

1 **Doubly robust estimator of risk in the presence of censoring dependent on time-**  
2 **varying covariates: application to a primary prevention trial for coronary events**  
3 **with pravastatin**

4

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19 **Abstract**

20 **Background:** In the presence of dependent censoring even after stratification of  
21 baseline covariates, the Kaplan–Meier estimator provides an inconsistent estimate of  
22 risk. To account for dependent censoring, time-varying covariates can be used along  
23 with two statistical methods: the inverse probability of censoring weighted (IPCW)  
24 Kaplan–Meier estimator and the parametric g-formula estimator. The consistency of the  
25 IPCW Kaplan–Meier estimator depends on the correctness of the model specification of  
26 censoring hazard, whereas that of the parametric g-formula estimator depends on the  
27 correctness of the models for event hazard and time-varying covariates.

28 **Methods:** We combined the IPCW Kaplan–Meier estimator and the parametric g-  
29 formula estimator into a doubly robust estimator that can adjust for dependent  
30 censoring. The estimator is theoretically more robust to model misspecification than the  
31 IPCW Kaplan–Meier estimator and the parametric g-formula estimator. We conducted  
32 simulation studies with a time-varying covariate that affected both time-to-event and  
33 censoring under correct and incorrect models for censoring, event, and time-varying  
34 covariates. We applied our proposed estimator to a large clinical trial data with  
35 censoring before the end of follow-up.

36 **Results:** Simulation studies demonstrated that our proposed estimator is doubly robust,

37 namely it is consistent if either the model for the IPCW Kaplan–Meier estimator or the  
38 models for the parametric g-formula estimator, but not necessarily both, is correctly  
39 specified. Simulation studies and data application demonstrated that our estimator can  
40 be more efficient than the IPCW Kaplan–Meier estimator.

41 **Conclusions:** The proposed estimator is useful for estimation of risk if censoring is  
42 affected by time-varying risk factors.

43

#### 44 **Keywords**

45 Double robustness; dependent censoring; prediction; time-varying covariate

46

#### 47 **Background**

48 Establishment of the long-term effectiveness of primary prevention treatments often  
49 requires large randomized controlled trials (RCTs) over a long time period. In such  
50 RCTs, survival functions and risks between randomized groups are compared using the  
51 Kaplan–Meier estimator because censoring before the end of the follow-up cannot be  
52 avoided. This approach assumes independent censoring, such that censoring occurs  
53 randomly in each treatment group. The standardization approach can provide a  
54 consistent estimate of risks in each group even if censoring is not unconditionally

55 independent, but the conditionally independence of potential survival time after  
56 stratification of treatment groups and baseline covariates [1–4]. In this paper, we call  
57 this type of censoring as baseline-conditional independent censoring.

58 Even a baseline-conditional independent censoring assumption can be dubious.

59 Our motivating study is the Management of Elevated Cholesterol in the Primary  
60 Prevention Group of Adult Japanese (MEGA) study, which is a large primary  
61 prevention RCT for coronary heart disease (CHD) using pravastatin, where censoring  
62 before the end of follow-up occurred in about 10% of patients [5]. Patients enrolled in  
63 the MEGA study had hypercholesterolemia (total cholesterol [TC] level: 220–270  
64 mg/dl), ~~and~~ were 40–70 years old, and received daily clinical care during the follow-up  
65 period. When a patient with hypercholesterolemia received a medical checkup and  
66 found that their plasma lipids were worsening (e.g., increasing TC), they may have  
67 required other drugs that were not allowed in the study protocol. Patients who observed  
68 worsening of their symptoms might go to see a doctor other than their primary care  
69 doctor. These cases may have led to censoring dependent on mid-course clinical  
70 characteristics, and the censoring was correlated with future CHD events. ~~Because they~~  
71 ~~received daily clinical care, censoring can be dependent on mid-course clinical~~  
72 ~~characteristics that were predictive of CHD prognosis.~~ If censoring is dependent on

73 potential survival time even after stratification of treatment groups and baseline  
74 covariates, the Kaplan–Meier estimator provides inconsistent estimates of survival  
75 function [6]. In such a situation, one possibility to mitigate the dependency is to use  
76 time-varying covariates measured during the follow-up period.

77           The inverse probability of censoring weighted (IPCW) Kaplan–Meier  
78 estimator is a semiparametric method for estimation of risk that adjusts for censoring  
79 that may depend on the observed past, ~~but not on the future prognosis~~ [7]. It requires  
80 fitting a model for the probability of censoring at each time conditional on past  
81 covariates. Calculation of the IPCW Kaplan–Meier estimate needs to update censoring  
82 probability at each time and to weight each subject in the risk set. The weight depends  
83 on the time-varying covariates, but not on the future prognosis. The drawback of the  
84 IPCW estimator is that it can be statistically inefficient [8].

85           An alternative to IPCW methods is a g-formula-based estimator, which can be  
86 estimated using two different principles. First representation of the g-formula is an  
87 iterated conditional expectation, and targeted maximum likelihood estimation can be  
88 applied, which was first introduced by Bang and Robins [9]. Their method uses the  
89 weight of the IPCW method and regression models for the outcome process. It can  
90 produce doubly robust estimates, meaning that the estimator is consistent if either the

91 regression model for the hazard of censoring or a regression model for the outcome  
92 process is correctly specified, but necessarily both [10–12]. However, only a few  
93 researchers have applied this method. One of the reasons may be that they are  
94 unintuitive because it requires recursive regression models for an iterated conditional  
95 expectation; first, we regressed the outcomes measured at  $t = K$  on the covariates  
96 measured up to  $t = K - 1$ , second, we regressed the predicted outcome on the covariates  
97 measured up to  $t = K - 2$ , and we continue these procedures until  $t = 1$ . The second  
98 representation of the g-formula is the generalized version of standardization [1, 2], and  
99 the parametric g-formula estimator (g-computation algorithm formula) is an alternative  
100 method can be applied. The parametric g-formula estimator that also requires models  
101 for the outcome and covariate process [13]. It can be regarded as a sequential, non-  
102 recursive imputation-based methodology [14, 15], so it is intuitive for applied  
103 researchers. It is flexible because it can easily compare dynamic treatment regimens  
104 [16]. However, it requires a specification of full-model likelihood, and robustness  
105 regarding model correctness can be a concern. A doubly robust estimator for the  
106 parametric g-formula estimator, involving the time-varying covariates, has not been  
107 proposed.

108 In this paper, we propose an extension of the parametric g-formula estimator

109 that is more robust at modeling misspecification. The key idea is to combine the IPCW  
110 estimator and the parametric g-formula estimator into doubly robust estimators [9, 17–  
111 19] while incorporating time-varying covariates to adjust for dependent censoring.

112 The paper is organized as follows. In the next section, we briefly describe the  
113 MEGA study and introduce notations and assumptions. We also describe our proposed  
114 estimator, and we give settings and the results of simulation studies. Finally, the  
115 proposed estimator is applied to the MEGA study data.

116

#### 117 **Data, notations, and assumptions**

118 The MEGA study is a prospective, randomized, open-label, blinded-endpoint-designed  
119 controlled trial conducted in Japan to evaluate the primary preventive effect of  
120 pravastatin against CHD in daily clinical practice. A total of 7832 men and  
121 postmenopausal women aged 40–70 years with hypercholesterolemia and no history of  
122 CHD or stroke were randomized to dietary therapy only (diet group) or dietary therapy  
123 plus 10–20 mg daily pravastatin (diet plus pravastatin group) between February 1994  
124 and March 1999.

