Measuring Correlated Qubit Errors on Quantum Computers

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Research Article

Keywords: quantum computing, error correction, crosstalk

Posted Date: November 3rd, 2021

DOI: https://doi.org/10.21203/rs.3.rs-1035320/v1

Abstract

Quantum computing is an emerging technology that, with its exponential computational speedup, will allow us to solve a new level of hard problems. From rapid vaccine development, efficient discovery of exotic materials, and complex AI models, to quickly cracking RSA encryption, quantum computers have the potential to revolutionize science. However, for quantum computing to solve these problems, quantum error correction methods are needed to eliminate qubit errors. An underlying assumption of current quantum error correction is that qubit errors are not correlated. If correlated qubit errors are prevalent, quantum error correction will be difficult, if not impossible. In this work, I developed a method to identify correlated errors between pairs of qubits and used this method to determine if correlations exist on current small-scale quantum computers. My method uses independence of error probabilities $P(q_a) \times P(q_b) = P(q_a q_b)$ to determine correlation. After running 40 million experiments to collect error probabilities by varying the quantum gates, number of gates, and pairs of qubits on multiple quantum computers, I found that these computers have a high occurrence of correlated qubit errors when qubits are topological neighbors. For example, I found that Hadamard gate operations can cause correlated errors, supported by a p-value of 0.0160. My work shows that the assumption that qubit errors are not correlated is incorrect for current quantum computers. In the future, scientists can use these methods to test for correlated errors as they develop new quantum computers, enabling quantum computing to achieve its full potential.

Introduction

Quantum computing, a revolutionary technology that uses the properties of quantum mechanics, such as superposition and entanglement, promises to allow extremely difficult computational problems[1][2][7][9][12][15][25][26][30][31] to be solved in minutes rather than eons. Large companies like IBM, Google, and Microsoft, as well as new tech startups like Rigetti, have poured millions of dollars into the quantum race[6].

Due to their abilities, quantum computers will allow us to calculate at unprecedented speeds. As problems become more complex, the time it takes for classical computers to solve them grows exponentially large (Figure 1). Fortunately, quantum computers can have a linear time growth to solve these problems.

Quantum computers use gates as their “programs”. These gates are analogous to the classical logic gates like “AND”, “OR”, “NOT”. Though quantum computers have a large number of gates, I focus on three in my study: Identity, Hadamard, and Pauli-X. The Identity (ID) gate keeps the state of the qubit the same, acting as a delay. The Hadamard (H) gate puts a qubit in and out of superposition. And finally, the Pauli-X (X) gate flips a qubit from |0> to |1> and |1> to |0>, similar to a boolean “NOT” gate. Below is a quantum circuit, in which you place the gates for the specific qubits on the line. Quantum programs use the circuits as instructions as to which gates apply to which qubits. The wires represent qubits, boxes are gates and the computation runs left to right. By convention, we assume qubit states are initialized to |0>. At the end of every circuit, there is a measurement gate, which measures the state of the qubit as either 0 or 1.

In Figure 2a, qubit *a* has five gates applied. First, the X gate flips the state from |0> to |1>. Then the four H gates put the qubit in and out of superposition, and the qubit ends up back in state |1>.

Figure 2b involves gates acting on qubit *b*. Like qubit *a*, the X at the beginning flips the qubit from |0> to |1>. Then, the four ID gates does not affect the state, leaving the qubit in state |1> until the end.

In both these examples of Figure 2, the qubits should be measured at the end as classical bits with a value of 1. However, if this is not the case, then the quantum computer made an error.

$$ID = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Quantum computing uses the language of linear algebra to model gates, as shown in the matrices above. Each qubit is represented as a vector, in which the matrices of gates can be applied. Quantum states are represented by vectors. Applied gates correspond to matrix multiplication. Therefore, we can simulate these actions on a classical computer. However, it is limited to small instances because simulations scale poorly with system size causing the matrices to grow exponentially.

