamma} &= \frac{1}{I_z} (a(\mu_p F_{x1} + \mu_p F_{y1} + \mu_i F_{x1} + \mu_i F_{y1}) \\
&- c(-\mu_p F_{x1} + \mu_p F_{y1} - \mu_i F_{x1} + \mu_i F_{y1}))
\end{align*}
\]

The details of UKF are displayed in Fig. 4, where system states are the four tire road adhesion coefficients, namely \( x = [\mu_p, \mu_p, \mu_i, \mu_i] \); the inputs to the system are eight nominal tire forces, namely \( u = [F_{x1}, F_{y1}, F_{x1}, F_{y1}, F_{x1}, F_{y1}, F_{x1}, F_{y1}] \); the system outputs are \( z = [a_x, a_y, \dot{\gamma}] \).

### 3.3 Optimal energy efficiency torque allocation

After obtaining the estimated road adhesion coefficients, the lateral tire forces can be calculated by the Dugoff model. The estimation results of front left tire are shown as Fig. 5.

In Fig. 5, the blue curve is the tire force obtained by the linear tire model, which can be expressed as tire slip angle times tire lateral stiffness; the black curve is the reference value output by CarSim®; the red curve is the estimation result based on method aforementioned. The result obtained by linear model is larger than the reference is because the lateral stiffness will decrease with the increase of the tire slip angle. It can be shown in Fig. 5 that the estimation value matches the reference curve pretty well so it can be applied in the whole control strategy.

The inverter power consumption and the mechanical friction power consumption are considered uncontrollable,
so the total power consumption can be described as

\[ P_{\text{con}} = P_{\text{ou}} + P_{\text{iron}} + P_{\text{copper}} \]  

(35)

where \( P_{\text{ou}} \) is the motor mechanical output; \( P_{\text{iron}} \) is the motor iron loss; \( P_{\text{copper}} \) is the motor copper loss. More specifically,

\[
P_{\text{copper}} = R_e (i_a^2 + i_q^2) = R_e \left( (i_a - \frac{\omega_e L_q i_q}{R_e})^2 + (i_q + \frac{\omega_e (\varphi_f + L_d i_d)}{R_e})^2 \right) \]

(36)

\[
P_{\text{iron}} = R_i (i_a^2 + i_q^2) = \frac{\omega_e^2 L_q^2 i_q^2}{R_i} + \frac{\omega_e^2 (\varphi_f + L_d i_d)^2}{R_i} \]

(37)

\[
P_{\text{ou}} = T_e \omega \]

(38)

\[
i_q = \frac{T_e}{P_e \varphi_f} \]

(39)

In Section 2, it is assumed that \( L_d = L_q \). According to the Ref [21],

\[
i_a = \frac{\omega_e^2 L_q (R_e + R_i) \varphi_f}{R_e R_i^2 + \omega_e^2 L_d^2 (R_e + R_i)} \]

(40)

Then the object function of motor power consumption is derived as

\[
J_1 = \sum_{i} P_{\text{con}}_{i}^{2} \]

(41)

The tire slip energy is considered to be important dissipation energy from driving axles to wheels. To make most use of driving torque, an objective function is introduced to minimize the tire slip energy, which is

\[
\min J_2 = \left\| P_{\text{slip}} \right\|^2 = \sum_{i=1}^{n} (T_i (k) \omega_t (k) - F_{\text{ou}} (k) v_t (k))^2 \]

(42)

The IWM can work in both driving and braking mode, under certain conditions the IWM braking will not be enough. To fully use the braking recovery energy, a parallel braking energy recovery strategy based on feedback braking is applied where mechanical braking system will start to work when the IWM braking force is saturated. This can be described as

\[
T_i = \max \{-4T_{\text{max}}, u_i \} \]

(43)

\[
T_{\text{ou}} = u_i - \max \{u_i, -4T_{\text{max}} \} \]

(44)

where \( T_i \) is total torque provided by IWMs; \( T_{\text{ou}} \) is IWM output saturation; \( T_{\text{bin}} \) is braking torque provided by mechanical braking system;

IWM torque output \( T_i \) complies with its physical saturation, which is

\[
-T_{\text{max}} \leq T_i \leq T_{\text{max}} \]

(45)

\[
T_{\text{max}} (\omega) = \begin{cases} 
T_{\text{max}}, & \text{if } \omega \leq P_{\text{max}} / T_{\text{max}} \\
T_{\text{max}} / \omega, & \text{else} 
\end{cases} \]

(46)

where \( P_{\text{max}} \) and \( T_{\text{max}} \) are max power and max torque output of the IWM; \( \omega \) is IWM rotational speed.

