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Estimation of Effect on Gross Domestic Product of Production Factors Using CES and Translog Production Functions: An Application to China Economy

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Abstract

In this paper, we investigate the effect on economic growth (GDP) for China's economy with capital, labor and energy input factors by using CES production function and Translog production function.

The empirical findings of the study showed that CES, consisting of capital and labor factors, is less efficient than the Translog function consisting of capital, labor and energy input factors for GDP estimation.

The Ridge regression method is used to the parameter estimation of Translog production function using historical data because there is collinearity between variables. Then, based on the fitted Translog production model including capital, labor and energy input factors, the results of the output elasticities for each of the factors and the substitution elasticities between input factors have been dynamically estimated. To predict the future economic growth of the China economy, the inputs of Translog production model are predicted by using Holt-Winter's method. The elasticities of the output of all input factors are positive. According to degrees of the effect on GDP, we can list the factors as labor, capital and energy, respectively. This situation represents the China economy is labor and capital intensive.

Keywords: Gross Domestic Product, CES production function, Translog production function, Holt-Winter's method, Ridge regression.

Subject classification codes: C51, C53, E23, 047

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1. Introduction

The production process transforms the inputs into outputs. Gross Domestic Product (GDP) in general economy can be described by a production function. Thus, the determination of the functional form of the production function is an important issue in the empirical analysis. There are four different production function models commonly used in the literature. These are the Cobb-Douglas production function for which elasticity of substitution is equal to one, Constant Elasticity of Substitution (CES) production function, Leontief production function and Translog production function. In this study, we used CES production function and Translog production function to establish the impact of capital investments, labor and energy on economic development.

Analyzing the factor substitution is very important to develop the economic policies. In this study, firstly, we used the classic assumption which the elasticities of substitutions are constant and the CES model. But in fact, this assumption may not be realistic. It may change over time. Secondly, we use the Translog function as a model for variable elasticity substitution. In this model, it is avoided using the data of factor prices.

CES production function has a lot of applications in macroeconomics as international trade (Lloyd ve MacLaren, 2002), economic growth (Papageorgiou ve Saam, 2008) and energy economics (McFarland vd., 2004). CES functions are non-linear and they can also not be linearized. So for reason, conventional linear estimation methods cannot be implemented to them. There are some difficulties in estimating the parameters of CES production function, such as the surface of the objective function having a large flat area and local minimum (Henningsen ve Henningsen, 2012). The popular estimation methods for CES production function are linear Taylor series approximation and non-linear least squares method.

Christensen, Jorgenson, and Lau evaluated the Translog production function as an extension of the Cobb-Douglas production function. (Christensen vd., 1973).

Then, (Romer, 1986), (Lucas Jr., 1988), (Mankiw, N.G., Romer, D. and Weil, 1992) and (Benhabib ve Spiegel, 1994) have made important contributions. Then, Grey Cobb-Douglas production function is used to estimation of GDP (Onalan ve Basegmez, 2018).

There are many factors that have contributed to GDP, such that capital, labor, energy, optimal allocation of technology sources, innovations, etc. In this study, CES production function and Translog production function models are established to confirm that capital, labor and energy have an important effect on GDP. When fitted to the real data of the production function models, it is generally used multiple regression models. In a multiple regression application, we need to know the future values of the inputs (independent variables) to the prediction of future values of the response variable. Thus, future values of capital, labor and energy input factors are forecasted based on Holt-Winter's method in this study.

The aim of this paper is twofold. Firstly, it is to investigate the output elasticity for each input factor and the elasticity of substitution between capital, labor and energy input factors. Secondly, it is to estimate the future values of GDP which is an indicator of China's Economic growth.

The CES production function and Translog production function are implemented in real data. To predict the future values of GDP, capital, labor and energy input factors are predicted using by Holt-Winter's method, then based on the fitted model is analyzed the variation of the elasticities according to levels of input factors within the considered time period.

The remaining part of this paper is organized as follows. Section II reviews the fundamental properties of CES production function, nested CES function and parameter estimation methods. Section III describes the model framework and the estimation procedures such as Translog production function, Ridge regression method and Holt-Winter's method. In Section IV, we describe the data set, how the data is processed and it is presented the empirical results and discussion. Section V includes the conclusion of the study.

2. Constant Elasticity of Substitution (CES) Production Function

GDP can be considered as a measure of the economic growth of countries. The main factor affecting the GDP is industrial output. The industrial output can be considered as a function of capital and labor inputs (Hossain vd., 2013). In substance, economics production is affected by various environmental factors as capital, labor, agricultural activities, technology, industry, energy, raw materials, etc. Production is a process that converts production factors (inputs) into finished products (outputs).

In the economics literature, the empirical relationship between given the quantity of economics inputs and specified outputs is represented as the production function. One of the main problems for economic governance in the production process is determination of the functional relationship between the production output and input factors. The general form of the production function is described by $f: D \rightarrow R_+$, $D \in R_+^n$ and $Q = f(X_1, X_2, \dots, X_n)$ where X_1, X_2, \dots, X_n are inputs and Q is production level. This function is a differentiable function of all its inputs. Variables are mostly available capital stock in terms of the value of machinery and buildings and labor in terms of people employed or land in terms of acres. Therefore, it is assumed that the inputs of this function are homogeneous. The production function with n input factors is called h – homogeneous degree, if

$$f(kx_1, kx_2, \dots, kx_n) = k^h f(x_1, x_2, \dots, x_n), \quad (h > 0)$$

where $k \in R$ and if $h > 1$, per percent increase in input levels would result greater than per percent increase in the output level, if $h < 1$, per percent increase in input levels would result in less than per percent increase in output and $h = 1$ represent the constant return to scale.

The production functions are characterized according to technology elasticity of substitution and return to scale.

Hicks elasticity of substitution measure to a production function with more than two inputs is described as (Stern, 2011),

$$\sigma_{ij} = - \frac{\partial \ln(X_i/X_j)}{\partial \ln((\partial Q/\partial X_i)/(\partial Q/\partial X_j))}.$$

2.1.Functional form of CES production function

The Constant - Elasticity of Substitution (CES) production function originally is given by (Arrow vd., 1961). It is a generalization of the Cobb-Douglas production function. The CES function puts a restriction on elasticity substitution of being constant along the whole isoquant. The CES production function assumes that the elasticities of substitution between any two inputs are the same.

The CES production function has many forms. In this study, we will use the CES production function which is proposed by Kmenta with capital and labor inputs (Kmenta, 1967). It is given as follows,

$$Q = A[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-\vartheta/\rho} \quad (1)$$

where Q denotes the total value of output, L denotes input of labor which is measured in men-per (and hour) year, K represents the capital input which is measured in money term, $A \in [0, \infty)$ represents the productivity (technologic progress level), $\delta \in [0, 1]$ denotes the inputs' optimal distribution, $\rho \in [-1, 0) \cup (0, \infty)$ represents the elasticity of substitution and $\vartheta \in (0, \infty)$ ($\vartheta = K/L > 0$) is the function's homogenous order or rate of return to scale (degree of homogeneity). In the original form of CES, parameter ϑ was taken as $\vartheta = 1$. If $\vartheta = 1$, constant return to scale, if $\vartheta < 1$, decreasing return to scale and if $\vartheta > 1$, increasing return to scale.

The elasticity of substitution (EOS) of CES function is written as,

$$\sigma = \frac{\partial \ln(K/L)}{\partial \ln(MP_L/MP_K)} = \frac{1}{1 + \rho} \geq 0. \quad (2)$$

The logarithmic form of CES function is given by

$$\ln Q = \ln \gamma - \frac{\vartheta}{\rho} \ln[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]. \quad (3)$$

Uzawa (1962) and McFadden (1963) tried to extent the CES function to n –input factor production functions. Marginal productivity of labor for CES function is given by

$$\frac{\partial Q}{\partial L} = (1 - \delta) A^{-\rho} \left(\frac{Q}{L}\right)^{\rho+1}.$$

Then,

$$\frac{Q_L}{Q} = (1 - \delta) A^{-\rho} \left(\frac{Q}{L}\right)^{\rho+1}. \quad (4)$$

In Eq. (4), taking the logarithm of both side, we obtained

$$\ln(Q_L/Q) = \ln((1 - \delta)A^{-\rho}) + (\rho + 1) \ln(Q/L). \quad (5)$$

Eq. (5) represents the relationship between nominal wage, the market price of production and average production.