125

126

127 Table 1. Type and number of events within 5 years in the MEGA study

	Diet group		Diet + pravastatin group	
	<i>n</i>	%	<i>n</i>	%
CHD event	85	2.1	57	1.5
Follow-up completed	3498	88.2	3353	86.7
Refusal of follow-up	259	6.5	364	9.4
Death by causes other than CHD	60	1.5	42	1.1
Loss to follow-up	64	1.6	50	1.3
Total	3966	100.0	3866	100.0

128

129

After randomization, laboratory tests were conducted at months 1, 3, and 6, and

130

annually thereafter. The follow-up period was initially scheduled for 5 years. Table 1

131

shows the types and number of events within 5 years. Although there were three reasons

132

for censoring during the study period (refusal of follow-up, death by causes other than

133

CHD, and loss to follow-up), we collectively treated them as censoring before the end

134

of the follow-up period.

135

Let  $t = 1, \dots, T$  denote month of follow-up where  $T + 1 = 60$  months is the

136

follow-up of interest. There were 7832 patients at baseline, and observations of patients

137 were assumed to be independently identically distributed.  $R$  denotes the treatment  
138 assigned ( $R = 1$  for assignment to the diet plus pravastatin group, and  $R = 0$  for  
139 assignment to the diet group).  $C_t$  and  $Y_t$  denote the indicator of censoring and  
140 occurrence of a CHD event by time  $t$ , respectively, with  $C_0 = Y_0 = 0$  by definition.  $L_t$   
141 denotes time-varying covariates measured at time  $t$ , and  $V$  denotes baseline covariates  
142 that are time-independent (e.g. sex, current smoker). We assumed that baseline  
143 covariates  $V$  and  $L_0$  are always observed. We denoted the history of a variable using  
144 overbars. For example,  $\bar{L}_t = (L_0, \dots, L_t)$  is the covariate history through time  $t$ . We  
145 assumed the order  $(C_t, Y_t, L_t)$  within each interval  $(t - 1, t)$ ; therefore,  $Y_t$  and following  
146 variables are missing if  $C_t = 1$ . We defined  $C_{T+1} = 1$  if  $C_T = Y_T = 0$  (follow-up  
147 completed).

148 We wanted to estimate the marginal event-free survival in each treatment group  
149 if any censoring was absent in the study population. However, measured-observed data  
150 contains censoring, as in the MEGA study. The usual the Kaplan–Meier estimator  
151 assumes independent censoring, that is, the hazard of  $Y_t$  among subjects at risk is the  
152 marginal hazard of  $Y_t$  given the treatment group. The standardization approach, or a g-  
153 formula that adjusts for baseline covariates, assumes baseline-conditional independent  
154 censoring, that is, the hazard of  $Y_t$  among subjects at risk is the conditional hazard of  $Y_t$

155 given the treatment group and baseline covariates [1–4].

156 Even when these two assumptions are attainable, estimators discussed in the  
157 next section provide a consistent estimate of the marginal survival in each treatment  
158 group if any censoring was absent in the study population. These estimators assume  
159 positivity (equation 1) and are conditionally independent of censoring (equation 2);

$$160 \Pr\left(C_t = 0 \mid \bar{C}_{t-1} = 0, \bar{Y}_{t-1} = 0, R, V, \bar{L}_{t-1}\right) > 0 \text{ for } t=1, 2, \dots, T \text{ (1)}$$

$$161 \Pr\left(C_t = 0 \mid \bar{C}_{t-1} = 0, \bar{Y}_T = 0, R, V, \bar{L}_{t-1}\right) = \Pr\left(C_t = 0 \mid \bar{C}_{t-1} = 0, \bar{Y}_{t-1} = 0, R, V, \bar{L}_{t-1}\right)$$

162 for  $t = 1, 2, \dots, T$  (2).

163 The conditional independence of censoring assumption, equation (2), states that  
164 for  $t = 1, 2, \dots, T$ , the variables  $(Y_t, \dots, Y_T)$  is independent of  $C_t$ , in other words, the  
165 distribution of  $(Y_t, \dots, Y_T)$  is the same between  $C_t = 1$  and  $C_t = 0$  among subjects who  
166 had a similar history of the covariates. The conditionally independence of ~~the~~ censoring  
167 assumption is also referred to as no unmeasured confounders for the censoring  
168 assumption [7], which states that conditional on the treatment groups, baseline  
169 covariates, and the time-varying covariates measured until time  $t - 1$ , the hazard of  
170 censoring at time  $t$  does not further depend on ~~possibly unobserved CHD events after~~  
171 ~~time~~ no unmeasured confounders for censoring and unobserved CHD. In the next  
172 section, we describe the existing estimators and our proposed estimator for the hazard of

173  $Y_t$ .

174

## 175 **Existing estimators and proposed estimator**

176 Due to randomization, baseline factors are balanced between treatment groups. In this  
177 section, we focus on the diet plus pravastatin group ( $R = 1$ ) and suppress  $R$  for  
178 notational simplicity. A similar argument holds for the diet group ( $R = 0$ ).

179

### 180 *Estimators of hazard of $Y_1$*

181 At time  $t = 1$ , the observed data is  $n$  copies of  $(V, L_0, C_1, (1 - C_1)Y_1)$ . We show three  
182 types of estimators for  $\Pr(Y_1 = 1)$ ; the IPCW estimator, the parametric g-formula  
183 estimator, and the doubly robust estimator.

184 To obtain the IPCW estimate, we need to fit a model for  $C_1$  such as the logistic  
185 model  $\Pr(C_1 = 0|V, L_0; \alpha) = e(V, L_0; \alpha) = \{1 + \exp(-\alpha_0 - \alpha_1 V - \alpha_2 L_0)\}^{-1}$ . After fitting  
186 the model, the IPCW estimator for  $\Pr(Y_1 = 1)$  is expressed as

187  $n^{-1} \sum_i (1 - C_{i1})Y_{i1}/e(V_i, L_{i0}; \hat{\alpha})$ . The consistency of the IPCW estimator relies on the

188 correct specification of  $e(V, L_0; \alpha)$ . If no censoring is observed at  $t = 1$ , we set

189  $e(V_i, L_{i0}; \hat{\alpha}) = 1$ ; therefore, the IPCW estimator equals the empirical risk.

190 To obtain the parametric g-formula estimate, we need to fit a model for  $Y_1$  such

191 as the logistic model  $\Pr(Y_1 = 1|C_1 = 0, V, L_0; \beta) = p(V, L_0; \beta) = \{1 + \exp(-\beta_0 - \beta_1 V -$   
 192  $\beta_2 L_0)\}^{-1}$ . After fitting the model ~~in~~ for the subjects not censored at  $t = 1$  (subjects with  
 193  $C_1 = 0$ ), the parametric g-formula estimator for  $\Pr(Y_1 = 1)$  is expressed as  
 194  $n^{-1} \sum_i p(V_i, L_{i0}; \hat{\beta})$ . The consistency of the parametric g-formula estimator relies on the  
 195 correct specification of  $p(V, L_0; \beta)$ .