The first task can be accomplished by distributing the IWM torque to meet following constrains:

\[
\begin{aligned}
\Sigma T (k) &= T_p (k) + T_r (k) + T_a (k) + T_v (k) \\
\Delta T (k) &= -T_p (k) + T_r (k) - T_s (k) + T_v (k)
\end{aligned} \]

(47)

\[
J_i (k) = (u_i (k) - \Sigma T (k))^2 + (u_i (k) - \Delta T (k))^2
\]

(48)

Then the overall cost-function of the optimal allocation is defined as

\[
J (k) = \xi_i J_i (k) + \xi_j J_j (k) + \xi_k J_k (k)
\]

(49)

where \( \xi_i, \xi_j, \xi_k \) are weights.

Altogether, the optimal torque allocation problem can be restated as:

Minimize the cost function Eq. 49, subject to the constrains Eq. 43-45.

This problem can be solved by the Sequential Quadratic Programming (SQP) proposed in [23] which can be summarized as follows. For optimal problem:

\[
\begin{aligned}
\min & \quad f (x) \\
\text{s.t.} & \quad h_i (x) = 0, i \in E = [1,2, \ldots, l] \\
& \quad g_i (x) \geq 0, i \in I = [1,2, \ldots, m]
\end{aligned} \]

(50)

The Lagrange function is defined as

\[
L (x, \mu, \lambda) = f (x) - \sum_{i \in E} \mu_i h_i (x) - \sum_{i \in I} \lambda_i g_i (x)
\]

(51)

And then

\[
A^E (x) = \nabla h (x) \quad , \quad A^I (x) = \nabla g (x) \quad , \quad A (x) = [A^E; A^I] \quad ,
\]

\[
W (x, \mu, \lambda) = \nabla^2 L (x, \mu, \lambda).
\]

Step 0, given initial pair \( (x_0, \mu_0, \lambda_0) \in \mathbb{R}^n \times \mathbb{R}^l \times \mathbb{R}^m \) and symmetric positive definite matrix \( B_0 \in \mathbb{R}^{n \times n} \), calculate \( A^E_k (x) = \nabla h (x) \quad , \quad A^I_k (x) = \nabla g (x) \quad , \quad A_k = [A^E_k; A^I_k] \). Choose parameter \( \eta \in (0, 1 / 2) \) and the allowable errors

\[
0 \leq \varepsilon_1, \varepsilon_2 \leq 1. \quad \text{Set } k := 0.
\]

Step 1, solve the subproblem

\[
\min \quad \frac{1}{2} d^T B_k d + \nabla f (x_k)^T d
\]

s.t.

\[
h (x_k) + A^I_k d = 0
\]

(52)

\[
g (x_k) + A^E_k d \geq 0
\]

to get the optimal solution \( d_k \).

Step 2, if \( \left\| d_k \right\| \leq \varepsilon_1 \) and \( \left\| h_k \right\| + \left\| g_k \right\| \leq \varepsilon_2 \), stop and get an approximate KT point of the original problem \( (x_k, \mu_k, \lambda_k) \).

Step 3, for a certain cost function \( \phi (x, \sigma) \), choose a penalty parameter \( \sigma_k \) to make \( d_k \) is in the falling direction of \( \phi (x, \sigma) \) at the point of \( x_k \).

Step 4, Armijo searching. Make \( m_k \) to be the minimum nonnegative integer \( m \) satisfying following inequality:

\[
\phi (x_k + \rho^m d_k, \sigma_k) - \phi (x_k, \sigma_k) \leq \eta \rho^m \phi (x_k, \sigma; d_k).
\]

(53)
An Energy Efficient Control Strategy for Electric Vehicle Driven by In-Wheel-Motors Based on Discrete Adaptive Sliding Mode Control

then choose \( \alpha_k := \rho^\lambda \), \( x_{k+1} := x_k + \alpha_k d_k \).