2.2. Parameter estimation of CES production function

CES production function is nonlinear in parameters, so it cannot be linearized to estimate the parameters using the traditional linear estimation methods. In the parameter estimation of CES production function is used to non-linear fitting techniques. In the estimation process, we assume that input variables are non-stochastic or if they are stochastic, independent of disturbance term (Hoff, 2002).

In general, two approaches widely used in CES parameter estimation are linear Taylor series approach and non-linear least squares method. We can use a linear approach with respect to ρ .

2.3. Estimating the CES function using Kmenta Approximation

In Q series expanding the Taylor series around the $\rho = 0$ and discarding the terms of third and higher order expanding leads the following form (Kmenta, 1967),

$$\ln Q_i = \ln \gamma + \vartheta \delta \ln K_i + \vartheta(1 - \delta) \ln L_i - \left(\frac{1}{2}\right) \rho \vartheta \delta(1 - \delta) [\ln K_i - \ln L_i]^2 + \varepsilon_i \quad (6)$$

Let we define the new variables as,

$$Y^* = \ln Q_i, \quad X_1^* = \ln K_i, \quad X_2^* = \ln L_i, \quad X_3^* = [\ln K_i - \ln L_i]^2,$$

$$\beta_0 = \ln \gamma, \quad \beta_1 = \vartheta \delta, \quad \beta_2 = \vartheta(1 - \delta), \quad \beta_3 = -\left(\frac{1}{2}\right) \rho \vartheta \delta(1 - \delta).$$

Then, Eq. (6) can be written as follows according to this new parameterization.

$$Y^* = \beta_0 + \beta_1 X_1^* + \beta_2 X_2^* + \beta_3 X_3^* + \varepsilon. \quad (7)$$

The application of the ordinary least squares method yields the following parameter estimations (Mahaboob vd., 2017),

$$\hat{\gamma} = e^{\hat{\beta}_0}, \quad \hat{\delta} = \frac{\hat{\beta}_1}{\hat{\beta}_1 + \hat{\beta}_2}, \quad \hat{\vartheta} = \hat{\beta}_1 + \hat{\beta}_2, \quad \hat{\rho} = -2 \left(\frac{\hat{\beta}_1 + \hat{\beta}_2}{\hat{\beta}_1 \hat{\beta}_2} \right) \hat{\beta}_3. \quad (8)$$

The disadvantage of this approach is that it gives reliable results if ρ is close the point zero (Thursby ve Lovell, 1978).

The substitution elasticity of (EOS) σ is a function of ρ and estimated as,

$$\hat{\sigma} = \frac{1}{1 + \hat{\rho}}.$$

2.4.Nested CES Production Function

A nested production function is given by (Henningsen vd., 2019), (Kemfert, 1998),

$$Q_t = e^{mt} A \left[a(bK_t^{-\alpha} + (1-b)E_t^{-\alpha})^{\beta/\alpha} + (1-a)L_t^{-\beta} \right]^{-\gamma/\beta}$$

where a and b denote the share parameters denoting input factor contributions to output. α and β denote the substitution parameters, $A \geq 0$ represents the productivity (efficient), m denotes the annual rate of technological change, t is the time in years and Q denotes the output. In the single level CES function, all input factors have the same substitution elasticity with each other. However, the nested CES function allows the different substitution elasticities between the factors.

When the model is applied the real data, firstly capital and energy combined into an intermediate input using a two-input CES function with share parameter b and substitution parameter α . Secondly, this intermediate input is combined with labor L into output Q using another CES function with share parameter a and substitution parameter β .

$$Q_t = Ae^{mt} X^{-\gamma/\beta}$$

$$X = aY^{\beta/\alpha} + (1-a)L^{-\beta}$$

$$Y = bK^{-\alpha} + (1-b)E^{-\alpha}$$

$$\rho_{in} = \frac{1}{1+\alpha} \quad \text{and} \quad \rho_{out} = \frac{1}{1+\beta}$$

ρ_{in} and ρ_{out} are inner and outer nested substitution parameters, respectively.

The parameters of nested CES function is estimated using non-linear least squares method as,

$$\min_{A,m,a,b,\alpha,\beta} \sum_t \left(\ln Q_t - mt - \ln A + \frac{1}{\beta} \ln \left[a(bK_t^{-\alpha} + (1-b)E_t^{-\alpha})^{\beta/\alpha} + (1-a)L_t^{-\beta} \right] \right)^2.$$

To estimate the parameters of CES function, Kemfert (1998) is used the statistical software SHAZAM.

3. Translog Production Function

The Translog function belongs to the quadratic response surface methodology. The transcendental logarithmic function (Translog production function) was first introduced by (Christensen vd., 1973). The Translog production function is a more general form of variable elasticity of substitution. It does not impose any restrictions on the substitutability between different input factors. A Translog production function can be view as a second-order Taylor approximation for arbitrary function (Lin ve Liu, 2017). The Translog function is widely used to detect the substitution effect of explanatory variables. The interactions between the components of interested system may be described as a quadratic response surface as follows,

$$\hat{y} = a_0 + a_1x_1 + a_2x_2 + a_3x_3 + a_{12}x_1x_2 + a_{12}x_1x_2 + a_{13}x_1x_3 + a_{23}x_2x_3 + a_{11}x_1^2 + a_{22}x_2^2 + a_{33}x_3^2$$

Generally, a Translog production function is described as follows;

$$\ln Q_t = \beta_0 + \sum_i \beta_i \ln X_i + \sum_i \beta_{ii} (\ln X_i)^2 + \frac{1}{2} \sum_{i \neq j} \beta_{ij} (\ln X_i) (\ln X_j) + \varepsilon \quad (9)$$

Using the symmetry property of partial derivatives, i.e. $\frac{1}{2} (\beta_{ij} + \beta_{ji}) = \beta_{ij}$, we will use the following form of the Translog production function in practice,

$$\ln Q_t = \beta_0 + \sum_{i=1}^n \beta_i \ln X_i + \sum_{i=1}^n \beta_{ii} (\ln X_i)^2 + \sum_{j>1}^n \beta_{ij} (\ln X_i) (\ln X_j) + \varepsilon \quad (10)$$

where Q is the output, X_i are inputs and β_0 , β_i and β_{ij} are parameters to be estimated. Parameter β_0 denotes the technology level.

Generally, response surface function with n – variables is estimated as follows. Polynomial coefficients are determined based on least squares regression method. We assume that there are k observation points.

$$Q = \begin{bmatrix} \ln Q_1 \\ \ln Q_2 \\ \vdots \\ \ln Q_k \end{bmatrix}$$

$$\mathbf{X} = \begin{bmatrix} 1 & \ln X_{11} & \cdots & \ln X_{1n} & (\ln X_{11})^2 & \cdots & (\ln X_{1n})^2 & \ln X_{11} \ln X_{12} & \cdots & \ln X_{1n} \ln X_{1n-1} \\ 1 & \ln X_{21} & \cdots & \ln X_{2n} & (\ln X_{21})^2 & \cdots & (\ln X_{2n})^2 & \ln X_{21} \ln X_{22} & \cdots & \ln X_{2n} \ln X_{2n-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \ln X_{kn} & \cdots & \ln X_{kn} & (\ln X_{kn})^2 & \cdots & (\ln X_{kn})^2 & \ln X_{kn} \ln X_{kn-1} & \cdots & \ln X_{kn} \ln X_{kn-1} \end{bmatrix}$$

$$\boldsymbol{\beta} = [\beta_0 \quad \beta_1 \quad \cdots \quad \beta_n \quad \beta_{11} \quad \cdots \quad \beta_{nn} \quad \beta_{12} \quad \cdots \quad \beta_{n,n-1}]^T$$

$$\boldsymbol{\varepsilon} = [\varepsilon_1 \quad \varepsilon_2 \quad \cdots \quad \varepsilon_n]^T$$

$$\mathbf{Q} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{Q}$$

The elasticity of output with respect to input X_i is given as,

$$f_i = \frac{\partial Q}{\partial X_i} \frac{X_i}{Q} = \beta_i + \sum_{j=1}^n \beta_{ij} \ln X_j \quad (11)$$

The marginal product of input X_i is given as,

$$y_i = \frac{\partial Q}{\partial X_i} = \left[\beta_i + \sum_{j=1}^n \beta_{ij} \ln X_j \right] \cdot \frac{Q}{X_i} \quad (12)$$

A Translog production function with ($n = 3$) three input factors $Q = f(K, L, E)$ which includes the capital stock, labor force and energy consumption is defined as;

$$\begin{aligned} \ln(Q_t) = & \beta_0 + \beta_K \ln K_t + \beta_L \ln L_t + \beta_E \ln E_t + \beta_{KK} (\ln K_t)^2 \\ & + \beta_{LL} (\ln L_t)^2 + \beta_{EE} (\ln E_t)^2 + \beta_{KL} (\ln K_t \ln L_t) \\ & + \beta_{KE} (\ln K_t \ln E_t) + \beta_{LE} (\ln L_t \ln E_t) \end{aligned} \quad (13)$$

In order for a production function to have consistent and desired properties the following three conditions are required, *i*) monotonically increasing, *ii*) continuity, *iii*) exact concave. A Translog function generally satisfies these conditions. The exact concavity property of Translog function is tested using the Hessian matrix.