196 To obtain the doubly robust estimate, we need to fit a model for  $C_1$  and  $Y_1$   
 197 similarly as conducted for the IPCW estimator and the parametric g-formula estimator,  
 198 respectively. After fitting the models  $e(V, L_0; \alpha)$  and  $p(V, L_0; \beta)$ , the doubly robust  
 199 estimator for  $\Pr(Y_1 = 1)$  is expressed as

$$200 \quad n^{-1} \sum_i \left[ \frac{(1-C_{i1})Y_{i1}}{e(V_i, L_{i0}; \hat{\alpha})} - \frac{(1-C_{i1}) - e(V_i, L_{i0}; \hat{\alpha})}{e(V_i, L_{i0}; \hat{\alpha})} p(V_i, L_{i0}; \hat{\beta}) \right] \quad (3).$$

201 The contributions of censored patients or patients with an event are different; for censored  
 202 patients, their contribution is  $p(V_i, L_{i0}; \hat{\beta})$  like the g-formula estimator, and for patients  
 203 with an event, their contribution is  $Y_{i1}/e(V_i, L_{i0}; \hat{\alpha}) -$   
 204  $\{1 - e(V_i, L_{i0}; \hat{\alpha})\}p(V_i, L_{i0}; \hat{\beta})/e(V_i, L_{i0}; \hat{\alpha})$ . The **doubly** robust estimator is consistent if  
 205 either the model  $e(V, L_0; \alpha)$  or  $p(V, L_0; \beta)$  is correctly specified [9, 17–19]. Intuitively,  
 206 when the model for censoring is correctly specified, the term  $(1 - C_{i1}) - e(V_i, L_{i1}; \hat{\alpha})$   
 207 should be zero, so (3) reduces to the IPCW estimator and is, therefore, consistent. Inside  
 208 the summation can be expressed as  $(1 - C_{i1})\{Y_{i1} - p(V_i, L_{i0}; \hat{\beta})\}/e(V_i, L_{i0}; \hat{\alpha}) +$

209  $p(V_i, L_{i0}; \hat{\beta})$ , and when the model for an event is correctly specified, the term  $Y_{i1} -$   
 210  $p(V_i, L_{i0}; \hat{\beta})$  should be zero, so (3) reduces to the g-formula estimator and is, therefore,  
 211 consistent. Our proposed estimator utilizes this doubly robust estimator for the hazard of  
 212  $Y_1$ . In the next subsection, we show how to extend it to estimate the hazard of  $Y_t (t > 1)$   
 213 incorporating time-varying covariates.

214 We noted that with one categorical baseline covariate and no parametric model  
 215 is needed for outcomes and censoring, it can be shown that the IPCW estimator, the g-  
 216 formula estimator, and the doubly robust estimator are equivalent. Specifically, given  $n$   
 217 subjects, all of whom may be stratified into  $j$  levels of a baseline covariate, such that  $a_j,$   
 218  $m_j,$  and  $n_j$  are the number of observed (i.e. not censored), number of events, and overall  
 219 number at level  $j$  of the covariate, respectively. The IPCW estimator can be written as  
 220  $(1/n) \sum_j m_j / (a_j / n_j) = (1/n) \sum_j n_j m_j / a_j$ , because  $\Pr(C_1 = 0 | \text{level } j) = a_j / n_j$ . The g-formula  
 221 estimator can be written as  $(1/n) \sum_j n_j (m_j / a_j)$ , because  $\Pr(Y_1 = 1 | \text{level } j) = m_j / a_j$ . Finally,  
 222 the doubly robust estimator can be written as  $(1/n) \sum_j [m_j / (a_j / n_j) - \{(n_j - a_j) (0 - a_j / n_j) /$   
 223  $(a_j / n_j) + a_j (1 - a_j / n_j) / (a_j / n_j)\} (m_j / a_j)] = (1/n) \sum_j n_j m_j / a_j$ , which is exactly a common  
 224 form of the IPCW estimator and the g-formula estimator.

225

226 *Estimators of hazard of  $Y_t$  ( $t > 1$ )*

227 In this subsection, we show the estimators of the hazard of  $Y_t$  ( $t > 1$ ), which are  
 228 extended versions of the IPCW estimator and the parametric g-formula estimators for  
 229  $\Pr(Y_1 = 1)$ . Finally, we propose a doubly robust estimator that extends equation (3).

230 To obtain the IPCW Kaplan–Meier estimate, we need to fit a model for  $C_t$  such  
 231 as the pooled logistic model,

$$232 \quad \text{logit } \Pr\left(C_t = 0 \mid \bar{C}_{t-1} = 0, \bar{Y}_{t-1} = 0, V, \bar{L}_{t-1}\right) = \alpha_{0t} + \alpha_1 V + \alpha_2 L_{t-1} \quad (4).$$

233 In the model, it is possible to include  $L_0, \dots, L_{t-2}$ , but in some cases, it may cause  
 234 multicollinearity due to the correlation between  $L_0, \dots, L_{t-1}$ . After fitting the model  
 235 using the maximum likelihood estimation, the IPCW Kaplan–Meier estimator for the  
 236 hazard of  $Y_t$  is expressed as  $\hat{\Pr}\left(Y_t = 1 \mid \bar{Y}_{t-1} = 0\right) = \sum_i Y_{it} \pi_{it}(\hat{\alpha}) / \sum_i X_{it} \pi_{it}(\hat{\alpha})$ , where

237  $\pi_t(\hat{\alpha})$  is obtained as

$$238 \quad \pi_t(\hat{\alpha}) = \prod_{j=1}^t \Pr\left(C_{tj} = 0 \mid \bar{C}_{tj-1} = 0, \bar{Y}_{tj-1} = 0, V, \bar{L}_{tj-1}; \hat{\alpha}\right),$$

239 and  $X_t$  is the at-risk indicator, which is 1 if the patient is at-risk at time  $t$  and is 0

240 otherwise. Finally, the risk at  $t$  can be obtained as  $1 - \prod_{j=\theta_1}^t \left\{ 1 -$

241  $\hat{\Pr}\left(Y_j = 1 \mid \bar{Y}_{j-1} = 0\right)$ . The consistency of the IPCW Kaplan–Meier estimator relies on

242 the correct specification of the model for  $C_t$  (equation 4) [7]. Note that the IPCW

243 Kaplan–Meier estimator reduces to the usual Kaplan–Meier estimator when  $\alpha_1$  and  $\alpha_2$

244 of equation (4) are 0, that is, the independent censoring assumption is true [20].

245 To obtain the parametric g-formula estimate, we need to fit a model for  $Y_t$ .

246 Unlike baseline covariates, time-varying covariates will not be measured for patients

247 who were censored before time  $t$ . Thus, we need to specify the full-model likelihood

248 (likelihood for conditional event probability and time-varying covariates) by fitting

249 models for  $Y_t$  and  $L_t$  such as

250 
$$\text{logit Pr}\left(Y_t = 1 \mid \bar{C}_t = 0, \bar{Y}_{t-1} = 0, V, \bar{L}_{t-1}\right) = \beta_{0t} + \beta_1 V + \beta_2 L_{t-1} \quad (5), \text{ and}$$

251 
$$E\left(L_t \mid \bar{C}_t = 0, \bar{Y}_{t-1} = 0, V, \bar{L}_{t-1}\right) = \gamma_{0t} + \gamma_1 V + \gamma_2 L_{t-1} \quad (6).$$

252 After fitting the models using the maximum likelihood estimation, we sequentially

253 imputed the conditional probability of CHD event and time-varying covariates from  $t =$

254 1 to  $T$ . The parametric g-formula estimator for the hazard of the risk at  $t$  can be obtained

255 as  $Y_t$  is expressed as  $n^{-1} \sum_{j=1}^t \sum_i m_{j,i}(\hat{\beta}, \hat{\gamma})$ , where  $m_t(\hat{\beta}, \hat{\gamma})$  is obtained as  $m_t(\hat{\beta}, \hat{\gamma}) =$

256 
$$\Pr\left(Y_t = 1 \mid \bar{Y}_{t-1} = 0, V, \bar{L}_{t-1}; \hat{\beta}, \hat{\gamma}\right) \prod_{j=1}^{t-1} \left\{ 1 - \Pr\left(Y_j = \theta 1 \mid \bar{Y}_{j-1} = 0, V, \bar{L}_{j-1}; \hat{\beta}, \hat{\gamma}\right) \right\}.$$

257 The consistency of the parametric g-formula estimator relies on the correct specification

258 of the model for  $Y_t$  (equation 5) and the model for  $L_t$  (equation 6) [16, 21, 22].