While development of full scale quantum computing faces many challenges[11], one of the major ones is quantum errors. These errors can be caused by “cross talk” which means qubit errors are correlated. Quantum error correction[3][13][19] is an active field of research. Many different schemes[10][16][18] have been developed based on the assumption that there are no correlated qubit errors in quantum computers. However, if correlated qubit errors are proven to occur at a high rate, then these assumptions either will not be met, or the resource requirements increase dramatically[4][17]. Thus, a thorough understanding of correlated qubit errors is essential.

To improve understanding of correlated qubit errors, this paper provides two contributions. First, I present a method for testing for correlated qubit errors using independence of probabilities. Second, I present initial results of using this method on existing quantum hardware. Correlated qubit errors have had little research. This is most likely because quantum error correction researchers assume that these errors will be eliminated by researchers developing the quantum computers.

Research Question and Hypothesis

In the project, the question asked was whether qubit adjacency affected independence of error rates. Adjacency means two qubits are directly connected and can be entangled. To determine this, the following method was used:

Suppose $P(q_a)$ is the probability that qubit *a* has an error, $P(q_b)$ is the probability that qubit *b* has an error and $P(q_a q_b)$ is the probability that both qubits *a* and *b* have an error at the same time. Then, errors are independent if $P(q_a) \times P(q_b) \cong P(q_a q_b)$. If this is not true, then the errors are not independent, but correlated.

The hypothesis is that $P(q_a)$ and $P(q_b)$ are only correlated when two qubits are adjacent (connected). When qubits are non-adjacent, there will not be any correlated errors.

The null hypothesis is that qubit errors are independent for both adjacent and non-adjacent qubits. To test this, I need to determine the individual error rates for each qubit operating individually and the error rates for each pair of qubits operating simultaneously.

Methods

Experimental Setup

I used the IBM Quantum Experience[27] to perform all of my experiments. It provided me with three quantum computers (*ibmqx2, ibmq_valencia, and ibmq_santiago*) as well as a quantum computer simulator. I used IBM's Qiskit[29] Python[28] package to create and execute the circuits. The variables I used in my project were the three quantum computers, three different gates, and the number of gates. The quantum computers are open to the public on the Internet, although *ibmq_valencia* was retired by IBM shortly after the experiments were completed.

For each experiment I ran, I chose a quantum computer, a gate, and a number of gates. I tested each combination of the three computers and three gates. In order to test the cumulative error rate of the quantum computers, I started with 2 gates and doubled up to 256, measuring the error rates for each gate count.

Each quantum computer has a different connection topology (shown below in Figure 4), which is used to define adjacent and non-adjacent qubits. The circles with numbers represent individual qubits and the lines represent which qubits are connected (adjacent) which allows them to be entangled.

Method of Analysis - Testing Qubit Error Probabilities

Figure 4 shows the method that was used to calculate $P(q_a)$, $P(q_b)$ and $P(q_a q_b)$. Steps A, B and C were used to determine $P(q_a)$ and $P(q_b)$ for the individual qubits. Step A used a quantum computer simulator to determine what the correct answer for a circuit should be. Steps B and C executed the same circuit 1,000 times for qubits *a* and *b*, and compared each circuit execution to the expected answer obtained from the simulator to identify incorrect answers. This was used to compute $P(q_a)$ and $P(q_b)$.

Similarly, steps D and E were used to determine **$P(q_a q_b)$** by running a circuit that operated on both qubits *a* and *b.*

$P(q_a q_b)$ was compared to **$P(q_a) \times P(q_b)$** to see if they were approximately equal, indicating independence. For each combination of quantum computer, quantum gate and number of gates, as well for each possible pair of qubits *a* and *b*, this procedure was repeated, counting how many times the results were correct or incorrect between adjacent and non-adjacent qubits for each of the gate operators tested. These counts were used in a one-way chi-square test to compute p-values at 256 gates.

In addition, the overall values of **$P(q_a q_b)$** and **$P(q_a) \times P(q_b)$** were calculated for each gate operation and number of times the gate was applied. This data was used to create graphs of the results in which **$P(q_a)$**, **$P(q_b)$**, **$P(q_a q_b)$** and **$P(q_a) \times P(q_b)$** are plotted as a function of the number of gates applied.