The output elasticity respect to input factors,

- The output elasticity of Capital,

$$f_K = \frac{dQ/Q}{dK/K} = \beta_K + \beta_{KL} \ln L_t + \beta_{KE} \ln E_t + 2\beta_{KK} \ln K_t$$

- The output elasticity of Labor force,

$$f_L = \frac{dQ/Q}{dL/L} = \beta_L + \beta_{LK} \ln K_t + \beta_{LE} \ln E_t + 2\beta_{LL} \ln L_t$$

- The output elasticity of Energy,

$$f_E = \frac{dQ/Q}{dE/E} = \beta_E + \beta_{KE} \ln K_t + \beta_{LE} \ln L_t + 2\beta_{EE} \ln E_t .$$

The elasticity of substitution is described by the ratio between the percentage changes in the proportion of input factors. Substitution elasticity indicates how hard to substitute one input factor with the other. It is varying zero to infinity. The substitution elasticity between different factors is given as (Lin ve Ahmad, 2016), (Lin ve Liu, 2017),

- The substitution elasticity between Capital and Labor,

$$\sigma_{KL} = \left[1 + \left[-\beta_{KL} + \left(\frac{f_K}{f_L} \right) \beta_{LL} \right] (-f_K + f_L)^{-1} \right]^{-1}$$

- The substitution elasticity between Capital and Energy,

$$\sigma_{KE} = \left[1 + \left[-\beta_{KE} + \left(\frac{f_K}{f_E} \right) \beta_{EE} \right] (-f_K + f_E)^{-1} \right]^{-1}$$

- The substitution elasticity between Labor and Energy,

$$\sigma_{LE} = \left[1 + \left[-\beta_{LE} + \left(\frac{f_L}{f_E} \right) \beta_{EE} \right] (-f_L + f_E)^{-1} \right]^{-1} .$$

3.1. Estimation of Translog Production Function

The Translog function can be considered as a second-order Taylor approximation of an arbitrary function. It is also similar to a linear regression model according to coefficients. Thus, the parameters of Translog production function in Eq.(13) can be estimated using the Ordinary Least Square method (OLS).

3.2.Ridge Regression

The linear regression model is given by,

$$Y = \beta X + e$$

where Y is a vector of the dependent variable, X describes $n \times p$ explanatory variables. e is random error. The assumptions of regression model are that $E(e) = 0$, $Var(e) = \sigma^2 I_n$, e is a random variable that has a normal distribution. There is not a relationship between independent variables and residuals.

The collinearity phenomena occur when there is linear or approximately linear relationship between two or more independent variables. In the existence of collinearity in the regression model, The OLS method is to be biased and singular. To obtain sensitive parameter estimation, the collinearity must be eliminated. In this study, the independent variables without removing from the model, to eliminate the collinearity, we will use the ridge regression estimator. We may detect the existing collinearity using one of the following methods:

- (1) Correlation coefficient: If the correlation coefficient $r > 0,75$, we suspect from the collinearity
- (2) Variance Inflation Factor (VIF): VIF value is calculated as,

$$C_{ij} = \frac{1}{1 - R_{ij}^2}$$

where, R_{ij} is partial correlation. If $C_{ij} > 10$, there is collinearity.

Ridge regression is a biased regression estimation method. Ridge regression estimator has proposed by (Hoerl ve Kennard, 1970). Ridge regression method is used to eliminate the collinearity problem in data analysis. Ridge regression is an improvement version of OLS method.

In the presence of multicollinearity, a constant small value of k is added to the diagonal elements in correlation matrix $(X'X)$ to reduce the dependence on independent variables. Ridge estimator is given by,

$$\hat{\beta}(k) = (X'X + kI)^{-1}X'Y \quad (14)$$

where k ($0 < k < 1$) is ridge parameter (shrinkage parameter) and I is the identity matrix. When $k = 0$ the ridge estimator $\hat{\beta}$ is the OLS estimation (Hrishikesh D. Vinod, 1978). Covariance matrix for ridge estimator is,

$$Var(\hat{\beta}) = \sigma^2(X'X + kI)^{-1}X'X(X'X + kI)^{-1}$$

where $\hat{\sigma}^2 = \frac{1}{n-p} \sum_{i=1}^n e_i^2$ and $e_i = y_i - \hat{y}_i$. Total MSE of the ridge estimator is given by (Hoerl ve Kennard, 1970).

$$MSE(\hat{\beta}) = \sigma^2 \sum_{i=1}^p \lambda_i(\lambda_i + k)^{-1} + k^2 \beta'(X'X + kI)^{-2} \beta$$

where λ_i are the eigenvalue of $(X'X)$ correlation matrix. The choice of appropriate k value (a) the regression coefficients must be stable, (b) Variance Inflation Factor (VIF) must be small. In the literature, there are different methods to obtain optimal value of ridge parameter k . The most popular of them is that the $\hat{\beta}(k)$ values are plotted with respect to the values of k , then the value of $\hat{\beta}(k)$ that is stable is selected as the optimal ridge parameter k .

Let the Eigenvalues of $(X'X)$ correlation matrix $\lambda_{\max} = \lambda_1 > \lambda_2 > \dots > \lambda_p = \lambda_{\min}$, then

$$k \leq \frac{\lambda_{\max} - 100\lambda_{\min}}{99}$$

To estimate the value of k is given by (Muniz ve Kibria, 2009),

$$k = \left(\prod_{i=1}^p \frac{1}{m_i} \right)^{1/p}, \quad m_i = \left(\frac{\hat{\sigma}^2}{\hat{\beta}_i^2} \right)^{1/2}$$

VIF show how the variance of an estimator is inflated by the presence of multicollinearity. When the variance of an estimator R_{ij} approaches 1, collinearity increases. Any value of VIF that is greater than 10 strongly indicates the presence of collinearity.

3.3.Holt-Winter's Method

We will use the Holt-Winter's exponential smoothing method to get the basic forecast values of the Capital, Labor and Energy input variables. This method is a common seasonal forecasting method. The multiplicative Holt-Winter's method is

described following three basic equations (Dhakre vd., 2016), (Hamilton J. D., 1994) and (Prado ve West, 2010),

$$\text{Exponential trend for long-term trend: } S_t = \alpha \left(\frac{y_t}{I_{t-p}} \right) + (1 - \alpha)(S_{t-1} + b_{t-1})$$

$$\text{Trend smoothing} : b_t = \beta(S_t - S_{t-1}) + (1 - \beta)b_{t-1}$$

$$\text{Seasonal smoothing} : I_t = \gamma \left(\frac{y_t}{S_t} \right) + (1 - \gamma)I_{t-p}$$

$$\text{Forecasting} : F_{t+m} = (S_t + b_t)I_{t+m-p}$$

where α, β and γ are parameters takes the values in interval $[0,1]$, t is the time period, y_t is observed value of series at time t .

4. Empirical Analysis

In this study, we use the annual GDP series as a proxy of aggregate national economic level for China economy. We estimated the future economic growth pattern of China by focusing on the 1994-2014 period. We use the annual data gross domestic product Q (0.1 billion yuan) as an output, fixed asset investment K (0.1 billion yuan), the number of employees L (10,000 people) and total energy consumption E (10,000 tons of standard coal) as input factors to analyze the China economy between 1994-2014. Data are obtained from World Bank's World development indicators (WDI) China Statistical Yearbook and China Energy Yearbook (CSYD) as published by China's National Bureau of Statistic and by (Cheng ve Han, 2017).

It was estimated the future values of GDP which is a function of capital, labor and energy input factors by using Improved GM(1,1) Model and Grey CES production function. Firstly, we investigated the effect on the economic growth of capital and labor by using CES production function. The model parameters are estimated using Kmenta approach and we predicted the economic growth.