259 We propose an estimator of the hazard of  $Y_t$  that extends the doubly robust

260 estimator (equation 3). To obtain the estimate, we need to fit models for  $C_t$ ,  $Y_t$ , and  $L_t$  as

261 conducted for the IPCW Kaplan–Meier estimator (equation 4) and the parametric g-

262 formula estimator (equations 5 and 6). After fitting these models, the proposed doubly  
 263 robust estimator for the hazard of  $Y_t$  is expressed as,

$$\begin{aligned}
 264 \quad & \widehat{\Pr} \Pr \left( Y_t = 1 \mid \bar{Y}_{t-1} = 0 \right) = (\sum Z_t)^{-1} \sum_i \left[ \frac{(1-C_{it})Y_{it}}{\pi_{it}(\hat{\alpha})} - \right. \\
 265 \quad & \left. \frac{(1-C_{it})-\pi_{it}(\hat{\alpha})}{\pi_{it}(\hat{\alpha})} \Pr \left( Y_{i,t} = 1 \mid \bar{Y}_{i,t-1} = 0, V_i, \bar{L}_{i,t-1}; \hat{\beta}, \hat{\gamma} \right) \right] \frac{(1-C_{it})Y_{it}}{\pi_{it}(\hat{\alpha})} - \\
 266 \quad & \frac{(1-C_{it})-\pi_{it}(\hat{\alpha})}{\pi_{it}(\hat{\alpha})} m_{\bar{Y}}(\hat{\beta}, \hat{\gamma}) \quad (7).
 \end{aligned}$$

267  
 268 where  $Z_t$  is the at-risk or censored indicator, which is 1 if the patient is at-risk or  
 269 censored at time  $t$  and is 0 otherwise. The contributions of patients censored patients-by  
 270  $t$  or patients with an event at  $t$  are different; for censored patients, their contribution is  
 271  $\Pr \left( Y_t = 1 \mid \bar{Y}_{t-1} = 0, V, \bar{L}_{t-1}; \hat{\beta}, \hat{\gamma} \right)$ . For patients with an event, their contribution of an  
 272 event is weighted by the inverse of probability uncensored until  $t$ . Finally, the risk at  $t$  is  
 273 obtained as  $1 - \prod_{j=0}^t \left\{ 1 - \widehat{\Pr} \left( Y_j = 1 \mid \bar{Y}_{j-1} = 0 \right) \right\}$ . The weights and predicted event  
 274 probabilities are similar as to the ones used in the IPCW Kaplan–Meier estimator and the  
 275 parametric g-formula estimator, but we need to calculate the function (7) and risk at  $t$ .

276 As demonstrated in the Additional file (Appendix A), this estimator is consistent if  
 277 either the model for  $C_t$  (equation 4) or models for  $Y_t$  and  $L_t$  (equation 5 and 6) is  
 278 correctly specified. In the IPCW Kaplan–Meier estimator, patients with  $C_t = 1$  were out  
 279 of the risk set; therefore, they do not contribute to the estimation of the hazard of  $Y_t$ . On

280 the other hand, patients with  $C_t = 1$  contribute to the estimation of the hazard of  $Y_t$  in  
281 equation (7), because the function inside the summation reduces to  $m_t(\hat{\beta}, \hat{\gamma})$ , which  
282 might lead to statistical efficiency. The variance estimate of the proposed estimator can  
283 be obtained through a nonparametric bootstrap [23]. We have provided SAS code for  
284 the proposed estimator in an additional file 2-(Appendix B).

285

### 286 *Comparison with existing doubly robust estimators*

287 In this subsection, we briefly compare our proposed estimator (7) with existing doubly  
288 robust estimators [9, 24, 25]. Zhang et al. [24] and Bai et al. [25] proposed doubly  
289 robust estimators for survival functions, which can be summarized as follows:

290 *Confounding between treatment groups: present due to the observational study*  
291 setting

292 *Censoring mechanism: baseline-conditional independent censoring (censoring*  
293 may depend only on the baseline covariates)

294 On the other hand, we proposed an estimator for survival functions,

295 *Confounding between treatment groups: absent due to randomization*

296 *Censoring mechanism: conditional independent censoring (censoring may*  
297 depend on time-varying covariates)

298 In RCT settings considered here, where no baseline confounding occurs between the  
299 treatment groups, the proposed estimator that specifies an empty set as  $L_t$  (thus models  
300 are unnecessary for the joint density of  $L_t$ ) results in the existing doubly robust  
301 estimators provided in [24, 25]. In other words, these existing estimators assume a  
302 baseline-conditional independent censoring mechanism, although they also attempt to  
303 adjust for baseline-confounding between the groups in observational-study settings.

304 Bang et al. [9] proposed a doubly robust estimator for the g-formula  
305 represented by an iterated conditional expectation. The estimator needs recursive fitting  
306 of the iterative conditional expectation. However, as Bang et al. [9] noted, the  
307 parametric models can be incompatible with each other, so it is difficult to specify all  
308 the models correctly.

309

310

## 311 **Simulation study**

### 312 *Simulation designs*

313 To evaluate the performance of the proposed estimator, we carried out simulation  
314 studies with dependent censoring due to a time-varying covariate. We simulated data  
315 from two treatment groups, coded as  $R = 0$  (control treatment) and  $R = 1$  (test

316 treatment). The simulations were based on 1000 replications. We considered the  
 317 situation where baseline covariates were measured at time  $t = 0$ , and time-varying  
 318 covariate and censoring were investigated at time  $t = 1, \dots, 4-2$ , on the other hand, event  
 319 time was measured from time  $t = 0$  to  $t = 3-5$  on a continuous time scale. We were  
 320 interested in the treatment group-specific risks and the risk ratio at  $t = 3$  and  $t = 5$ .

321 For each patient  $i$  ( $= 1, \dots, 1000$ ), a baseline covariate  $V$  was generated from  
 322 the Bernoulli distribution of success probability 0.5. Independently, the time-varying  
 323 covariate at  $t$  ( $= 0, 1, 2$ ) was generated from the following mixed effect model,

$$324 \quad L_{it} = 2 - 0.1(1 - R_i)t - 0.51-1R_it + b_{i0} + b_{i1}t + \epsilon_{it}.$$

325 Random variables  $(b_{i0}, b_{i1})$  were generated from a bivariate normal distribution with  
 326 means of 0 and variance of 1.0 and 0.5, respectively, with a covariance of 0.5. The  
 327 random error  $\epsilon_{it}$  was generated from the standard normal distribution. Distributions of  
 328  $L_t$  were the same in both treatment groups at  $t = 0$  but declined more steeply in the test  
 329 treatment group such that  $L_t$  mimicked TC in the MEGA study.