Step 5, calculate
\[
A_{k+1}^E = \nabla h(x_{k+1}), \quad A_{k+1}^I = \nabla g(x_{k+1}), \quad A_{k+1} = [A_{k+1}^E; A_{k+1}^I]
\]
and the least-squares multiplier
\[
\begin{bmatrix}
\hat{\mu}_{k+1} \\
\hat{\lambda}_{k+1}
\end{bmatrix} = \left[A_{k+1}^E A_{k+1}^I \right]^{-1} A_{k+1} \nabla f_{k+1} \tag{54}
\]

Step 6, correct \( B_k \) to \( B_{k+1} \). Set
\[
s_k = \alpha_k d_k, \quad \gamma_k = \nabla \mathcal{L}(x_k, \mu_k, \lambda_k) - \nabla_s \mathcal{L}(x_k, \mu_k, \lambda_k)
\]
\[
B_{k+1} = B_k - \frac{B_k s_k^T B_k}{s_k^T s_k} + \frac{z_k \gamma_k}{s_k^T \gamma_k}, \tag{55}
\]
where
\[
z_k = \theta \gamma_k + (1 - \theta) B_k s_k \tag{56}
\]
\[
\theta_k = \begin{cases} 
0.8s_k^T B_k s_k & \text{if } s_k^T y_k \geq 0.2s_k^T B_k s_k \\
0.8s_k^T B_k s_k & \text{if } s_k^T y_k \geq 0.2s_k^T B_k s_k 
\end{cases} \tag{57}
\]

Step 7, set \( k = k+1 \), switch to step 1.

4 Simulation results and analyses

Simulations are conducted to verify the effectiveness of the proposed control strategy based on the co-simulation platform of CarSim® and Matlab/Simulink. The model is established based on a high-fidelity full-vehicle model in CarSim® and all the powertrain components are replaced by four IWMs. Meanwhile, simulations results by torque distribution method shown in Eq. 59 are used as comparisons to validate the improvement of integrated optimal torque allocation. Parameters of the vehicle used in simulations are shown as Table 1.

### Table 1 Vehicle parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>Vehicle total mass</td>
<td>1259.98 kg</td>
</tr>
<tr>
<td>( I_z )</td>
<td>Vehicle yaw moment of inertia</td>
<td>4607 kg·m²</td>
</tr>
<tr>
<td>( a )</td>
<td>Distance from CG to front axle</td>
<td>1.14 m</td>
</tr>
<tr>
<td>( b )</td>
<td>Distance from CG to rear axle</td>
<td>1.64 m</td>
</tr>
<tr>
<td>( R_t )</td>
<td>Tire rolling radius</td>
<td>0.36 m</td>
</tr>
<tr>
<td>( k_f )</td>
<td>Cornering stiffness of the front tire</td>
<td>-143583 N/rad</td>
</tr>
<tr>
<td>( k_r )</td>
<td>Cornering stiffness of the rear tire</td>
<td>-111200 N/rad</td>
</tr>
<tr>
<td>( G_p )</td>
<td>Transmission ratio of steering system</td>
<td>17</td>
</tr>
<tr>
<td>( I_w )</td>
<td>Wheel yaw moment of inertia</td>
<td>1.33 Nm²</td>
</tr>
<tr>
<td>( K_m )</td>
<td>IWM control gain</td>
<td>80 Nm/V</td>
</tr>
</tbody>
</table>

Double line change and new European driving cycle (NEDC) are conducted in simulations to verify the motion control performance and the economic efficiency of the proposed control. The results are shown in the subsequent sections.

4.1 Double line change

The control target of this study is to coordinate the motion control of EV driven by IWMs in an effective way. The first simulation is the double line change with longitudinal velocity increase. The vehicle is set to accelerate from 15m/s to 20m/s in 40 seconds and the double line change track is shown as below. The air density is set as 1.206kg/m³ during the simulation. Two comparison simulations are also conducted; one is conducted by using the normal sliding mode control with proposed torque optimal allocation, the other is conducted by using the ADSMC and the allocation method based on tire load. Simulation results are displayed as Fig.5 and Fig.6.

![Fig.5 Simulation results of longitudinal velocity](image)

![Fig.6 Simulation results of yaw rate](image)

From Fig.5 and Fig.6, it can be concluded that the proposed control method can accomplish the longitudinal and yaw motion control successfully. Moreover, the ADSMC outperforms the DSMC in suppressing chattering according to Fig. 6.
The load-based allocation (LBA) method in Eq. 50 is applied as the comparison to the optimal allocation (OA) method. The results are shown in Fig. 7.