Capital (K_t): No capital stock data is reported in the China statistical system. The time series of capital is usually not found in economics databases. It is obtained from other sources. It is calculated using perpetual inventory method (Raymond W. Goldsmith, 1951).

$$K_t = I_t + (1 - \delta_t)K_{t-1}$$

where K_t is capital stock in year t , I_t is investment in year t . Initial capital stock K_0 is determined as follows,

$$K_0 = \frac{I_0}{\overline{GDP} + 0,05}$$

Labor (L): The number of employees. Data obtained from China Statistical Yearbook.

Energy consumption (E_t): The data on energy consumption taken from China Statistical Yearbook and APEC energy database.

4.1. Parameter Estimation of CES Production Function

The fit of the estimates of CES production function is given in Table1.

Table 1. The regression estimation results

	Coefficients	Standard Error	t-Stat	P-value
Intercept	-35.987	13.039	-2.760	0.013
$\ln K$	0.575	0.051	11.228	0.000
$\ln L$	3.708	1.213	3.058	0.007
$\left(\ln\left(\frac{K}{L}\right)\right)^2$	0.019	0.021	0.939	0.361

Adjusted R Square = 0.997 , F statistics = 2213.223

Table 2. Parameter values of CES production function

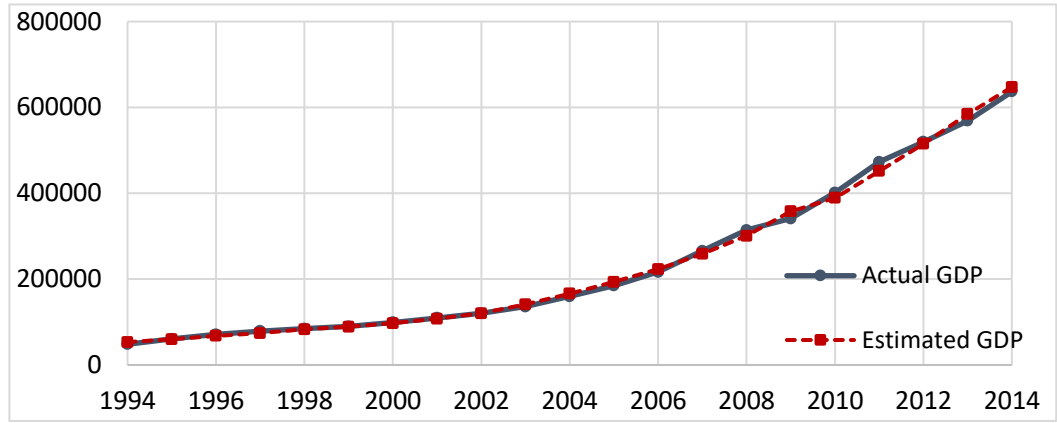
δ	$1 - \delta$	θ	ρ	σ	γ	$-\theta/\rho$
0.134	0.866	4.283	-0.078	1.085	$e^{-35.987}$	54.72

Using the estimated parameter values, CES function can be rewritten as,

$$\hat{Q}_{CES} = e^{-35.987} [0.134 K^{0.078} + 0.866 L^{0.078}]^{54.72} . \quad (15)$$

The actual and fitted values graphic is presented in Fig.1.

Figure 1. Actual GDP and Estimated GDP obtained from Kmenta Approach



The estimated value of elasticity of substitution $\hat{\sigma}_{CES} = 1.085$ denotes that CES production function may be a suitable model to describe the China economic growth, according to $\hat{\sigma}_{CES}$ estimation we can say that the general economy has a substitution potential between capital and labor inputs.

Substituting the input factor that has high unit price with the input factors that have relatively lower input price, the total production costs can be reduced.

The values of $\hat{\delta} = 0.134$ and $1 - \hat{\delta} = 0.866$ imply the distribution rate (share) of total output(Q) via capital (values of building, lands, machinery, other tools etc.) and labor (wages and salaries), respectively.

Parameter $\hat{\vartheta} = 4.283$ represents the elasticity of scale, this result implies that China economy has increasing return to scale.

4.2. Translog production model

In this section, we want to build a Translog production function with the capital, labor and energy input factors. Due to interaction and squared terms of the input factors in Eq. (13). The model is likely to suffer from severe multicollinearity. So before establishing a regression equation for Translog production function, we must detection the multicollinearity existing.

The condition number is above 30, the regression has significant multicollinearity. VIF is an important indicator to measure the severity of multicollinearity. If $VIF > 10$, then there is multicollinearity. In this case, Ridge regression is feasible method to deal with multicollinearity. In the parameter estimation process is used the software of NCSS 2007 and regression results are given in Table 4.

Table 3. Multicollinearity Test

Variable	Detection tolerance	R^2	VIF	Multicollinearity problem is present?	Condition Number
GDP	0.3%	99.7%	328.90	True	27112687.43
K	0.7%	99.3%	152.89	True	
L	13.4%	86.6%	7.45	True	
E	1.8%	98.2%	55.64	True	

Table 4 shows the estimation results of the Ridge regression with the Ridge parameter $k = 0.0006$.

Table 4. Ridge Regression Coefficient for $k = 0.006$

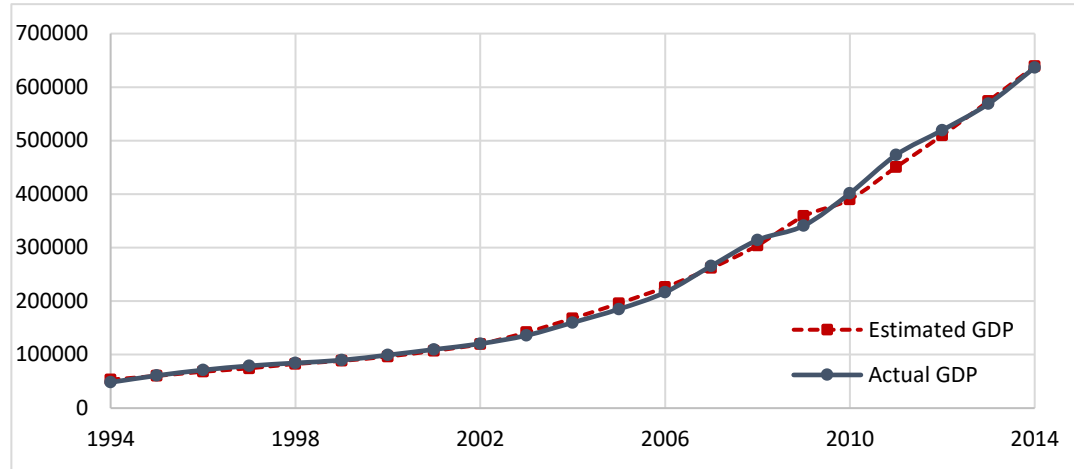
	Variables	Coefficient	Std. Error	Standardized Coefficient	VIF
R-Squared: 0.9962	β_0	-22.337470			
Sigma : 0.0669	$\ln K$	0.145865	0.027855	0.204000	4.372500
	$\ln L$	1.748346	0.474985	0.091300	1.771700
	$\ln E$	-0.013067	0.089362	-0.007000	6.517000
	$\ln K \ln K$	0.007516	0.001731	0.240500	8.840200
	$\ln L \ln L$	0.076086	0.021126	0.088900	1.755400
	$\ln E \ln E$	0.001052	0.003231	0.013800	5.151300
	$\ln K \ln L$	0.012340	0.002304	0.200100	4.022500
	$\ln K \ln E$	0.007475	0.001070	0.173800	1.784400
	$\ln L \ln E$	0.000447	0.006575	0.002900	5.340500

According to the result of Ridge regression, the fitting degree value of Translog equation is 99.62% and the multicollinearity of variables is effectively eliminated. The coefficients of technological progress input and energy input are negative and other variables have positive coefficients.

According to Table 4, the corresponding Ridge regression equation can be written as follows,

$$\begin{aligned} \ln \hat{Q}_t = & -22.337470 - 0.145865 \ln K_t + 1.748346 \ln L_t + -0.013067 \ln E_t \\ & + 0.007516 (\ln K_t)^2 + 0.076086 (\ln L_t)^2 + 0.001052 (\ln E_t)^2 \\ & + 0.012340 (\ln K_t \ln L_t) + 0.007475 (\ln K_t \ln E_t) \\ & + 0.000447 (\ln L_t \ln E_t) \end{aligned}$$

Figure 2. Actual GDP and Estimated GDP according to Ridge Regression.



The actual and the estimated GDP values obtained by using Ridge regression equation is shown in Fig.2. The output elasticities for each input are calculated and shown in Table 5.

