SUPPLEMENTARY INFORMATION

GEOMETRIC MEDIATOR STRUCTURES AND FORCE CONSTANTS

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For the presently described spherically symmetric Maxwellian case, ϕ, the electrostatic potential, is a function of *r* alone, and the Maxwellian electromagnetic tensor and the associated field tensor $\mathbf{F_{1\mu}}$ can be constructed according to equation (SI.1), where the only surviving field tensor components are (following the symbolism and development of Tolman [SI-1]):

$$ds^2 = g_{11}\left[dr^2 + r^2 d\Omega \right] + g_{44}dt^2 = - e^{\mu}\left[dr^2 + r^2 d\Omega \right] + e^{\nu}dt^2 ,$$

$$\mathbf{F_{21} = - F_{12}}\ , \mathbf{F_{13} = - F_{31}} \quad \text{and} \quad \mathbf{F_{14} = - F_{41}}\ , \quad \text{i.e.}$$

$$\mathbf{T^{\mu\nu} = - g^{\nu\beta}F^{\mu\alpha}F_{\beta\alpha} + \tfrac{1}{4}\, g^{\mu\nu}F^{\alpha\beta}F_{\alpha\beta}} \quad \text{or} \quad \mathbf{T^{\mu\mu} = - g^{\mu\mu}F^{\mu\alpha}F_{\mu\alpha} + \tfrac{1}{4}\, g^{\mu\mu}F^{\alpha\beta}F_{\alpha\beta}}\ , \qquad \text{(SI.1)}$$

then

$$\mathbf{T^4_4 = \frac{\left(F_{12}F^{12} + F_{13}F^{13} - F_{14}F^{14}\right)}{2}}, \quad \mathbf{T^1_1 = \frac{\left(- F_{12}F^{12} - F_{13}F^{13} - F_{14}F^{14}\right)}{2}},$$

$$\mathbf{T^2_2 = \frac{\left(- F_{12}F^{12} + F_{13}F^{13} + F_{14}F^{14}\right)}{2}} \quad \text{and} \quad \mathbf{T^3_3 = \frac{\left(F_{12}F^{12} - F_{13}F^{13} + F_{14}F^{14}\right)}{2}}\ .$$

The resultant field quantities are

$$\mathbf{(F_{14})^2 = - \left(T^4_4 + T^1_1\right)g_{11}\, g_{44} = \left(T^2_2 + T^3_3\right)g_{11}\, g_{44}}\ ,$$

$$\mathbf{(F_{12})^2 = - \left(T^2_2 + T^1_1\right)g_{11}\, g_{11}} \qquad \text{and} \qquad \mathbf{(F_{13})^2 = - \left(T^3_3 + T^1_1\right)g_{11}\, g_{11}}\ .$$

Therefore, we see that the static-spherically-symmetric Maxwellian tensors exhibit the same stress and energy relationship as the geometric tensors [SI-1],

$$\mathbf{T^4_4 = - \left(T^1_1 + T^2_2 + T^3_3\right)}\ . \qquad \text{(SI.2)}$$

The present geometric-modeling endeavor, with its Maxwellian-tensor-form mimicking-component, has produced the fundamental and limiting agent for the currently-studied distorted geometry, namely a particular constraining functional relationship between the geometry-defining tensors (for an empty-space geometry, all of the components of the energy-momentum tensor are zero). In using this simple equation-of-state, equation (SI.2), as a restricting distortional-model tensor relationship, we thereby elicit the metric-defining differential equations for such a family of geometric distortions.

The geometric-energy-density or field equations (SI.4-SI.7), after using solution Eq. (SI.3), are repeated here (from [SI_2]); also see [SI_1];

$$\mu' = \frac{2(1 - u^3)}{(Iu - \gamma)}\frac{u^2}{R0}, \qquad \text{(SI.3)}$$

$$Iu = -u\left[\frac{3}{7}u^6 - \frac{3}{4}u^3 + 1\right],$$

$$8\pi\kappa\, \mathbf{Td_1^1} = -e^{-\mu}\frac{1}{(Iu - \gamma)}\left(\frac{u^2}{R0}\right)^2\left[2u^2 + (3u^3 - 1)\frac{1 - u^3}{(Iu - \gamma)}\right],$$

$$8\pi\kappa\, \mathbf{Td_2^2} = e^{-\mu}\frac{1}{(Iu - \gamma)}\left(\frac{u^2}{R0}\right)^2\left[4u^2 + (3u^3 - 1)\frac{(1-u^3)^2}{(Iu - \gamma)}\right],$$