In all, this phase was executed more than 15,000 times over the course of six months, resulting in over 40 million circuit executions on quantum hardware.

Results, Analysis, And Discussion

ID Gate Experiment Results

Because the Identity (ID) gate does not change the state of the qubit, and no actual operation is performed on the qubit(s), the ID gate acts as a delay operation. This allows us to use it as a control for comparison with actual gate operations. It also allows us to see whether there are any correlated environmental errors[5]. Figures 5a and 5b show that, while the rate of errors trends upwards over time, the measured 2-qubit error rate is almost identical to the expected 2-qubit error rate for both adjacent and non-adjacent. Thus environmental correlated errors are not prevalent. The p-value of 0.3950 provides strong confidence that my null hypothesis is true.

H Gate Experiment Results

In this experiment, Figures 6a and 6b show that the H gates show that there are correlated errors in the adjacent qubits. The non-adjacent qubits show significantly less error rates than the adjacent ones, and the correlation is statistically backed up with a p-value of 0.0160. In addition, comparison to the rate of increase for the ID gates indicates that the H gate operation is the source of the qubit errors.

X Gate Experiment Results

In the X gate experiment, Figures 10 and 11 show the graphs illustrate a clear distinction between adjacent and non-adjacent qubits. Both graphs' error rates continue to increase as more gates are applied, but adjacent qubits increase faster. The p-value of 0.0004 is a strong indication that adjacency caused the correlated errors, as shown in the graph on the right.

For the non-adjacent graph (Figure 7a), there is a small but noticeable divide between paired qubits and individual qubits. The reason for this is unknown, but physical qubit proximity might explain the divide. Instead of being directly connected to each other, they are possibly physically close to each other (non-adjacent but still close). This may cause some crosstalk that causes correlations. Combined with the rapid increase in error probability overall, this most likely is evidence that X gate creates more cross talk.

For the adjacent graph (Figure 7b), the error rate hits 0.25 at around 128 gates and levels off. This is because at this point, there are so many errors that the results are essentially random. Since two qubits go into the calculation, there are four possible measurements: 00, 01, 10, and 11. Each has a 0.25 (25%) chance of being the output result, so the two qubit error probability is capped at 0.25.

Related Work

There have been many published results from experiments regarding qubit errors. Michielsen et al[20] tested the correctness of various small circuits on existing quantum hardware (IBM). Their results showed that only extremely simple circuits were usable. The error rates were too high for more complex circuits. This matches the observations in my experiments in that as more gates are applied, the number of errors increases rapidly.

Burnett et al[21] studied the error rates of superconducting qubits with many different variables. Unlike many of the other studies, this was performed directly on qubits, and not through a quantum system. They reported varying results dependent on the variables.

Proctor et al[22] created a framework for testing how real world problems will perform on quantum computers. They performed testing on several algorithms, which, as is expected on current generation quantum computers, were not able to reliably provide correct answers.

Sarovar et al[23] defined a protocol for detecting cross-talk errors on quantum computers. They validated the protocol using a cross-talk simulator, but did not report results from testing on actual quantum computers.

Conclusion, Application, And Future Research

The lack of error correction is one of the obstacles keeping scientists from developing full-scale quantum computers. Correcting correlated errors requires large and complex error correction schemes that are difficult to implement and costly to build. In my experiment, I studied the prevalence of correlated errors in quantum computing on real IBM quantum computers to improve understanding of error correction. I hypothesized that qubits would exhibit correlated errors when adjacent but not when non-adjacent. The results show my hypothesis was correct that adjacency appeared to be the key variable that causes correlation in qubits. The only case where this was not true was when only environmental noise was acting on the qubits (ID Gates). To unlock the true power of quantum computing, we need to develop quantum computers that are free of correlated errors for adjacent qubits.

In the future, My methods can be used by scientists to test their machines for correlated errors. If we can mitigate these errors, we will have achieved another leap in our progress toward the full scale quantum computing revolution.

In the future, I plan on running my experiment on newer quantum computers, as well as quantum computers from different companies (like Google) to gather more inclusive data. In addition, I plan on investigating the difference between the correlated error rates of the X and H gates by testing other quantum gates.