Table 5. Output elasticities for each input

Year	f_K	f_L	f_E
1994	0.51713	1.89821	0.08939
1995	0.52015	1.90034	0.09074
1996	0.52277	1.90213	0.09187
1997	0.52414	1.90317	0.09250
1998	0.52593	1.90469	0.09339
1999	0.52690	1.90534	0.09379
2000	0.52875	1.90662	0.09460
2001	0.53096	1.90820	0.09559
2002	0.53382	1.91026	0.09688
2003	0.53864	1.91358	0.09901
2004	0.54342	1.91683	0.10111
2005	0.54770	1.91989	0.10304
2006	0.55167	1.92273	0.10484
2007	0.55562	1.92563	0.10666
2008	0.55981	1.92867	0.10858
2009	0.56418	1.93201	0.11064
2010	0.56637	1.93354	0.11162
2011	0.57013	1.93631	0.11336
2012	0.57325	1.93868	0.11482
2013	0.57622	1.94093	0.11622
2014	0.57927	1.94289	0.11751
Average	0.54556	1.91860	0.10220

The output elasticities of variables of capital, labor and energy are given by years. We see that the output elasticity change with the time period t . These values

represent that the output variables value fluctuation according to the sensitivity of input variables. Elasticities of substitution for the various inputs are calculated and shown and given in Table 6.

The elasticity of substitution between the pairs of capital, labor and energy is positive. It means that these variables can be substitute. Technological development increases economic growth. Technological progress is described by the output mines the contribution of capital, labor and energy. We can calculate the contribution rate of technological progress to output growth as,

$$\alpha_A(t) = 1 - \alpha_K(t) - \alpha_L(t) - \alpha_E(t)$$

where,

$$\alpha_K(t) = f_K(t) \frac{K_t}{K_t} / \frac{Q_t}{Q_t}, \quad \alpha_L(t) = f_L(t) \frac{L_t}{L_t} / \frac{Q_t}{Q_t}, \quad \alpha_E(t) = f_E(t) \frac{E_t}{E_t} / \frac{Q_t}{Q_t}$$

In the investigated time period, the contribution rate of technological progress to total GDP growth is on average 3.52%.

Table 6. Substitution elasticity of input factors

Year	σ_{KL}	σ_{KE}	σ_{LE}
1994	0.99396	0.99677	1.01225
1995	0.99389	0.99665	1.16627
1996	0.99382	0.99656	1.09318
1997	0.99379	0.99651	1.00486
1998	0.99375	0.99643	0.99975
1999	0.99373	0.99640	0.99975
2000	0.99368	0.99634	0.99975
2001	0.99363	0.99627	0.99975
2002	0.99356	0.99618	0.99975
2003	0.99344	0.99603	0.99975
2004	0.99332	0.99590	0.99975
2005	0.99322	0.99578	0.99975
2006	0.99313	0.99568	0.99975
2007	0.99303	0.99558	0.99975
2008	0.99293	0.99548	0.99975
2009	0.99283	0.99537	0.99975
2010	0.99278	0.99532	0.99975
2011	0.99269	0.99524	0.99975
2012	0.99262	0.99518	0.99975
2013	0.99255	0.99511	0.99975
2014	0.99247	0.99507	0.99976
Average	0.99328	0.99590	1.01297