$$8\pi\kappa\, \mathbf{Td_4^4} = -8\pi\kappa\,(\mathbf{Td_1^1} + \mathbf{2\,Td_2^2}) \text{ since } \mathbf{Td_3^3} = \mathbf{Td_2^2}$$

or

$$\mathbf{Td_4^4} = e^{-\mu}\frac{1}{8\pi\kappa(Iu - \gamma)}\left(\frac{u^2}{R0}\right)^2\left[-6u^2 - (3u^3 - 1)\frac{(2u^3 - 1)(u^3 - 1)}{(Iu - \gamma)}\right]$$

and

$$8\pi\kappa\,(\mathbf{Td_2^2} + \mathbf{Td_1^1}) = e^{-\mu}\frac{1}{(Iu - \gamma)}\left(\frac{u^2}{R0}\right)^2\left[2u^2 - (3u^3 - 1)\frac{(1 - u^3)u^3}{(Iu - \gamma)}\right]$$

leading to

$$(\mathbf{Fd_{14}})^2 = -g_{11}g_{44}(\mathbf{Td_4^4} + \mathbf{Td_1^1}) = g_{11}g_{44}(2\,\mathbf{Td_2^2}) \quad \text{and} \qquad \text{(SI.4)}$$

$$(\mathbf{Fd_{14}})^2(r \to \infty) \stackrel{\text{def}}{=} \left(\frac{Rs}{2}\right)^2\frac{2}{8\pi\kappa}\frac{1}{r^4} = \frac{Rs^2}{2}\frac{1}{8\pi\kappa}\frac{1}{r^4} \stackrel{\text{def}}{=} \left(\frac{q}{4\pi\varepsilon o\, r^2}\right)^2\frac{\varepsilon o}{2}. \qquad \text{(SI.5)}$$

$$(\mathbf{Fd_{12}})^2 + (\mathbf{Fd_{13}})^2 = \mathbf{2}\, g_{11}g_{11}\left(\frac{\mathbf{Td_4^4 - Td_1^1}}{\mathbf{2}}\right) \stackrel{\text{def}}{=} \mathbf{Fd_{mag}}^2 = \qquad \text{(SI.6)}$$

$$= -2\, g_{11}g_{11}(\mathbf{Td_1^1} + \mathbf{Td_2^2}) \quad \text{and}$$

$$(\mathbf{Fd_{12}})^2 + (\mathbf{Fd_{13}})^2(\mathbf{r} \to \infty) = 2\,Rs\,R0^3\frac{1}{8\pi\kappa}\frac{1}{r^6} \stackrel{\text{def}}{=} \qquad \text{(SI.7)}$$

$$\stackrel{\text{def}}{=} \frac{\mu o}{2}\left(\frac{\mu_{spin}}{2\pi}\right)^2\frac{1}{r^6}$$

where

$$\mu\_spin \stackrel{\text{def}}{=} \left(\frac{g_e}{2}\frac{Qe}{3M}\right) S\,\hbar \quad \text{and} \quad g_e = 2.0023193043\,6 \text{ (for the electron)}.$$

The field equations, in both the EM realm and the gravitational realm (Q = 0), exhibit r^{-6} geometric behavior which we have interpreted as constituting a “magnetic monopole” mimic (what is a “magnetic monopole” ?).

References

[SI-1]. Tolman, R. *Relativity, Thermodynamics and Cosmology*. Dover, NY, 248 (1987).

[SI-2]. Koehler, D. Geometric-Distortions and Physical Structure Modeling. *Indian J. Phys*. **87**, 1029 (2013).

SUPPLEMENTARY VIDEO

As a supplementary visualizing addition to the geometric modeling we include as a Supplementary Video an animated muon video file, or geo-muon decay.avi (simulating the muon to electron beta-decay, a higher-energy nuclear process), produced as a spherically symmetric representation with the following details:

frames 0-15; geometric-distortion mass-energy-density function for muon,

@ frame 15, "muon" transitions (morphs) to "W^-boson", and

@ frame 30-45, "W^-boson" transitions (morphs) to "electron + neutrinos";

neutrinos not displayed.

The beta-decay animation is constructed with linear radii but with logarithmic amplitudes and logarithmic normalizing radii R0 and is further normalized to a "neutrino" amplitude and an "electron" radius.