I also would like to expand my research beyond adjacent and non-adjacent qubits by testing error rates between entangled and non-entangled qubits. This would allow me to see if entanglement increases the likelihood of correlated errors.

Summary

In summary, my research has shown that independence of probabilities can be used to successfully detect correlated qubit errors and that the existing small scale quantum computers have a high level of correlated qubit errors. These correlated qubit errors will need to be eliminated for quantum computing to succeed.

Declarations

Competing interests:

The authors declare no competing interests.

Sources

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Figures

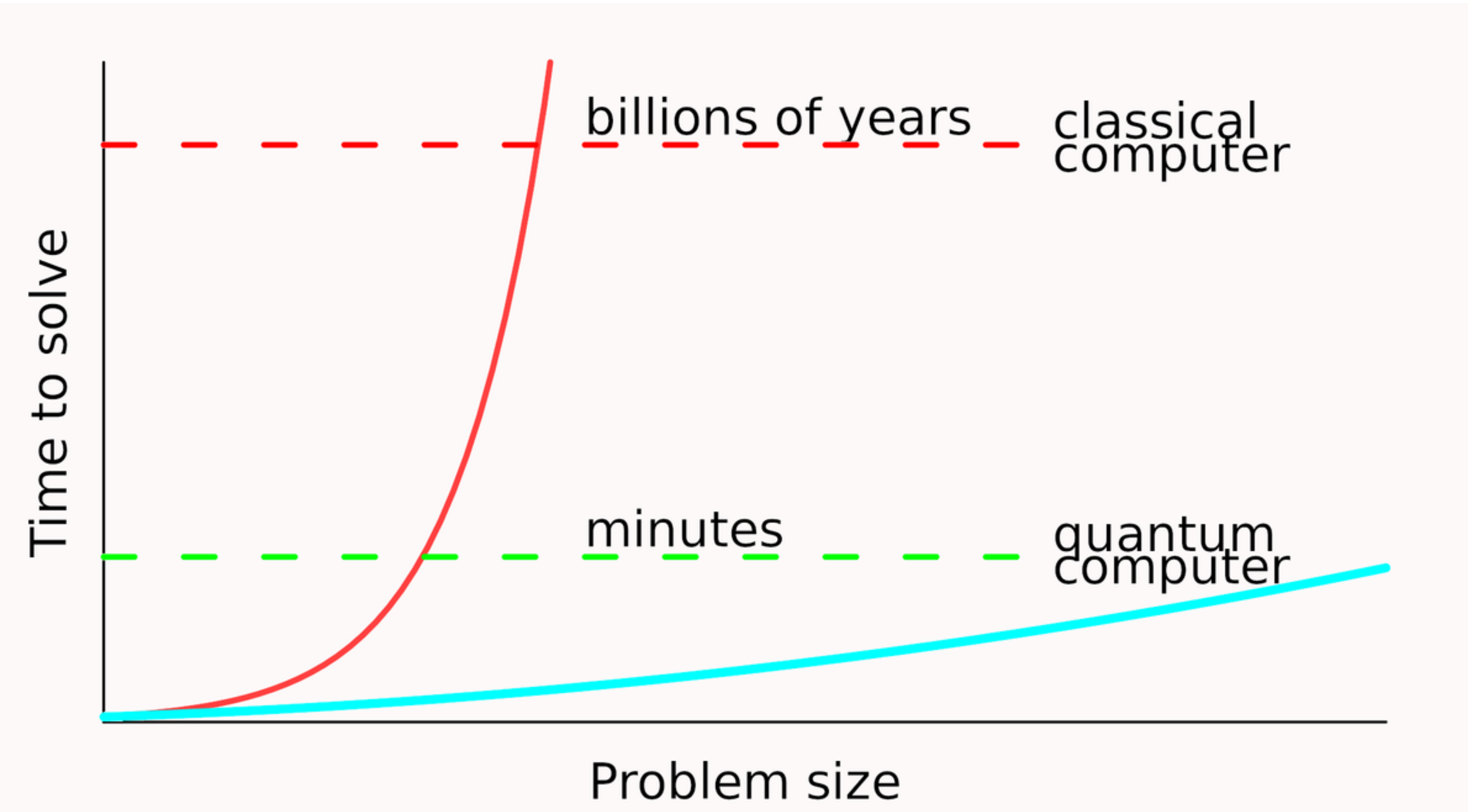

Figure 1

As problem size increases, the time it takes for classical computers to solve it grows exponentially while quantum computing time to solve grows linearly.