Figure 3. Estimated Capital according to Holt Winter's Method

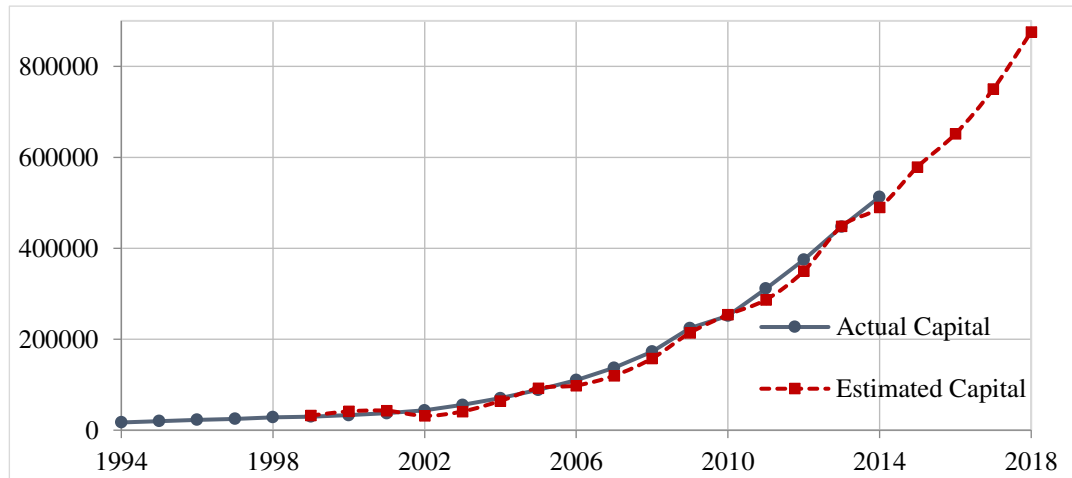


Figure 4. Estimated Labor according to Holt Winter's Method

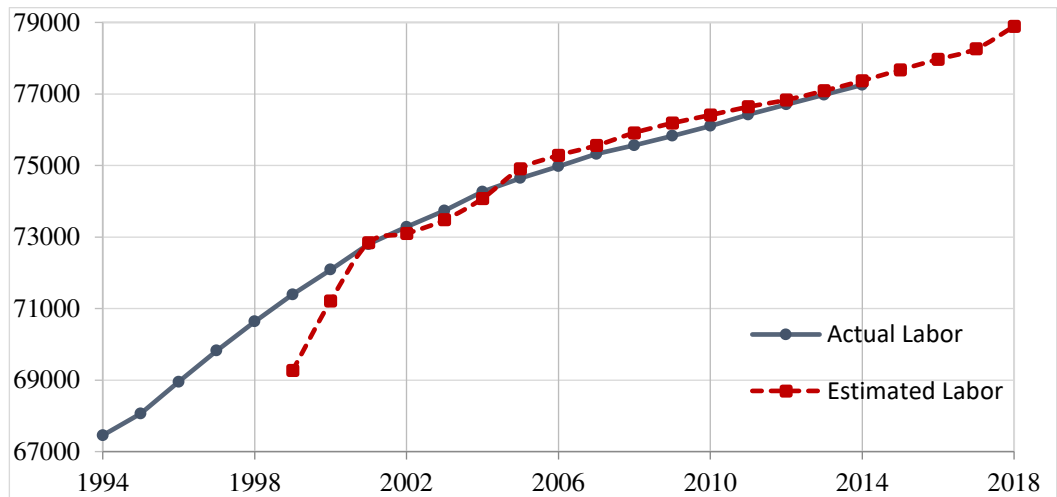


Figure 5. Estimated Energy according to Holt Winter's Method

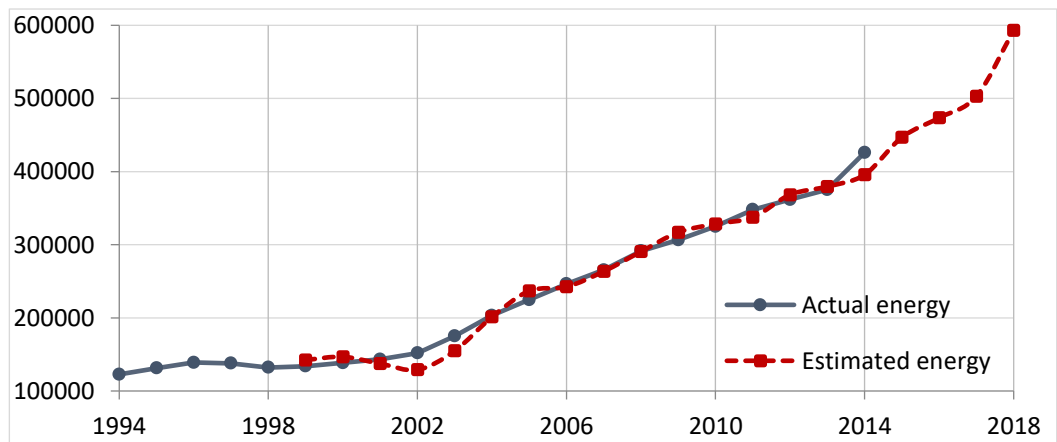
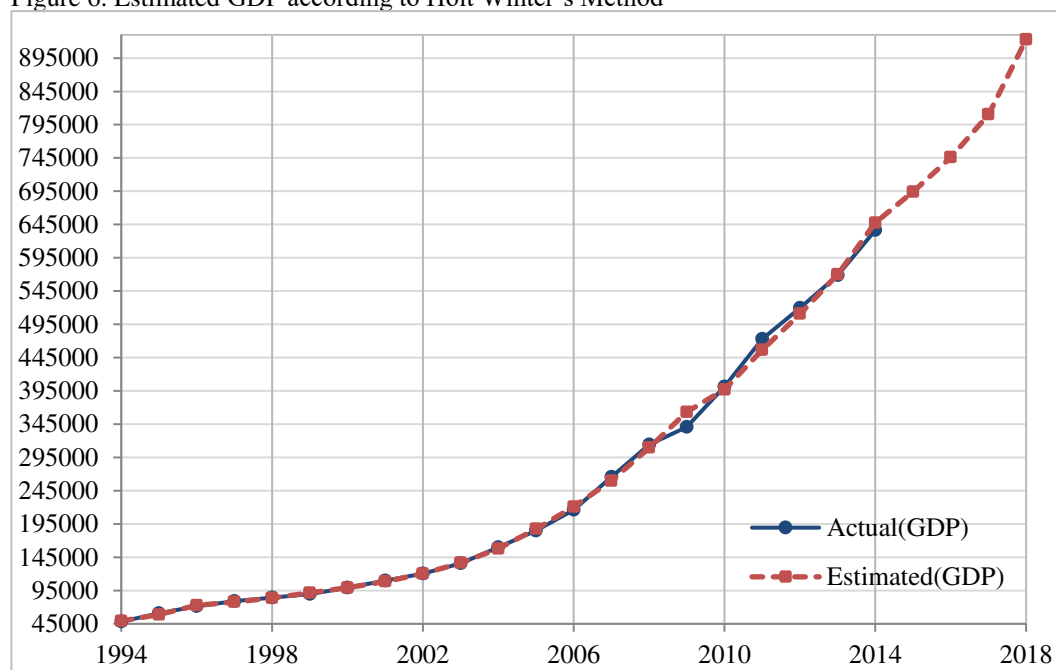


Figure 6. Estimated GDP according to Holt Winter's Method



The study showed that the triple exponential smoothing method can be an appropriate approach for level estimation of capital, labor and energy inputs. The estimated values of GDP using Translog production function with capital, labor and energy inputs and estimated GDP values by IMF are given in Table 7.

Table 7. Comparison with the estimated GDP by IMF and estimated GDP

Year	Estimated GDP (billion yuan)	Estimated GDP by IMF (billion yuan)	Actual GDP (billion yuan)
2012	510619.59	534123	519470.1
2013	570123.45	588018.8	568845
2014	570123.45	636139	636462.7
2015	694147.84	692391	
2016	745874.42	742082.3	
2017	810181.87	795409.2	
2018	922848.73	856923.6	

4.3.Error Analysis

Error Analysis is needed for examining the precision of forecasted results. The Mean Absolute Percentage Error (MAPE) is one of the most widely used methods which is evaluation of forecasting error. The MAPE is calculated as,

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|Q_t - \hat{Q}_t|}{Q_t}.$$

Where Q_t is the actual value. \hat{Q}_t is also forecasting value at time t. n is the number of periods forecasted. MAPE reference values for forecasting accuracy are shown in Table 8 (Lewis, 1982).

Table 8. MAPE reference values for forecasting accuracy

MAPE	Forecasting Accuracy
Less than 10%	Highly accurate
11% to 20%	Good forecast
21% to 50%	Reasonable forecast
51% or more	Inaccurate forecast

Table 9. MAPE values for Translog and CES production functions

	MAPE	Forecasting Accuracy
Translog production function	3.39%	Highly Accurate
CES production function	3.31%	Highly accurate

5. Conclusion

In this paper, CES production function model and Translog production function model are implemented to estimate the future values of the GDP which is an indicator of China economic growth with capital, labor and energy input factors.

Firstly, we implemented the CES production model with capital and labor input factors to observed data. The findings of CES model reveals that the elasticity of substitution between capital - labor bundle is $\sigma = 1.085$ and MAPE value is 0.03423. The CES model with capital and labor inputs gives a reliable forecast of GDP, the nested CES production model can give more reliable results for large data set. Because we have a small number of data, we did not use the nested CES model in the real application.

Secondly, we implemented the quadratic Translog production function with capital, labor and energy inputs to the dataset. In the empirical analysis, it is founded

the evidence of multicollinearity between variables. To eliminate the multicollinearity, it is used to the Ridge regression method.

The Translog function is more convenient than CES function because it has a very flexible structure. The output elasticities of input factors which are changing by time are calculated by using the fitted model. Empirical findings show that Ridge regression parameters are consistent with actual economic situation and Translog production function is a good explanatory model for China GDP values.

The output elasticity of each input factor and the elasticity of substitution between these input factors are analyzed. The output elasticities of capital, labor and energy are indicated that all of the input factors have a significant positive impact on China economy in the studied period. The output elasticity of labor is slightly larger than the capital and energy elasticities. This also shows that labor input has a leading role in promoting economic growth. The fluctuation range of output elasticities of capital and labor is stable by years, however, the output elasticity of energy is gradually increasing by years.

The contribution rate of technological progress to total GDP growth is found on average 3.52% in the investigated time period.

Finally, in general, the total production of the China economy highly depends on labor and capital inputs, besides the effect of energy input is relatively small.

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Hülya Başığmez and Ömer Önalın worked together in all stages of the article and shared the responsibilities equally in the analysis and writing stage. We take responsibility for our views. All authors have read and approved the final manuscript.

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Availability of data and materials

The data sources of all data analyzed during this study are introduced in the first part of “Empirical Analysis” section.

Competing interests

I declare no potential competing interests.