330 First, we generated a time to event  $T_i$  from the piecewise exponential model,  
 331 whose hazard function was,

$$\lambda(t|V, \bar{L}_t, R) = \exp(-5 + 1.5V + 1.2U_t + 1.2(1 - R)) \begin{cases} \exp(-6 + 1.5V + 1.2U_0 + 1.2(1 - R)), & 0 < t \leq 1 \\ \exp(-5 + 1.5V + 1.2U_1 + 1.2(1 - R)), & 1 < t \leq 2 \\ \exp(-4 + 1.5V + 1.2U_2 + 1.2(1 - R)), & 2 < t \end{cases}$$

where  $U_t = 1$  if  $L_t < 0$ , otherwise  $U_t = 0$ . Therefore, potential event time was shorter in the control treatment group through the effect of group and time-varying covariate.

Next, we generated censoring  $C_t$  at  $t (= 1, \dots, 4)$  from the Bernoulli distribution, whose probability was generated using the following logistic model,

$$\text{logit Pr}(C_t = 1 | \bar{C}_{t-1} = 0, T > t, V, \bar{L}_{t-1}, R) = \alpha_0 - 4 + t + 1.5V + 1.2U_{t-1}(t-1) + 1.2\alpha_R(1 - R).$$

Similarly to the event model, censoring occurred more frequently in the control treatment group through group effect and the effect of the time-varying covariate.  $C_+ = C_2 \bar{C}_{4-} = 0$  and  $T_i > 3$  indicates that the follow-up was completed. The direct dependence between the event and the censoring time is shown in additional file 1 (Appendix B).

We considered three scenarios for  $\alpha_0$  and  $\alpha_R$ : censoring probabilities in the control and test treatment groups are both 30% (scenario 1), both 20% (scenario 2), and 9% and 12%, respectively (scenario 3). The probabilities in scenario 3 were derived from Table 1.

In this setting, the observed event probabilities, censoring probabilities, and

349 ~~complete follow-up probabilities were approximately 20%, 60%, and 20% for the control~~  
350 ~~treatment group, and 15%, 40%, 45% for the test treatment group.~~  
351 We ~~used~~ created 20,000,000 simulated patients without censoring to calculate the true  
352 value of survival probability using their empirical distribution. To understand the  
353 performance of estimators, we considered eight situations: all combinations of correct or  
354 incorrect censoring models, event models, and covariate models. We defined correct  
355 models for censoring, event, and covariate as a model that specified the same covariates  
356 with the data-generating model. We defined incorrect models for censoring and event as  
357 a model that specified by replacing  $U_t$  by  $\exp(L_t)$  without incorporating  $V$ . An incorrect  
358 covariate model was specified without incorporating the interaction term of  $b_{i1}$  and  $t$ .

359 Simulations were evaluated in terms of the bias (mean difference between  
360 estimated and true parameter value) and relative efficiency (the ratio of the Monte Carlo  
361 standard deviation of the IPCW Kaplan–Meier estimator to that of the estimator) of the  
362 estimated survival probabilities at time  $t = 3$  and  $t = 5$ .

363

## 364 Results

365

366 *Simulation results*

367 We present our simulation results in Table 2. In Table 2, if the bias exceeded half of the  
368 standard error of the estimates, the printed bias was is shown in bold. In scenario 1, the  
369 bias for each group at  $t = 5$  was seen for the IPCW Kaplan–Meier estimator when the  
370 censoring model is incorrect, for the parametric g-formula estimator when one of the  
371 event model or covariate model is incorrect. However, our proposed estimator is unbiased  
372 when at least one of the censoring model or event model is correctly specified. This result  
373 reflected the double robustness of our proposed estimator; when the censoring model or  
374 set of event and covariate models are correct, the estimate is unbiased. Unexpectedly, our  
375 proposed estimator is less biased than the parametric g-formula estimator, even when the  
376 covariate model was incorrect. We consider that this property is only in this simulation  
377 because if the covariate model is incorrect, the estimated event probability is also  
378 incorrect for true probability. At  $t = 3$ , the parametric g-formula estimator showed less  
379 bias for the test treatment group even when the event model is incorrect. Regarding the  
380 bias, similar results can be seen in the other two scenarios.

381 Regarding the relative efficiency using the IPCW Kaplan–Meier estimator as the  
382 reference, both the parametric g-formula estimator and our proposed estimator were more  
383 efficient at  $t = 3$  As expected, the IPCW Kaplan–Meier estimator was biased when the

384 ~~censoring model was incorrectly specified, and the parametric g formula estimator was~~  
385 ~~also biased when the event and/or covariate models were incorrectly specified. On the~~  
386 ~~other hand, the proposed estimator was unbiased when the censoring model was correctly~~  
387 ~~specified, or the event and covariate models were correctly specified. However, it was~~  
388 ~~biased in the situation where both the IPCW Kaplan–Meier and parametric g formula~~  
389 ~~estimators were biased. than the reference in scenarios 1 and 2.The parametric g-formula~~  
390 ~~estimator was more efficient than the reference even at  $t = 5$ ,<sup>h</sup>. However, our proposed~~  
391 ~~estimator had a similar standard error as to the reference. In scenario 3, where the~~  
392 ~~censoring probability <sup>i</sup>was the lowest among the scenarios, our proposed estimator had a~~  
393 ~~similar standard error as the reference at both  $t = 3$  and 5. The coverage probability of the~~  
394 ~~proposed estimator using the bootstrap method with the correctly specified models was~~  
395 ~~close to the nominal level of 95 %. In summary, the efficiency recovery of our proposed~~  
396 ~~estimator may be affected by the censoring probabilities (comparing between the~~  
397 ~~scenarios) and the number of time points (comparing  $t = 3$  and  $t = 5$ ). When the censoring~~  
398 ~~probability is high but the number of time points is less than five, our proposed estimator~~  
399 ~~might be more efficient than the IPCW Kaplan–Meier estimator.When comparing Monte~~  
400 ~~Carlo standard deviations among these estimators, the parametric g formula estimator~~  
401 ~~was the most efficient. Our proposed estimator showed efficiency recovery from the~~

402 ~~IPCW Kaplan Meier estimator. The efficiency recovery was greater when estimating the~~  
403 ~~risk in the control treatment group.~~

404 [Table 2 is placed at the end of the document]

405

### 406 *Data applications*

407 Our proposed estimator was applied to the MEGA study data to estimate treatment  
408 group-specific risks at 5 years after randomization. As baseline covariates, we included  
409 age (years), gender, body mass index, history of hypertension and diabetes,  
410 hypercholesterolemia medication history, current smoking, current alcohol drinking,  
411 triglyceride, high-density lipoprotein cholesterol, and low-density lipoprotein  
412 cholesterol. As the time-varying covariate, we used recent TC.

413 After transforming our data into one record per person-time, we estimated the  
414 survival curve using our proposed estimator. First, we fitted the models for censoring  $C_t$ ,  
415 event  $Y_t$ , and covariate  $L_t$ . We fitted pooled logistic models for  $C_t$  and  $Y_t$ , where the  
416 time-varying intercept was included as a restricted cubic spline with 4 knots at 1–4  
417 years after randomization. We fitted a linear model for  $L_t$ . By fitting the pooled logistic  
418 model for  $Y_t$ , classical risk factors for CHD (age, male, hypertension, and diabetes) were  
419 found to be the prognostic factors (Additional file, Appendix C). By fitting the model

420 for  $C_i$ , those without hypertension, diabetes, or no history of medication for  
421 hyperlipidemia, tended to be censored before the end of the follow-up period  
422 (Additional file, Appendix D). Unexpectedly, time-varying TC hardly affected the event  
423 or censoring after adjusted for those important baseline covariates (Additional file 1,  
424 Appendix C and D); therefore, baseline-conditional independence assumption rather  
425 than conditional independence assumption might be plausible in the MEGA study.