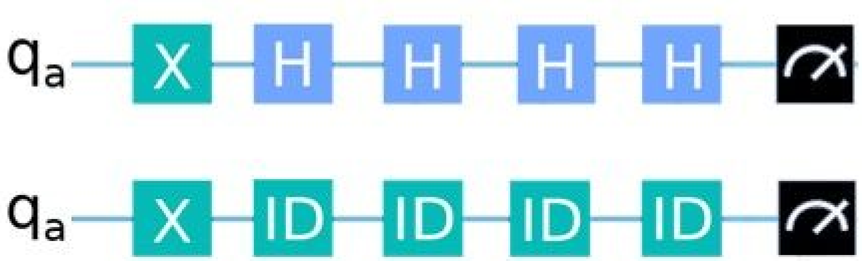

Figure 2

The quantum circuits pictured illustrate how gates are applied to qubits.

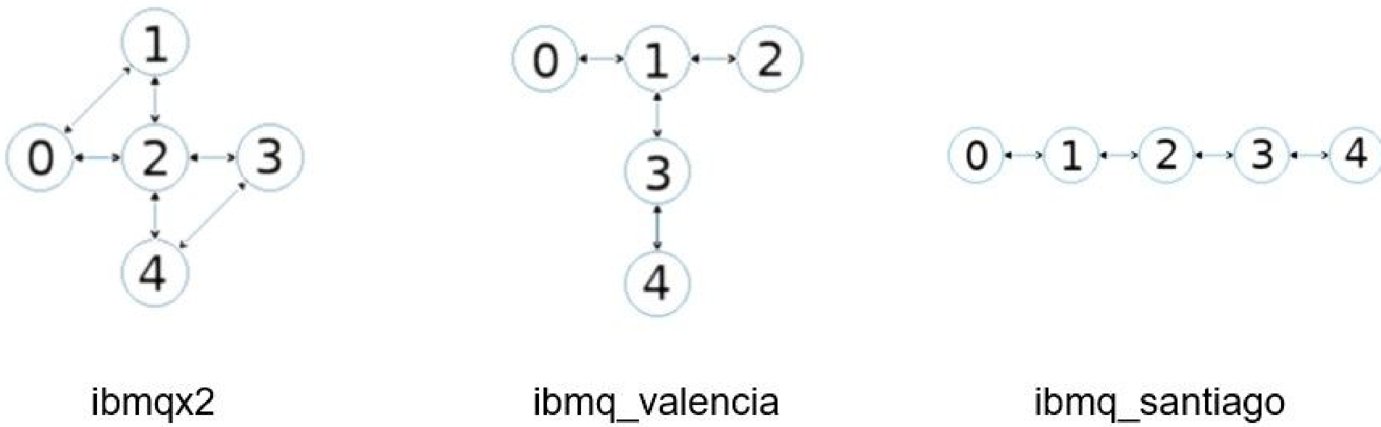

Figure 3

Connection topologies for ibmqx2, ibmq_valencia and ibmq_santiago.