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Figures

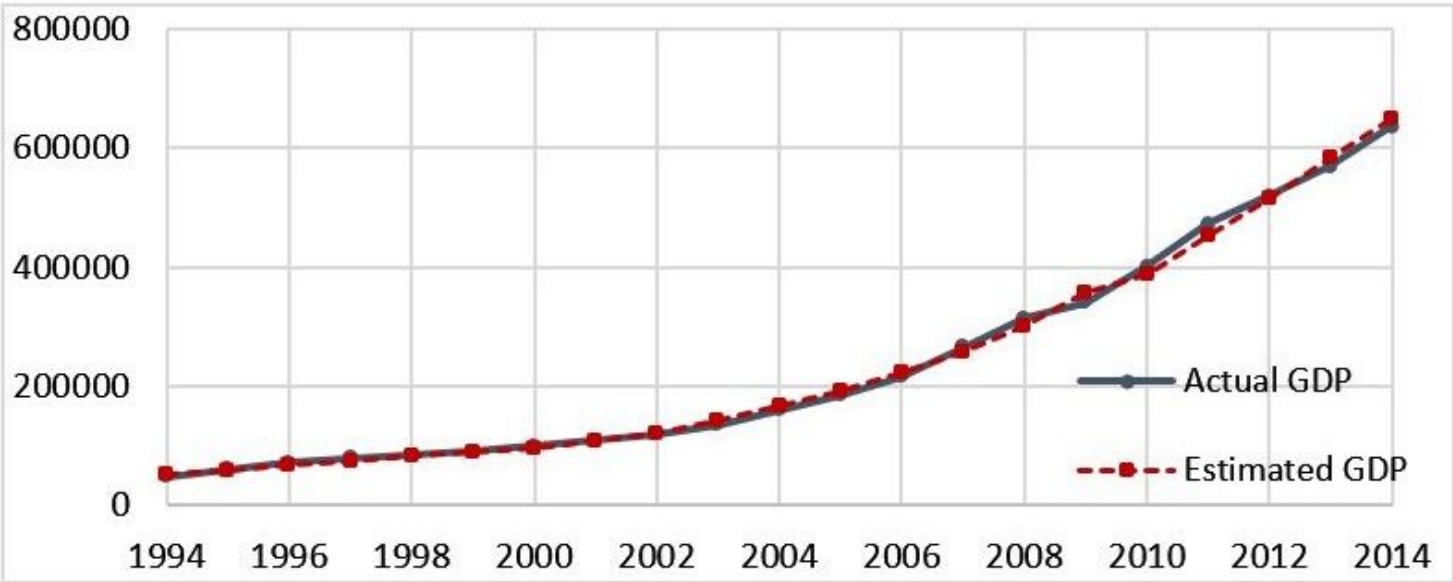


Figure 1

Actual GDP and Estimated GDP obtained from Kmenta Approach

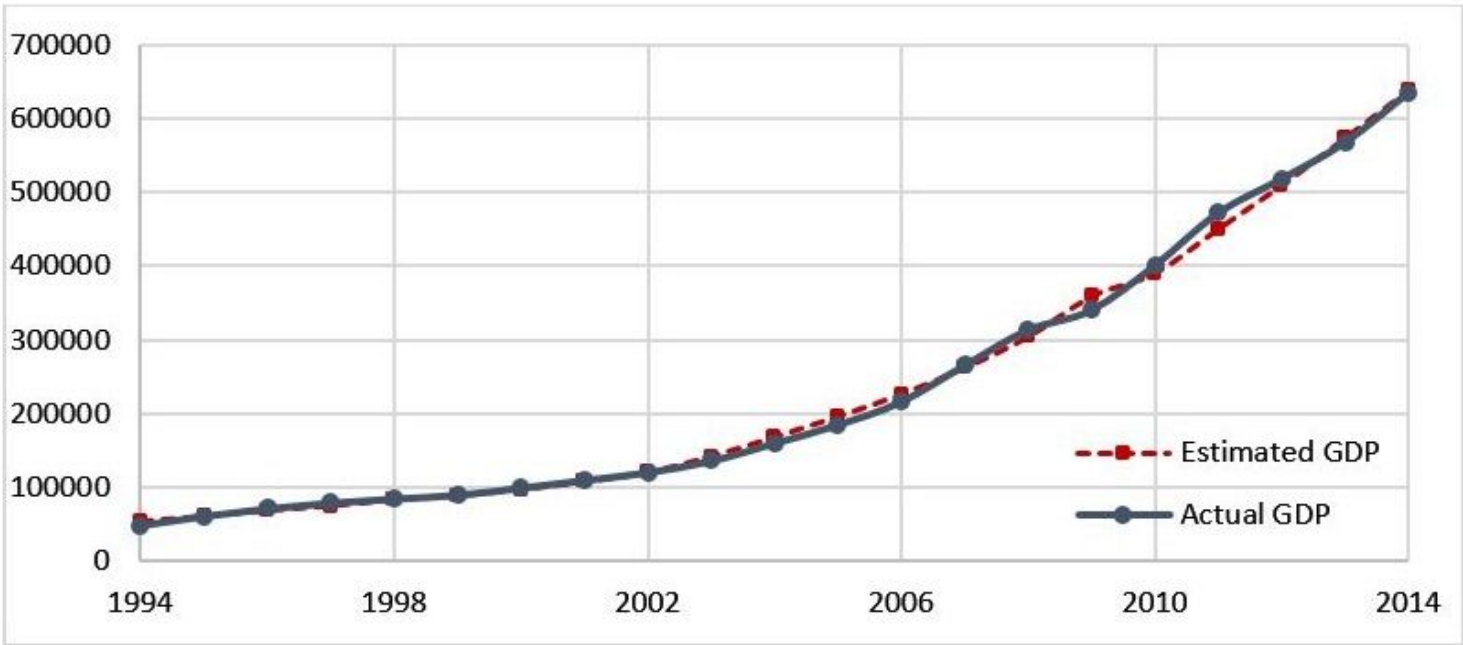


Figure 2

Actual GDP and Estimated GDP according to Ridge Regression.