426 We estimated the risk of CHD incidence at 5 years from randomization using  
427 the Kaplan–Meier estimator, IPCW Kaplan–Meier estimator, the parametric g-formula  
428 estimator, and our proposed estimator. The results are shown in Table 3. In the MEGA  
429 study dataset, the risk of CHD estimated using the usual Kaplan–Meier estimator and  
430 the risk estimated by other estimators were very similar. This may be due to the small  
431 impact of dependent censoring in the MEGA study and correctness of model  
432 specification for censoring and events. Because the ordinal Kaplan–Meier estimator  
433 showed similar results as the other three estimators that adjust for the possible  
434 dependent censoring, the impact of dependent censoring must be very mild. If the  
435 censoring model or event model was mis-specified, the results from the other three  
436 estimators might be more different. Therefore, the results from the three estimators may  
437 indicate that the postulated models were nearly correctly specified. The estimated

438 confidence interval of the parametric g-formula estimator was narrower than the other  
 439 estimators was similar or narrower than that of the IPCW Kaplan–Meier estimator and  
 440 our proposed estimator. The estimated confidence interval for the risk of diet +  
 441 pravastatin group of our proposed estimator was similar or narrower than that of was  
 442 narrower than the IPCW Kaplan–Meier estimator.

443  
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 447  
 448  
 449

450 Table 3. Risk of coronary heart diseases in the MEGA study at 5 years after randomization

Method	Diet group		Diet + pravastatin group		Risk Ratio	95% CI
	Risk (%)	95% CI	Risk (%)	95% CI		
Kaplan–Meier*	2.34	(1.90, 2.89)	1.63	(1.26, 2.11)		
IPCW Kaplan–Meier	2.39	(1.91, 2.95)	1.60	(1.19, 2.10)	0.68	(0.40, 1.06)

Parametric g-formula	2.36	(1.97, 3.01)	1.66	(1.30, 2.05)	0.71	(0.47, 0.98)
Proposed estimator	2.38	(1.91, 2.95)	1.61	(1.22, 2.06)	0.69	(0.42, 1.03)

451 \* The confidence intervals of the Kaplan–Meier estimator was obtained using the  
 452 Greenwood formula.

453

454 **Discussion**

455 In this paper, we proposed a doubly robust estimator of risk that adjusts for dependent  
 456 censoring due to time-varying covariates in RCT settings. The novelty of our proposed  
 457 estimator is as an extension of the existing estimator [9, 19] for more complex data with  
 458  $t > 1$  and time-varying covariates. The IPCW Kaplan–Meier estimator is routinely used  
 459 in the analysis of RCTs for the purpose of adjusting for dependent censoring with time-  
 460 varying covariates measured throughout the follow-up period. ~~The novelty of the~~  
 461 ~~proposed estimator is that we extended a well-known doubly robust estimator that is~~  
 462 ~~able to adjust dependent censoring due to only baseline covariates [24, 25] to one that is~~  
 463 ~~able to adjust dependent censoring due to time-varying covariates, by combining the~~  
 464 ~~IPCW Kaplan–Meier estimator and the parametric g-formula estimator.~~ We have also  
 465 provided SAS codes in Appendix B and an example of simulation data with additional  
 466 files, which can be easily implemented. The important property of our proposed

467 estimator is the double protection against model misspecification. Because risk factors  
468 for the endpoints are often identified before the beginning of the RCT, by measuring  
469 them longitudinally at as many time points as possible and by using them when  
470 constructing the models, we are in a better position to approximate the true regression  
471 function. able to approximate the true regression function.

472 The second property of our proposed estimator is the efficiency recovery over  
473 the IPCW Kaplan–Meier estimator, as shown in the simulation study. The degree of  
474 efficiency recovery could depend on either the censoring probability, event probability,  
475 the dependency of variables, or all of these factors combined. As studied previously [8],  
476 we considered that the censoring probability is an important factor. Further studies will  
477 involve understanding the factors that affect the degree of efficiency recovery using  
478 further simulations. In the simulation study and analysis of the MEGA study, the  
479 parametric g-formula estimator outperformed regarding efficiency. This phenomenon  
480 was expected because the asymptotic variance of the classical doubly robust estimator is  
481 no smaller than that of the g-formula estimator [26].

482 Our estimator relies on the assumption that censoring and event time are  
483 independent conditional on observed covariates including time-varying ones. However,  
484 in a situation that censoring and event time are not independent even if we condition on

485 time-varying covariates, our proposed estimator and other existing estimators cannot  
486 correct for selection bias. We also need the assumption of correct model specification.  
487 We need to incorporate the covariates that affect both event and censoring probabilities,  
488 and moreover, we need to specify the model form that approximates the true regression  
489 function.

490 In this study, we considered the estimation of a survival function in a specific  
491 group. If we compare two or more survival functions that may be observed with  
492 different interventions, we also need an additional exchangeability assumption (or the  
493 no-unmeasured confounders assumption) between the intervention groups [27]. In the  
494 simulations and data analysis, the exchangeability assumption is satisfied at baseline  
495 owing to the randomized design. In a future study, it will be interesting to extend our  
496 estimator into the observational study setting [24, 25].

497 All the estimators in this study can be applied to right-censored data. We  
498 consider that our proposed estimator cannot be applied to the data with interval or left-  
499 censored data in its current form. With those censoring, we know that an event has  
500 occurred only before a specific time. In this situation, how to predict event probability  
501 and how to weight uncensored subjects are not obvious. Note that the MEGA study  
502 corrects exact event time, so we consider that interval censoring or left censoring is

503 absent in the real data.

504 There are several reasons for censoring in the MEGA study, as shown in Table  
505 1. We treated refusal of follow-up, death by causes other than CHD, and loss to follow-  
506 up as reasons for censoring in the censoring model. Three estimators, including our  
507 proposed estimator, assessed the hypothetical survival function when there was no  
508 censoring. It may be meaningful to consider whether a survival function can be obtained  
509 if refusal to follow-up and loss to follow-up did not occur. When we separately  
510 accounted for the two reasons for dropouts, the survival curve was similar to the one  
511 using the Kaplan–Meier method [28]. It may be meaningful to consider whether a  
512 survival function would be obtained if refusal to follow-up and loss to follow-up did not  
513 occur. However, death by causes other than CHD needs additional consideration,  
514 because it is difficult to cease such competing risks for CHD without lowering the risk  
515 of CHD. Therefore, if there was no death by causes other than CHD, the survival  
516 function would be slightly lower than we estimated. Because in the MEGA study the  
517 proportion of censoring due to death by causes other than CHD was less than 1.5%, we  
518 believe the estimated survival functions are close to the true survival function, which  
519 would be obtained if these censorings had not occurred.