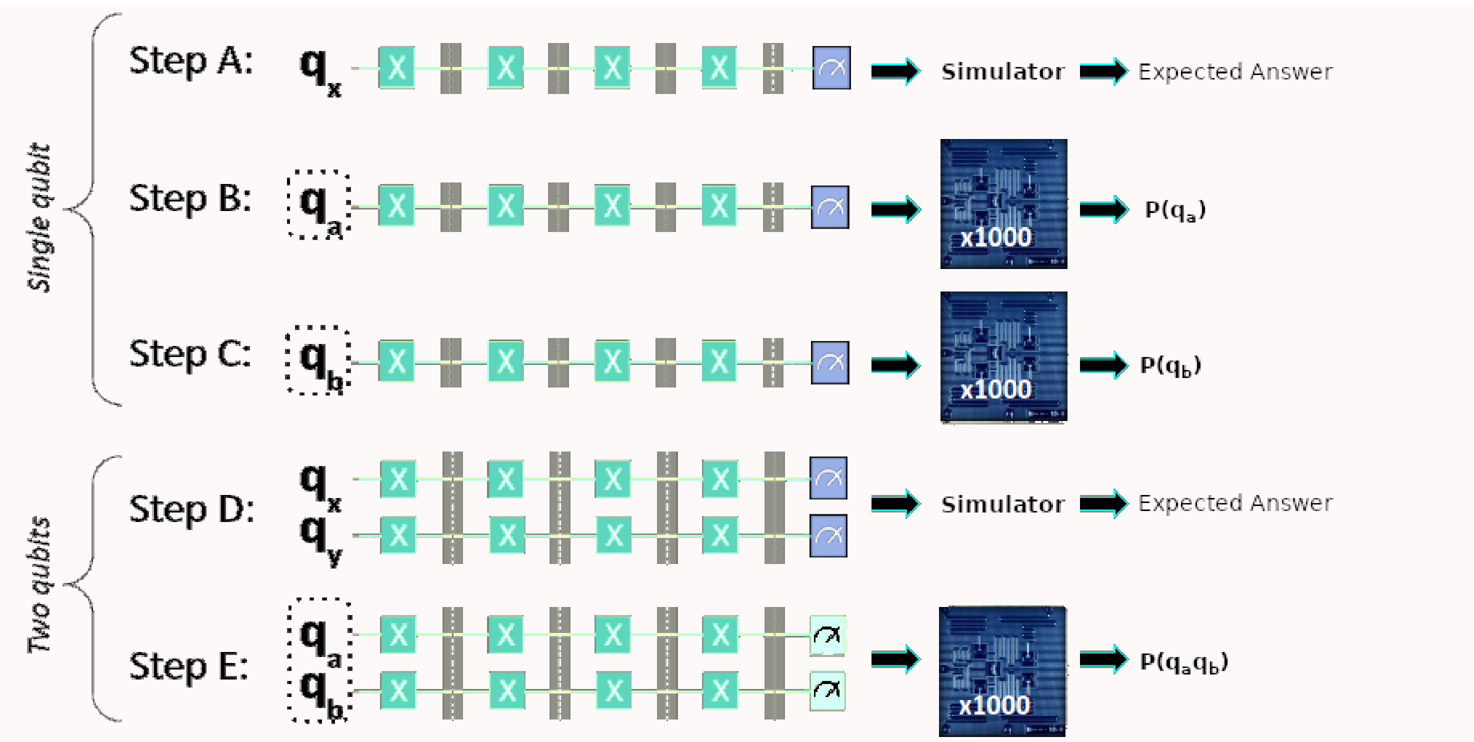

Figure 4

Procedure for computing P(qa), P(qb) and P(qaqb).