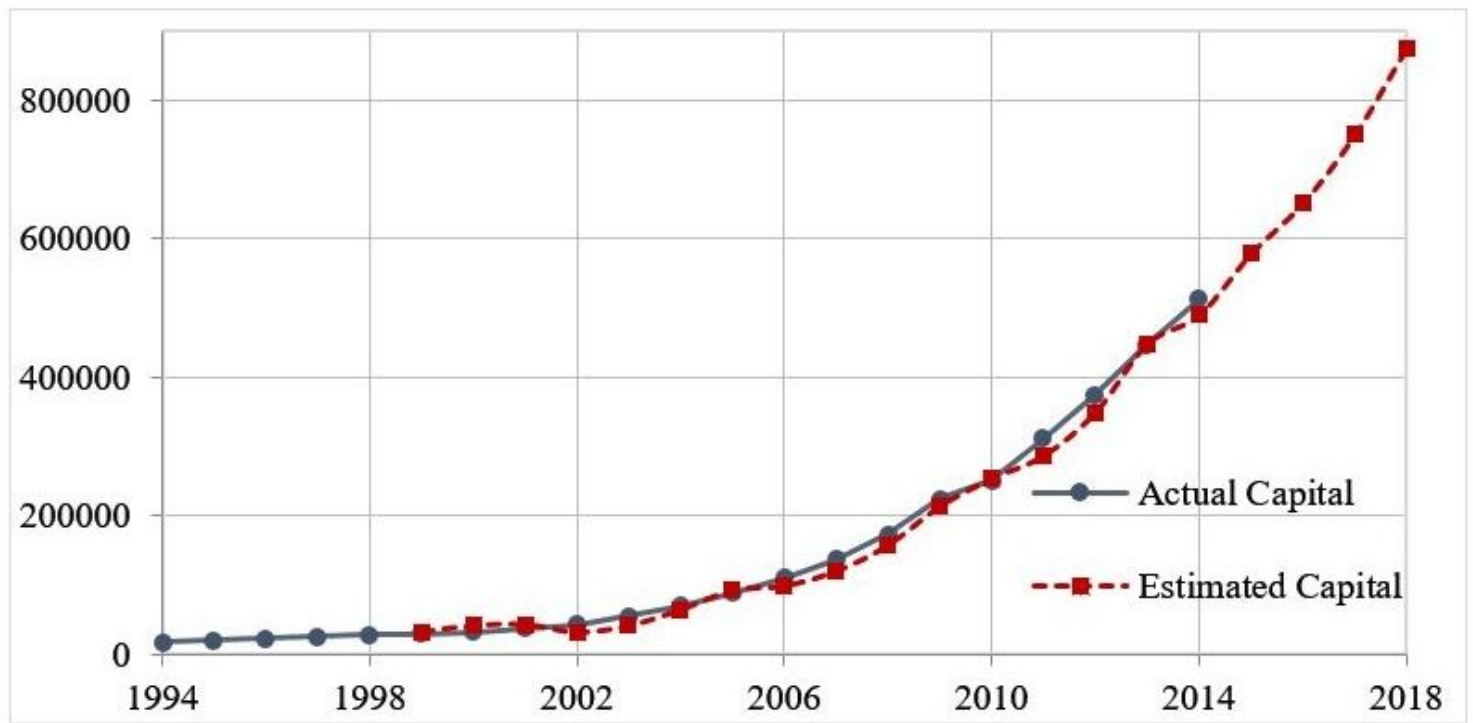


Figure 3

Estimated Capital according to Holt Winter's Method

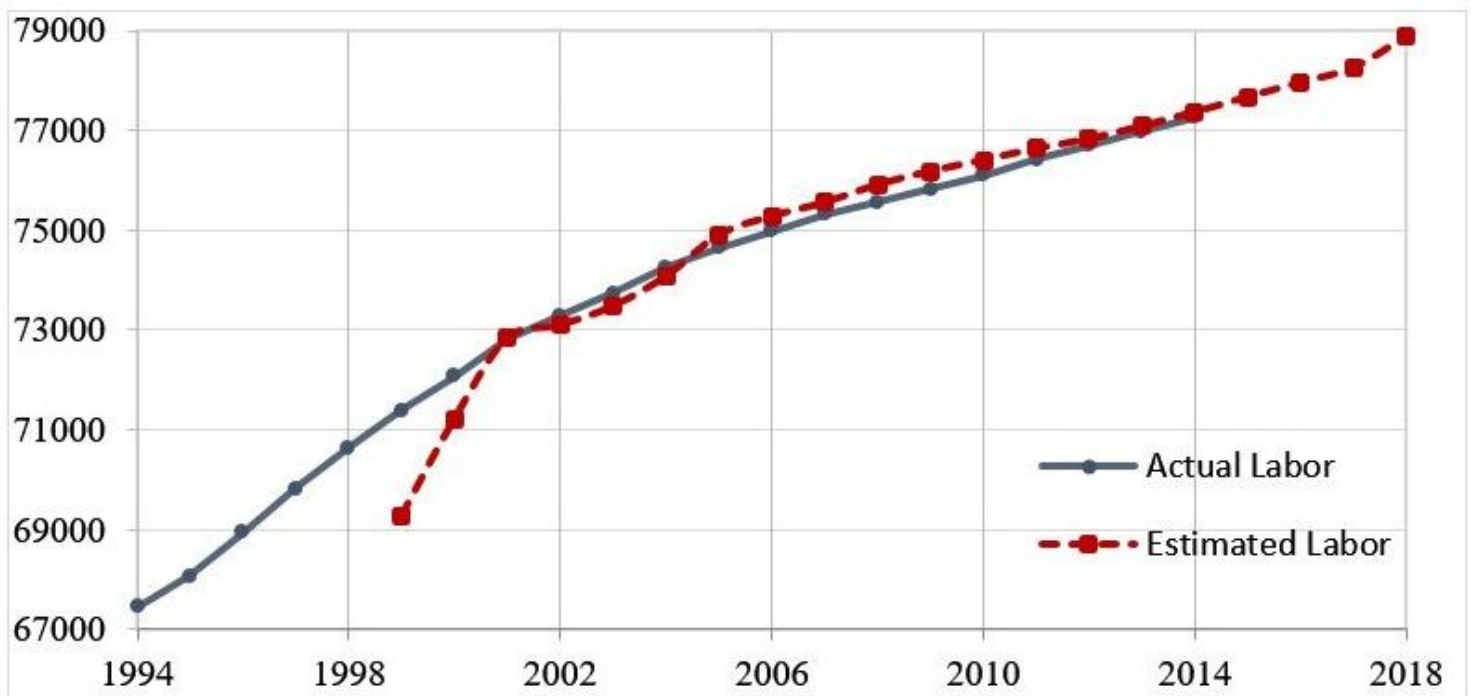


Figure 4

Estimated Labor according to Holt Winter's Method

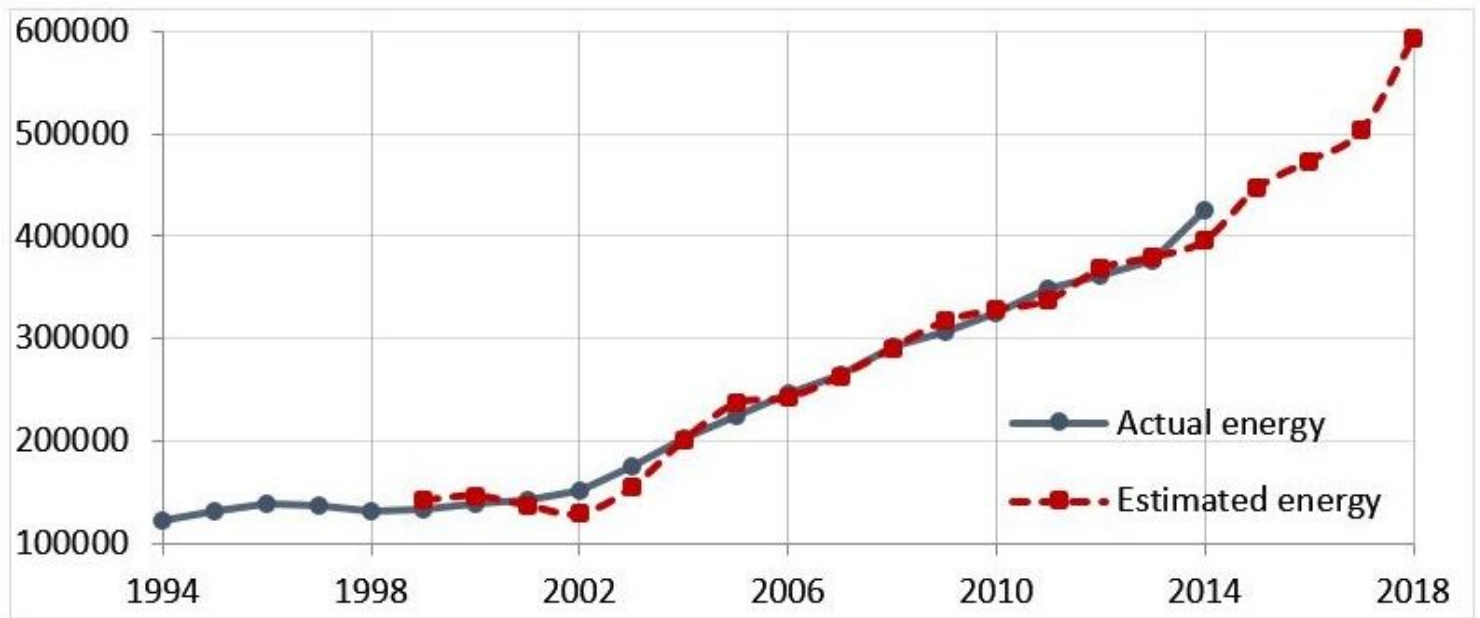


Figure 5

Estimated Energy according to Holt Winter's Method

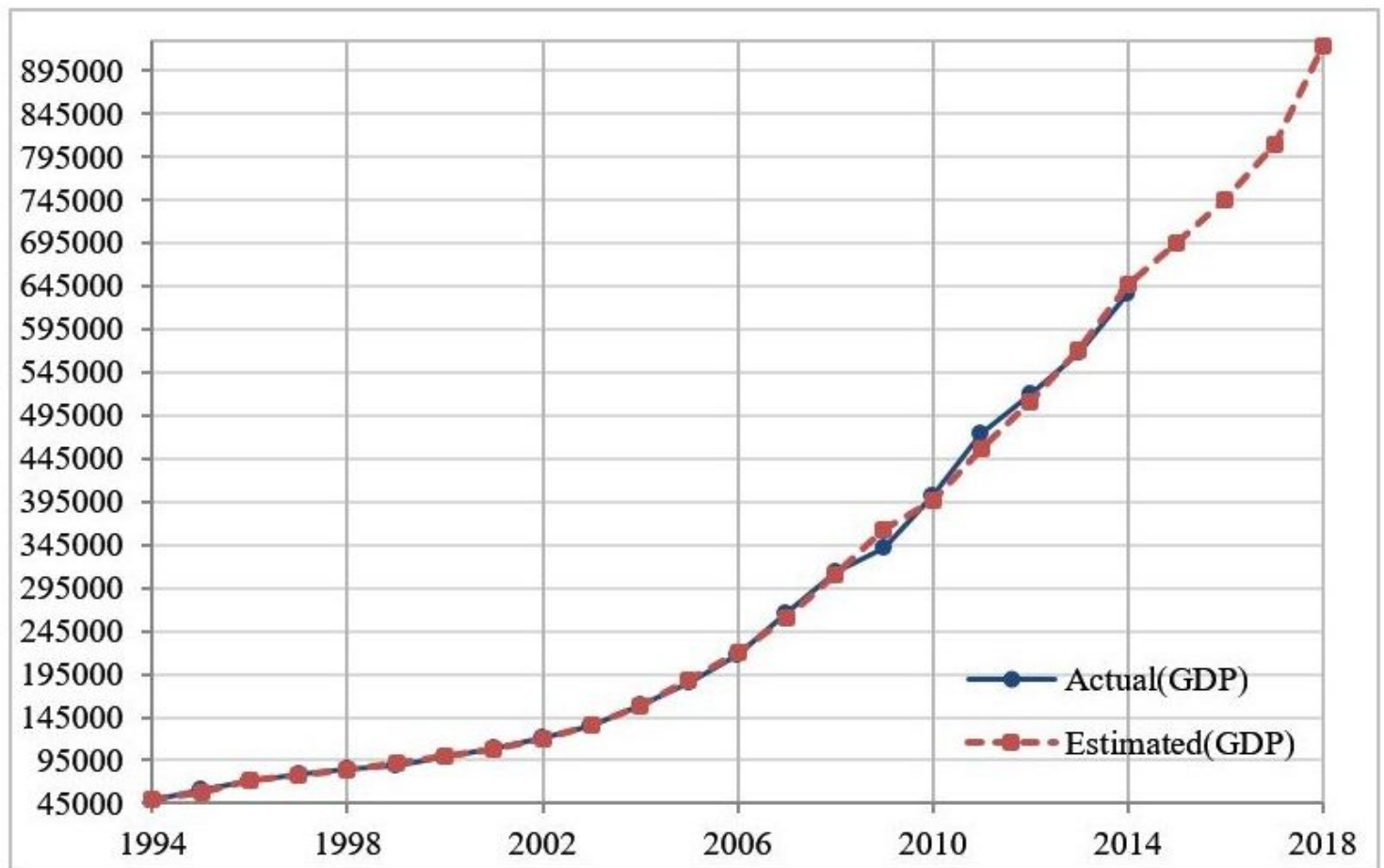


Figure 6

Estimated GDP according to Holt Winter's Method

Supplementary Files

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