520 There are two limitations in this study. First, we were not able to verify the

521 assumptions with the measured data. The positivity assumption will be satisfied unless  
522 the conditional probabilities ~~of~~ censoring are zero for all patients at  $t = 1, \dots, T$ . In the  
523 analysis of the MEGA study data, there were no patients who had an estimated  
524 probability of censoring near 1 (data not shown); therefore, we considered that the  
525 positivity assumption is acceptable. Conditional independence assumption implies that  
526 the treatment group, measured baseline, and time-varying covariates can completely  
527 explain censoring. However, given a rich collection of measured prognostic factors, the  
528 conditional independence assumption can be approximated. Several clinically important  
529 prognostic factors were measured in the MEGA study, and we used all of the baseline  
530 covariates and a time-varying covariate, TC. We considered time-varying TC was  
531 important for event and censoring probability, but the hazard ratio was close to 1;  
532 therefore, the impact of dependent censoring was very mild. In the future, we need to  
533 apply our estimator to data with censoring dependent on time-varying factors. The  
534 second limitation was the range of the simulation study. Because we were interested in  
535 the statistical properties of the estimators with fitted correct/incorrect models, the  
536 behavior of the estimators when other assumptions, such as positivity, were violated is  
537 unknown. We need further simulation studies to understand the performance of the  
538 estimators.

539

540 **Conclusions**

541 The proposed estimator is useful for the estimation of risk if censoring affected by time-  
542 varying risk factors occurred because of the doubly robust property and statistical  
543 efficiency over the IPCW Kaplan–Meier method.

544

545 **List of abbreviations**

546 CHD, coronary heart disease; CI, confidence interval; IPCW, inverse probability-of-  
547 censoring weighted; MEGA, Management of Elevated Cholesterol in the Primary  
548 Prevention Group of Adult Japanese; RCT randomized controlled trial; TC, total  
549 cholesterol.

550

551 **Declarations**

552 *Ethics approval and consent to participate*

553 Not applicable because this paper focuses on the development of statistical methods.

554 Real example data were originally published in Nakamura et al. [5].

555

556 *Consent for publication*

557 Not applicable.

558

559 *Availability of data and materials*

560 The SAS code [and an example of simulated dataset](#) is available in Additional File [24](#).

561

562 *Competing interests*

563 None declared.

564

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567

568 *Authors' contributions*

569 TK, TS, and YM designed the concept of this research. TK conducted the simulation

570 study and analyzed the MEGA study data. TK and TS drafted the manuscript. YM

571 supervised this study and critically reviewed the manuscript. All the authors have read

572 and approved the manuscript.

573

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578

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Table 2. Simulation results

Estimator	Model specification			Bias ( $\times 100$ ) at $t = 3$			Bias ( $\times 100$ ) at $t = 5$ Relative efficiency		
	Censoring	Event	Covariate	Control	Test	Log of Risk ratio	Control	Test	Log of Risk ratio
<i>Scenario 1: 30% censoring in both control and test groups.</i>									
IPCW	Correct	—	—	<u>0.0</u> - <u>0.10</u> - <u>1.1</u>	<u>0.00</u> - <u>0.00</u> - <u>0</u>	<u>-0.2</u> - <u>0.40</u> - <u>0.8</u>	<u>0.11</u> - <u>0.01</u> - <u>1.00</u>	<u>0.01</u> - <u>0.01</u> - <u>1.00</u>	<u>0.41</u> - <u>0.01</u> - <u>1.00</u>
<del>Kaplan-</del> <del>MeierKaplan-</del> Meier	Incorrect			<u>0.51</u> - <u>0.95</u> - <u>1.3</u>	<u>0.41</u> - <u>0.52</u> - <u>0.3</u>	<u>-0.2</u> - <u>0.91</u> - <u>2.2</u>	<u>1.9</u>	<u>1.5</u>	<u>-0.9</u>
Parametric	—	Correct	Correct	<u>0.0</u>	<u>0.0</u>	<u>-0.2</u> (1.23)-	<u>0.0</u>	<u>0.0</u>	<u>-0.3</u>
<del>g-formula</del> g-formula				(1.23) <u>0.00</u> - <u>0</u>	(1.22) <u>0.00</u> - <u>0</u>	<u>0.30</u> - <u>2</u>	(1.10) <u>1.20</u>	(1.08) <u>1.04</u>	(1.09) <u>1.17</u>
		<u>(relative efficiency)</u>							
		Correct	Incorrect	<u>0.11</u> - <u>0.22</u> - <u>1</u>	<u>0.10</u> - <u>0.42</u> - <u>0.7</u>	<u>0.13</u> - <u>0.9</u> - <u>11.6</u>	<u>1.2</u>	<u>0.4</u>	<u>3.9</u>
		Incorrect	Correct	<u>0.52</u> - <u>0.05</u> - <u>2</u>	<u>0.11</u> - <u>0.72</u> - <u>2</u>	<u>3.4</u> - <u>2.2</u> - <u>11.8</u>	<u>2.0</u>	<u>1.7</u>	<u>-2.2</u>

		Incorrect	Incorrect	<u>0.51.95.1</u>	<u>0.11.72.4</u>	<u>3.4-2.49.8</u>	<u>1.9</u>	<u>1.7</u>	<u>-2.4</u>
Proposed	Correct	Correct	Correct	<u>-0.1 (1.04)-</u>	<u>0.0 (1.01)-</u>	<u>-0.3 (1.02)-</u>	<u>-0.3</u>	<u>-0.1</u>	<u>-0.8</u>
				<u>0.30.1</u>	<u>0.10.1</u>	<u>0.8-0.2</u>	<u>(1.00)1.09</u>	<u>(0.99)1.05</u>	<u>(1.01)1.05</u>
<u>doubly robust</u>		<u>(relative efficiency)</u>							
<del>doubly robust</del>		Correct	Incorrect	<u>0.00.20.1</u>	<u>0.00.00.0</u>	<u>-0.10.80.5</u>	<u>0.2</u>	<u>0.0</u>	<u>0.8</u>
		Incorrect	Correct	<u>0.00.00.2</u>	<u>0.00.00.1</u>	<u>-0.2-0.40.4</u>	<u>0.0</u>	<u>0.0</u>	<u>-0.4</u>
		Incorrect	Incorrect	<u>0.00.00.1</u>	<u>0.00.00.1</u>	<u>-0.2-0.40.2</u>	<u>0.0</u>	<u>0.0</u>	<u>-0.4</u>
	Incorrect	Correct	Correct	<u>-0.1-0.3-0.1</u>	<u>0.0-0.10.0</u>	<u>-0.2-0.7-0.5</u>	<u>-0.3</u>	<u>-0.1</u>	<u>-0.7</u>
		Correct	Incorrect	<u>0.00.2-0.4</u>	<u>0.00.00.0</u>	<u>-0.10.9-1.6</u>	<u>0.2</u>	<u>0.0</u>	<u>0.9</u>
		Incorrect	Correct	<u>0.51.91.3</u>	<u>0.41.50.5</u>	<u>-0.2-1.03.6</u>	<u>1.9</u>	<u>1.5</u>	<u>-1.0</u>
		Incorrect	Incorrect	<u>0.51.91.3</u>	<u>0.41.60.4</u>	<u>-0.2-0.94.4</u>	<u>1.9</u>	<u>1.6</u>	<u>-0.9</u>

*Scenario 2: 20% censoring in both control and test groups.*

<u>IPCW</u>	<u>Correct</u>	<u>—</u>	<u>—</u>	<u>0.0-0.1</u>	<u>0.00.0</u>	<u>-0.3-0.3</u>	<u>-0.11.00</u>	<u>0.01.00</u>	<u>-0.31.00</u>
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Kaplan