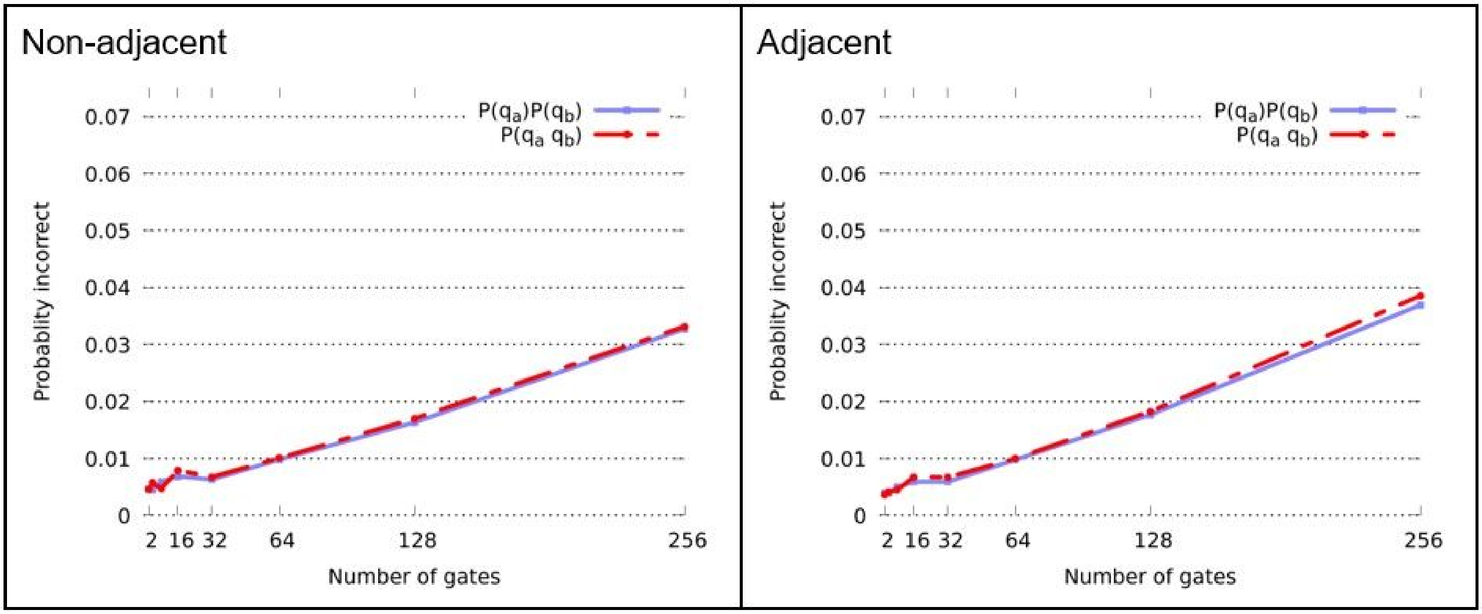

Figure 5

ID Gate experiment results differentiated between nonadjacent qubits to the left and adjacent qubits to the right.