![Load based allocation](image1)

(a) Load based allocation

![Optimal allocation](image2)

(b) Optimal allocation

**Fig. 7 Simulation results of IWM torques**

In Fig. 7, it can be seen that the yaw motion control is accomplished by differential torques between left and right IWMs in both two allocation methods. While the allocation ratios between front and rear IWMs are different. To quantitatively illustrate the improvement of the proposed control strategy more specifically, Table 2 shows the performances under different control strategy.

The evaluation results are obtained based on the aforementioned simulations. It can be concluded that with the ADSMC, the tracking performance has been improved comparing with the DSMC for its better chattering suppression performance. Furthermore, the power consumption is also decreased by ADSMC according to Table 2, because the frequent torque changes will cause more power consumption. According the second and third rows of Table 2, the power consumption by OA is decreased as expected but the tracking performances of yaw rate and longitudinal velocity are slightly worse than LBA. Combining with the Eq. 48, 49 and 50, it can be concluded that the OA sacrifices the tracking performance to obtain lower energy consumption.

**Table 2 Performances under different control strategies**

<table>
<thead>
<tr>
<th>Control Strategy</th>
<th>Error covariance of $v_x$</th>
<th>Error covariance of $\gamma$</th>
<th>Power consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSMC+LBA</td>
<td>$9.2134 \times 10^{-4}$</td>
<td>$1.1064 \times 10^{-4}$</td>
<td>$3.8009 \times 10^{-5}$</td>
</tr>
<tr>
<td>ADSMC+LBA</td>
<td>$4.1615 \times 10^{-4}$</td>
<td>$5.3861 \times 10^{-5}$</td>
<td>$2.6731 \times 10^{-5}$</td>
</tr>
<tr>
<td>ADSMC+OA</td>
<td>$4.5844 \times 10^{-4}$</td>
<td>$5.7721 \times 10^{-5}$</td>
<td>$2.2481 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

**4.2 NEDC**

Although the above simulation results can verify the coordinated motion control of proposed control strategy, they are still not enough to demonstrate the energy efficiency in such a short time interval. To fully prove the control performance of proposed strategy, the NEDC are conducted. The vehicle is set to drive in a single line without steering. Similarly, simulations by using the abovementioned control strategy ADSMC+LBA are conducted as the comparisons. The simulation results are shown as Fig. 8 and Figure 9. The control performance is displayed as Table 3.

![Simulation results of longitudinal velocity](image3)

(a) Optimal allocation

**Fig. 8 Simulation results of longitudinal velocity**
In Fig.8, it can be seen that both control strategy can track the reference longitudinal velocity in the whole-time range. While in Fig.9, the torque outputs with OA meet the torque output limit requirement and also have fewer sharp changes. According to Table 3, the control strategy with LBA shows better tracking performance at the cost of more power consumption and less energy efficiency. While even the control strategy with OA shows a bigger tracking error, it is still effective for the maximum absolute tracking error is 0.4960 which is totally acceptable. This indicates that the proposed control strategy can generate a proper control input to the system in an energy efficient way, precisely to track the reference outputs to accomplish vehicle coordinated motion control.

Based on all the simulation results exhibited above, the effectiveness of such an energy efficient control strategy for EV driven by IWMs based on DASMC are demonstrated.

5 Conclusion

An energy efficient control strategy for EV driven by IWMs based on DASMC is proposed in this study. Models are established firstly to demonstrate the operation mechanism of the whole system and two virtual control variables are used to describe the longitudinal and yaw control efforts to complete the vehicle coordinate motion control. Then DASMC method is applied to calculate the required total driving torque and yaw moment. A tire force estimator using UKF is designed to estimate real-time lateral tire forces used in the control scheme. Based on all abovementioned factors, energy efficient torque allocation method is developed to distribute the total driving torque and differential torque to each IWM. Simulation results of the proposed control strategy using co-platform of Matlab/Simulink and CarSim® demonstrate that this study can accomplish the vehicle motion control in a coordinated and economic way and improve the tracking performance as well as the system robustness.

6 Declaration

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Availability of data and materials
The datasets supporting the conclusions of this article are included within the article.

Authors’ contributions
The author’ contributions are as follows: Wangzhong Zhao and Chunyan Wang were in charge of the whole trial; Han Zhang wrote the manuscript and executed the research plan; Changzhi Zhou assisted with data processing and analyses.

Competing interests
The authors declare no competing financial interests.

Consent for publication
Not applicable

Ethics approval and consent to participate
Not applicable

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