MeierKaplan - Incorrect

Meier

0.31.3

0.21.0

-0.2-0.5

1.3

1.0

-0.5

Parametric

==

Correct

Correct

0.0

0.0

-0.2 (1.26)-

0.0

0.0

-0.3

(1.25)0.0

(1.24)0.0

0.3

(1.06)1.06

(1.04)1.04

(1.04)1.04

g-formula

(relative efficiency)

g-formula

Correct

Incorrect

0.01.1

0.00.3

0.23.9

1.1

0.3

3.9

Incorrect

Correct

0.11.4

-0.21.2

4.2-1.4

1.4

1.2

-1.4

Incorrect

Incorrect

0.11.3

-0.21.2

4.2-1.6

1.3

1.2

-1.6

Proposed

Correct

Correct

Correct

-0.1 (1.03)-

0.0 (1.03)-

-0.3 (1.05)-

-0.2

-0.1

-0.6

0.2

0.1

0.6

(1.00)1.00

(0.99)0.99

(1.01)1.01

doubly robust

(relative efficiency)

doubly robust

Correct

Incorrect

0.00.1

0.00.0

-0.20.5

0.1

0.0

0.5

Incorrect

Correct

0.00.0

0.00.0

-0.3-0.3

0.0

0.0

-0.3

Incorrect

Incorrect

0.00.0

0.00.0

-0.3-0.3

0.0

0.0

-0.3

	<u>Incorrect</u>	<u>Correct</u>	<u>Correct</u>	<u>-0.1-0.2</u>	<u>0.0-0.1</u>	<u>-0.3-0.4</u>	<u>-0.2</u>	<u>-0.1</u>	<u>-0.4</u>
		<u>Correct</u>	<u>Incorrect</u>	<u>0.00.1</u>	<u>0.00.0</u>	<u>-0.20.5</u>	<u>0.1</u>	<u>0.0</u>	<u>0.5</u>
		<u>Incorrect</u>	<u>Correct</u>	<u><b>0.31.3</b></u>	<u><b>0.21.0</b></u>	<u>-0.2-0.5</u>	<u><b>1.3</b></u>	<u><b>1.0</b></u>	<u>-0.5</u>
		<u>Incorrect</u>	<u>Incorrect</u>	<u><b>0.31.3</b></u>	<u><b>0.21.0</b></u>	<u>-0.2-0.5</u>	<u><b>1.3</b></u>	<u><b>1.0</b></u>	<u>-0.5</u>
<i>Scenario 3: 9% censoring in control group and 12% censoring in test group</i>									
<u>IPCW</u>	<u>Correct</u>	<u>==</u>	<u>==</u>	<u>0.0-0.1</u>	<u>0.00.0</u>	<u>-0.3-0.3</u>	<u>-0.11.00</u>	<u>0.01.00</u>	<u>-0.31.00</u>
<u>Kaplan-</u>									
<u>MeierKaplan-</u>	<u>Incorrect</u>								
<u>Meier</u>				<u>0.10.5</u>	<u>0.10.6</u>	<u>-0.8-1.6</u>	<u><b>0.5</b></u>	<u><b>0.6</b></u>	<u>-1.6</u>
<u>Parametric</u>	<u>==</u>	<u>Correct</u>	<u>Correct</u>	<u>0.0</u>	<u>0.0</u>	<u>-0.2 (1.28)-</u>	<u>0.0</u>	<u>0.0</u>	<u>-0.3</u>
				<u>(1.26)0.0</u>	<u>(1.25)0.0</u>	<u>0.3</u>	<u>(1.03)1.03</u>	<u>(1.02)1.02</u>	<u>(1.02)1.02</u>
<u>g-formula</u>		<u>(relative efficiency)</u>							
<u>g-formula</u>		<u>Correct</u>	<u>Incorrect</u>	<u>0.01.0</u>	<u>0.00.2</u>	<u>0.13.8</u>	<u><b>1.0</b></u>	<u>0.2</u>	<u><b>3.8</b></u>
		<u>Incorrect</u>	<u>Correct</u>	<u><b>-0.30.6</b></u>	<u><b>-0.50.8</b></u>	<u>3.4-2.2</u>	<u><b>0.6</b></u>	<u><b>0.8</b></u>	<u>-2.2</u>
		<u>Incorrect</u>	<u>Incorrect</u>	<u><b>-0.30.6</b></u>	<u><b>-0.50.7</b></u>	<u>3.3-2.4</u>	<u><b>0.6</b></u>	<u><b>0.7</b></u>	<u>-2.4</u>

<u>Proposed</u>	<u>Correct</u>	<u>Correct</u>	<u>Correct</u>	<u>0.0 (1.00)-</u>	<u>0.0 (1.00)-</u>	<u>-0.3 (1.00)-</u>	<u>-0.1</u>	<u>-0.1</u>	<u>-0.4</u>
				<u>0.1</u>	<u>0.1</u>	<u>0.4</u>	<u>(1.00)1.00</u>	<u>(1.00)1.00</u>	<u>(1.00)1.00</u>
<u>doubly robust</u>	<u>(relative efficiency)</u>								
<u>doubly robust</u>	<u>Correct</u>	<u>Incorrect</u>		<u>0.00.0</u>	<u>0.00.0</u>	<u>-0.30.1</u>	<u>0.0</u>	<u>0.0</u>	<u>0.1</u>
	<u>Incorrect</u>	<u>Correct</u>		<u>0.00.0</u>	<u>0.00.0</u>	<u>-0.3-0.3</u>	<u>0.0</u>	<u>0.0</u>	<u>-0.3</u>
	<u>Incorrect</u>	<u>Incorrect</u>		<u>0.00.0</u>	<u>0.00.0</u>	<u>-0.3-0.3</u>	<u>0.0</u>	<u>0.0</u>	<u>-0.3</u>
	<u>Incorrect</u>	<u>Correct</u>	<u>Correct</u>	<u>0.0-0.1</u>	<u>0.0-0.1</u>	<u>-0.30.0</u>	<u>-0.1</u>	<u>-0.1</u>	<u>0.0</u>
	<u>Correct</u>	<u>Incorrect</u>		<u>0.00.0</u>	<u>0.00.0</u>	<u>-0.30.1</u>	<u>0.0</u>	<u>0.0</u>	<u>0.1</u>
	<u>Incorrect</u>	<u>Correct</u>		<u>0.10.5</u>	<u>0.10.6</u>	<u>-0.8-1.6</u>	<u>0.5</u>	<u>0.6</u>	<u>-1.6</u>
	<u>Incorrect</u>	<u>Incorrect</u>		<u>0.10.5</u>	<u>0.10.6</u>	<u>-0.8-1.6</u>	<u>0.5</u>	<u>0.6</u>	<u>-1.6</u>

Numbers in parentheses are the relative efficiency compared with the IPCW Kaplan–Meier estimate with a correctly specified censoring model. If the bias exceeded half of the standard error of the estimates, the printed bias is shown in bold. True values calculated from a large simulated dataset were (0.89, 0.92, 0.69) (at  $t = 3$ ) and (0.810, 0.86, and -0.6774) (at  $t = 5$ ) for control group, test group, and risk ratio, respectively. The biases ( $\times 100$ ) from the method assuming the baseline-conditional independent censoring at  $t = 5$  for the control and test groups were (0.5, 0.4) (scenario 1), (0.4, 0.3) (scenario 2), and (0.2, 0.2) (scenario 3).