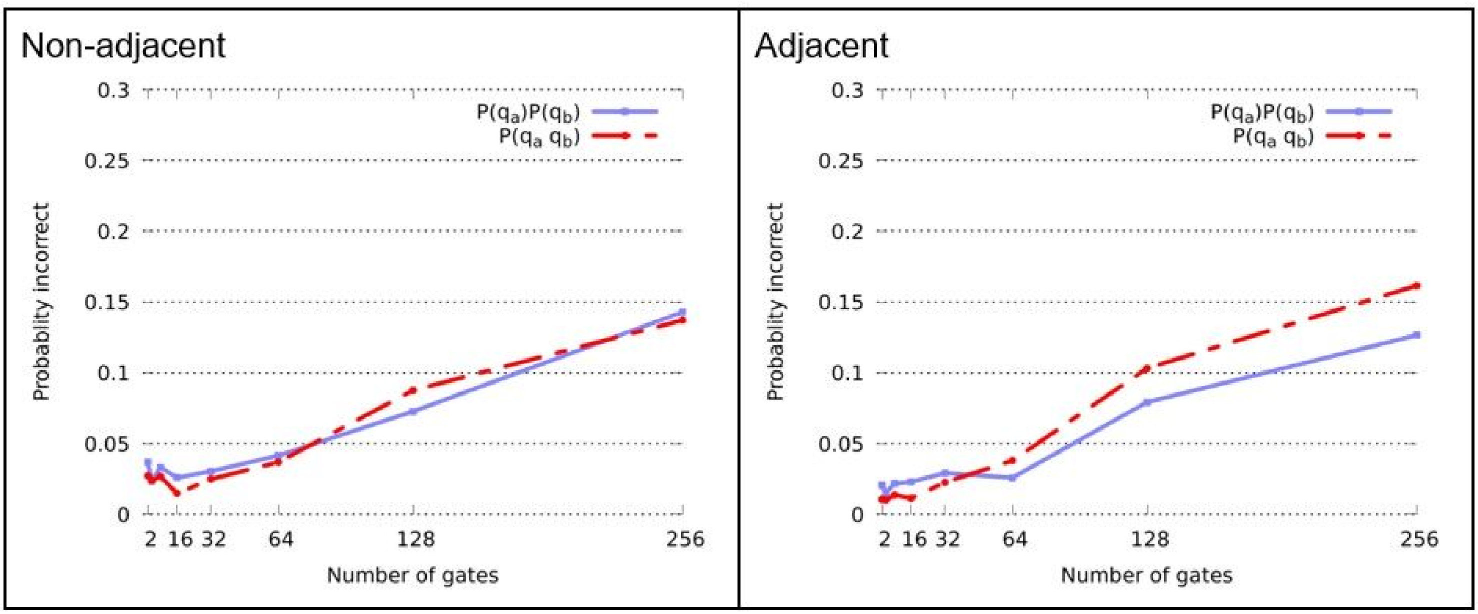

Figure 6

H Gate experiment results differentiated between nonadjacent qubits to the left and adjacent qubits to the right.

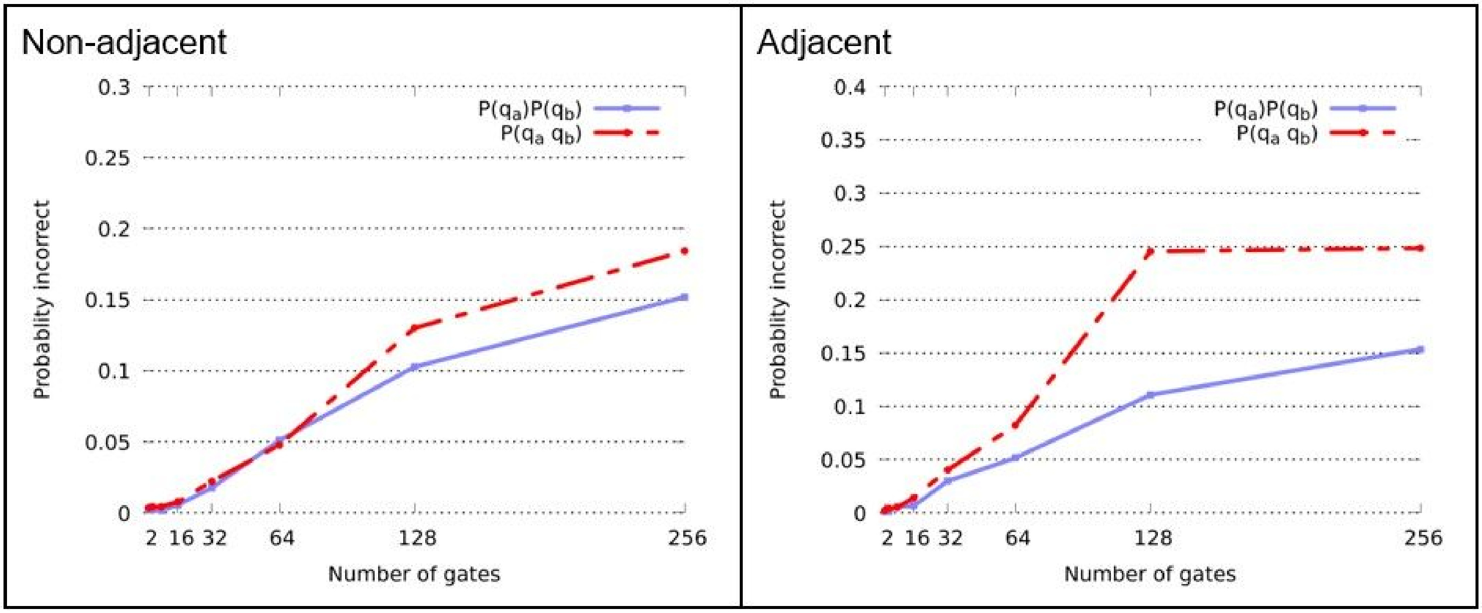

Figure 7

X Gate experiment results differentiated between nonadjacent qubits to the left and adjacent qubits to the right.